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On the Groenewold-Van Hove problem for \mathbf{R}^{2n} . (English. English summary)

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The classical results of Groenewold and Van Hove state, roughly speaking, that one cannot quantize every classical observable on the phase space \mathbf{R}^{2n} in a way consistent with Schrodinger quantization.

In the paper under review the author shows rigorously that there exists an obstruction to quantizing the Poisson algebra of polynomials on \mathbf{R}^{2n} in such a way that the Heisenberg algebra generated by canonical coordinates is represented irreducibly, thereby filling a gap in Groenewold's original proof. Moreover, for $n = 1$, the maximal quantizable Lie subalgebras of polynomials are determined and their possible quantizations are explicitly constructed. These are the extended symplectic algebra $\mathfrak{hsp}(2, \mathbf{R})$ with the extended metaplectic representation, and the coordinate (or position) algebra $C(2)$ with the representations

$$\sigma_a(f(q)p + g(q)) = -i\hbar \left(f(q) \frac{\partial}{\partial q} + \left(\frac{1}{2} + ia \right) \frac{\partial f}{\partial q}(q) \right) + g(q),$$

where $a \in \mathbf{R}$.

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