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Some Problems for Gauss Workshop.

1. Find the area of quadrilateral $ABCD$, where $AB = 12$,$BC = 14$, $CD = 13$, $DA = 9$, and the angles at A and B are right angles.

Height times average of top and bottom equals $12 \times 11.5 = 138$. *It would have been more interesting without the right angles being given, or just three sides being known.*

3. A litre of orange drink contains 10% orange juice. How much OJ must be added to have a mixture with 40% juice?

Let x be the amount of added OJ ... (yawn). What if you are unwilling (or unable) to use that approach? Your drink contains 100 ml of OJ — which would yield a 40% *mixture when combined with just 150 ml of the water. If you (mentally) extract 750 ml of pure water from your drink, how much OJ must you stir in to create a* $60 : 40$ *mixture?*

3. A dart board has three concentric circles. The inner circle is worth 5 points, the middle ring 3, the outer ring 2. Find the smallest number of darts that will earn exactly 21 points.

Since $5x + 3y + 2z =$

4. When filled, a ferry can carry: 500 cars and 400 bikes; or 600 cars; or 400 bikes and 3000 people. How many people can it carry without anything else?

3000 people and 400 bikes make a full load and so do 500 cars and 400 bikes. Hence 3000 people are equivalent to 500 cars. A full load consists of 600 cars — 1/5 more than 500 — or 3600 people. Just for fun: 1 car is equivalent to 6 people or 4 bikes.

5. After 30 games as goalie, Cy has a GAA (# goals/# games) of 3.00. For the rest of the season, he has 5 shut-outs (games without goals) and finishes with a GAA of 2.00. What is the minimum number of games to make this happen?

If L and M stand for the numbers of goaLs and gaMes, the GAA equals L/M, whence $L = 2M$ at end of season. During the first 30 games the GAA of 3 means a total of 90 goals. Including the 5 shut-outs, we have 90 goals in 35 games. In the remaining $M-35$ games (no more shut-outs), Cy allowed at least one goal per game, for a minimum $L = 90 + M - 35$. Since $L = 2M$, this makes $M = 55$.

8. Lee bought 38 CD's, each costing the same price over \$15. The total bill was \$$y29.2z$ with first and last digits illegible. What were the possible prices x for each CD?

In pennies: $38x = 10^4y + 2920 + z$, *and* $x \geq 1500 \Rightarrow y = 6, 7, 8$ *or* 9. *Clearly z is even, hence* ≤ 8. *Since* $10^4 = 38 \times 263 + 6$ *and* $2920 = 38 \times 77 - 6$, *the initial equation says that* 38 *divides* $6(y - 1) + z$. *No go for* $y = 8, 9$. *Possibilities: $y = 6$, $z = 8$, whence $x = 1656$; or $y = 7$, $z = 2$, whence $x = 1919$.*

13. B cycles to school along rail track at 6 km/h, and usually arrives at Xing simultaneously with train. Today she is 50 minutes late and meets train 6 km before Xing. How fast is the train?

She meets train 1 h before she gets to Xing (50 min late) hence 10 minutes before the usual time. In those 10 minutes, the train travels the 6 km to the Xing — at 36 km/h.

14. Alex rows downstream from A to B in three hours, upstream from B to A in four. How long does a log take to drift from A to B?

Say, the distance between A and B is 12 *"laps" — whatever that may be in traditional units. In laps per hour, Alex's speed is 4 downstream and 3 upstream. Hence river runs at 1/2 lap per hour — its speed being added and subtracted, respectively. Thus it takes 24 hours to do the 12 laps.*

15. Find the chance of the total point-sum in a roll of 5 dice is even.

For n dice, the answer (50%) is easily proved by induction. Beware of right answers based on faulty reasoning! Be ready with counter-examples.