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A multisymplectic framework for classical field theory and the calculus of variations. I. Covariant Hamiltonian formalism.


Let $Y$ be a fibred manifold over an $(n+1)$-dimensional base manifold $X$, and let $\Theta$ be the canonical $(n+1)$-form on $\Lambda^n T^* (J^{k-1} Y)$. Also, let us consider the subbundle $Z^{k-1}$ of the points $z \in \Lambda^{n+1} T^* (J^{k-1} Y)$ such that $i_\xi i_\eta z = 0$ for all tangent vectors $\xi, \eta$ in $T_{\varphi(x)} (J^{k-1} Y)$ vertical over $X$, where $\varphi$ stands for a local section of $J^{k-1} Y \to X$. The author proposes the pair $(Z^{k-1}, -d\Theta)$ as a covariant Hamiltonian counterpart of the Lagrangian system $(J^{2k-1} Y, -d\Theta_L)$, where $L$ is a $k$th order Lagrangian density on $Y$ and $\Theta_L$ stands for a Lepagean equivalent of $L$ in the sense of Krupka. To justify this, he explains that $J^{k-1} Y$-bundle maps $\sigma_L : J^{2k-1} Y \to Z^{k-1}$ exist (Legendre transformations) such that $\Theta_L = \sigma_L^* \Theta$. Let $J^1 (J^{k-1} Y)$ be the bundle over $J^{k-1} Y$ whose fibre over $\varphi(x)$ consists of the affine maps $J^1_{\varphi(x)} (J^{k-1} Y) \to \Lambda^{n+1} T^* (X)$. It is proved as the main result that $Z^{k-1}$ and $J^1 (J^{k-1} Y)$ are canonically isomorphic. Section 5 is devoted to reviewing the notion of regularity in higher-order variational calculus. Also, an interesting section of prospects is included. It should be noted that the Poincaré-Cartan form associated to a linear connection was introduced for the first time in a paper by P. L. Garcia and the reviewer [Atti Accad. Sci. Torino Cl. Sci. Fis. Mat. Natur. 117 (1983), suppl. 1, 127–147; MR 86i:58042]. The references on this matter quoted in the paper under review are from a later date.

{For the entire collection see MR 91k:58003}.

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