MOMENTUM MAPS & CLASSICAL FIELDS

4. Gauge Symmetries and IV Constraints

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Overview:

Here I discuss:

- gauge groups

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- the Vanishing Theorem

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- the Vanishing Theorem
- the relationship between the E-M map and (first class) secondary constraints

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- Assume that $\mathcal{G}$ is a covariance group, i.e., $\mathcal{L}$ is $\mathcal{G}$-covariant.

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Localizability

$\mathcal{G}$ localizable provided that for each pair of disjoint hypersurfaces Σ_1 and Σ_2 in X and each $\xi \in \mathfrak{g}$, there is a Lie algebra element $\chi \in \mathfrak{g}$ such that

$$\chi_Y|\pi_{XY}^{-1}(\Sigma_1) = \xi_Y|\pi_{XY}^{-1}(\Sigma_1) \text{ and } \chi_Y|\pi_{XY}^{-1}(\Sigma_2) = 0$$

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- The Poincaré group is not.

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- Later we'll require that the gauge group of a CFT be maximal.
- Some CFTs have no gauge symmetries: Proca on a frozen background. BUT: this is not generally covariant.
- Any generally covariant ("parametrized") CFT has nontrivial gauge symmetry, since for such theories the projection of $\mathcal{G} \subset \mathrm{Aut}(Y)$ in $\mathrm{Diff}(X)$ in "large".

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- The instantaneous gauge algebra is $\ker \varpi_\tau$
- Of course, $\mathcal{G}$ usually does not act in the instantaneous formalism.
- Nonetheless, using the instantaneous energy-momentum map $\mathcal{E}_\tau : \mathcal{P}_\tau \to \mathfrak{g}^*$, $\mathfrak{g}$ naturally defines a Lie subalgebra $\mathfrak{g}_{\mathcal{C}_\tau} \subset \mathfrak{X}(\mathcal{C}_\tau)$ which serves as an (infinitesimal) action. (Elements of $\mathfrak{g}_{\mathcal{C}_\tau}$ are the Hamiltonian VFs of the $\mathcal{E}_\tau(\xi)$.)

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- Recall that $\mathcal{G}$ is assumed known at the outset.

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- For strings, both $\mathrm{Diff}(X) \ltimes C^\infty(X, \mathbb{R}^+)$ and its subgroup $\mathrm{Diff}(X)$ are full.
- A sufficient condition for fullness will come later.

The Vanishing Theorem

Here we begin the program of relating initial value constraints to zero levels of the energy-momentum map by proving the Vanishing Theorem

- Let Σ_+ and Σ_- be two hypersurfaces in X which form the boundary of a compact region; we call Σ_+ and Σ_- an admissible pair of hypersurfaces.

  (We have in mind the case of a spacetime X with Σ_+ and Σ_- deformations of a given hypersurface to the future and past, respectively)

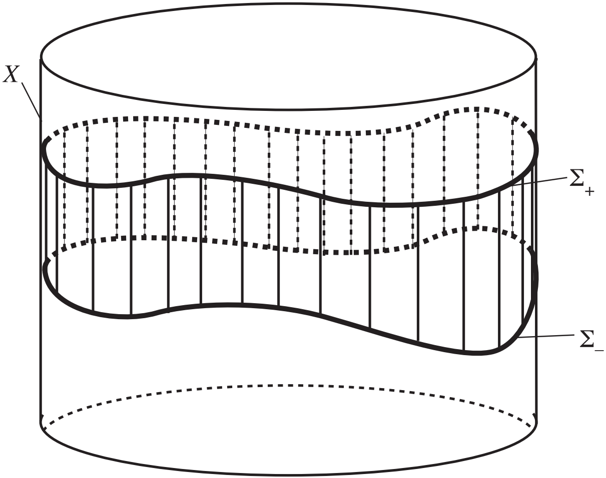

Figure: An admissible pair of hypersurfaces

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For every compact oriented hypersurface Σ there is a *disjoint* hypersurface Σ' such that (Σ, Σ') form an admissible pair.

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Now suppose that we have a field theory with gauge group $\mathcal{G}$ in which all fields are variational. Then from the first Noether theorem and Stokes' theorem,

$$\int_{\Sigma_+} \tau_+^*(j^1\phi)^* J^{\mathcal{L}}(\xi) = \int_{\Sigma_-} \tau_-^*(j^1\phi)^* J^{\mathcal{L}}(\xi).$$

for every solution ϕ of the Euler–Lagrange equations and admissible pair of hypersurfaces Σ_+ and Σ_-, where $\tau_\pm : \Sigma_\pm \to X$ are the inclusions.

By localizability, this integral vanishes.

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Vanishing Thm

Let $\mathcal{L}$ be the Lagrangian density for a field theory with gauge group $\mathcal{G}$. Then for any solution ϕ of the Euler–Lagrange equations and hypersurface Σ, the energy-momentum map on Σ in the Lagrangian representation vanishes:

$$\int_{\Sigma} \tau^*(j^1\phi)^* J^{\mathcal{L}}(\xi) = 0$$

for all $\xi \in \mathfrak{g}$, where $\tau : \Sigma \to X$ is the inclusion.

Bosonic String

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Using the expression already given for the Lagrangian multimomentum map given previously, the VT forces:

- $g_{AB}(h^{00}\phi^{A}{}_{,0}\phi^{B}{}_{,0} - h^{11}\phi^{A}{}_{,1}\phi^{B}{}_{,1}) = 0$

  —superHamiltonian constraint in Lagrangian disguise

- $g_{AB}h^{0\mu}\phi^{B}{}_{,\mu}\phi^{A}{}_{,1} = 0$

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Thm

Suppose that the Euler–Lagrange equations are well-posed. Then

$$\mathcal{C}_\tau \subset \mathcal{E}_\tau^{-1}(0)$$

Proof

Let $(\varphi, \pi) \in \mathcal{C}_\tau$. Then there is a solution $\phi \in \mathcal{Y}$ of the Euler–Lagrange equations with initial data (φ, π). Set $\sigma = \mathbb{F}\mathcal{L} \circ j\phi \circ \tau$ so that σ is a holonomic lift of (φ, π). Now apply the Vanishing Theorem to ϕ, obtaining for each $\xi \in \mathfrak{g}$,

$$0 = \int_\Sigma \tau^*(j\phi)^* J^{\mathcal{L}}(\xi) = \int_\Sigma \tau^*(j\phi)^* \mathbb{F}\mathcal{L}^* J(\xi)$$

$$= \int_\Sigma \sigma^* J(\xi) = \langle E_\tau(\sigma), \xi \rangle$$

$$= \langle \mathcal{E}_\tau(\varphi, \pi), \xi \rangle$$

as σ is a holonomic lift of (φ, π). Thus $(\varphi, \pi) \in \mathcal{E}_\tau^{-1}(0)$. ■

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- The well-posedness hypothesis can be substantially weakened: need only require $\mathrm{Sol} \neq \varnothing$
- Recall the standing assumption that all secondary constraints are first class

Energy-Momentum Theorem, Version I

If $\mathcal{G}$ is full and $\mathcal{E}_\tau^{-1}(0)$ is connected, then

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Energy-Momentum Theorem, Version II

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Thm

$$\mathcal{G} \text{ full } + \mathcal{C}_\tau \text{ coisotropic } \Leftrightarrow \mathcal{C}_\tau = \mathcal{E}_\tau^{-1}(0) \Leftrightarrow \mathcal{E}_\tau^{-1}(0) \text{ coisotopic.}$$

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- One has similar results for primary constraints vis-à-vis a certain momentum map

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Dynamic and Atlas Fields

The arena for dynamics is the primary constraint set $\mathcal{P}_\tau \subset T^*\mathcal{Y}_\tau$. Our first task is to invariantly split the fields and their momenta on $(\mathcal{P}_\tau, \omega_\tau)$ into the dynamic fields ψ and their conjugate momenta ρ and the nondynamic atlas fields α.

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- BUT the Thm does not hold if one replaces “atlas fields” by “kinematic fields” (subtle)

Combining these results with the Energy-Momentum theorem, we obtain item 4 of the introduction

Thm

$$H_{\tau,\zeta} = \int_{\Sigma_\tau} \alpha_i \Phi^i(\Psi, \rho)\, d^n x_0$$

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- So Hamilton's equations $X_H(\psi, \rho) \lrcorner\, \omega = dH(\psi, \rho)$ are

$$\begin{pmatrix} \dot{\psi} \\ \dot{\rho} \end{pmatrix} = \Pi(\psi, \rho) \cdot dh(\psi, \rho) = \Pi(\psi, \rho) \cdot \langle \alpha, D\Phi(\psi, \rho) \rangle.$$

- Introduce an almost complex structure $\mathbb{J}(\psi, \rho) : T_{(\psi,\rho)}\mathcal{D} \to T_{(\psi,\rho)}\mathcal{D}$ by

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- Finally, define the adjoint of $D\Phi(\psi, \rho)$ according to

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