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Tactical Versus Strategic Mathematical Problems.

In spite (or perhaps because) of my age and university position, I must admit that I am regularly intimidated by "elementary" problems of the kind that occur in the IMO and Putnam competitions – to mention only two examples. Sometimes I can solve them in the alotted time (plus-minus x%), but more often I cannot – and that fills me with terror and shame. As a defence, I have developed a way of distinguishing between "strategic" problems (good) and "tactical" ones (not so good). I am happy to say that most of the problems published by *Vector* fall into the first category – and will be glad to demonstrate this with reference to some recent examples. But first, please let me metaphorically explain my distinction between tactical and strategic.

If I were a mountain climber, I could imagine tactical poblems of marshalling pick axe and rope to scale walls or chimneys within the confines of an adult adventure playground. If I were a pianist, I would encounter poblems of fingering and phrasing within a Czerny étude. If I were a hockey player, I would have to solve problems of slap-shots and body-checking. In all these cases, I would still face the strategic problem of grasping a sense of the overall game and what it can mean to me and others. I might miss out on many of its joys.

As a mathematician, I might get wrapped up in the hunt for "perfect" numbers, of which only three dozen are known to date. Each of them is a power of two multiplied by a Mersenne prime, whence the search for the latter has become the main object of the hunt. Because of its very limitation – coupled with its transparency – this is a wonderful topic for initiation into number theory. However, it could also convey too a narrow view of a subject which is as vast as anything (known to humanity) under the sun.

Admittedly the distinction between "strategic" and "tactical" depends on what vistas are available at your particular place and time. The great Gauss, for instance, thought that the equation of Fermat's Last Theorem was just one of many he could gratuitously scribble down to stump his contemporaries. In hindsight, he was wrong objectively – but pedagogically he was right: until the later break-throughs of Kummer (1837) and Wiles (1994), the problem was a marginal oddity. Until a larger vista opens up, it seems unfair to coop up students in a tactical corral: they need and want to know what the world has is in store for them. In mathematics, that is a huge panorama.