# Combining individually inconsistent prioritized knowledge bases

**Guilin Qi, Weiru Liu**
School of Computer Science
Queen's University of Belfast
Belfast, Co.Antrim, BT7 1NN, UK
{G.Qi,W.Liu}@qub.ac.uk

**David H. Glass**
School of Computing and Mathematics
University of Ulster
Newtownabbey, Co.Antrim, BT37 0QB, UK
DH.Glass@ulster.ac.uk

## Abstract

It is well accepted that inconsistency may exist in a database system or an intelligent information system (Benferhat et al. 1993a; 1993b; 1997b; 1998; Benferhat & Kaci 2003; Elvang-Gøransson & Hunter 1995; Gabbay & Hunter 1991; Lin 1994; Priest et al. 1989; Priest 2001). Inconsistency can either appear in the given knowledge bases or as a result of combination or revision. In this paper, we will propose two different methods to combine individually inconsistent possibilistic knowledge bases. The first method, called an argument-based method, is a generalization of the merging method introduced in (Benferhat & Kaci 2003). When the knowledge bases to be merged are self-consistent, this method coincides with the original one. The second method, called a multiple-operator based method, combines the consistent and the conflict information using different operators. This method is more reasonable than the argument-based method because it differentiates the consistent and conflict information.

## Introduction

In some cases, we may confront the problem of merging individually inconsistent knowledge bases. It is well accepted that an agent may have inconsistent beliefs (Benferhat et al. 1993a; 1993b; 1997b; 1998; Benferhat & Kaci 2003; Elvang-Gøransson & Hunter 1995; Gabbay & Hunter 1991; Lin 1994; Priest et al. 1989; Priest 2001). Inconsistency can appear in a given knowledge base. Many researchers, especially those working on paraconsistent logic and argumentation, argued that inconsistency was not a bad thing and proposed some methods to deal with or handle the inconsistency (Elvang-Gøransson & Hunter 1995; Gabbay & Hunter 1991; Lin 1994; Priest et al. 1989). Even if the original knowledge bases $K_i$ are individually consistent, the result of combination by conjoining them (Benferhat et al. 1995) or merging them using the method introduced in (Benferhat & Kaci 2003) may be inconsistent.

As far as we know, there does not exist a merging method that handles individually inconsistent knowledge bases explicitly. It is always assumed (this assumption may be implicit) that the original knowledge bases are self-consistent when we confront the problem of merging (Baral et al. 1992; Benferhat et al. 1997a; 2001; Benferhat & Kaci 2003; Liberatore & Schaerf 1998; Lin & Mendelzon 1998;

Konieczny & Pino Pérez 1998). Since the original knowledge bases are individually inconsistent and may preserve some useful information about the real world, the result of combination is not required to be a consistent knowledge base. In the classical logic framework, given several individually inconsistent knowledge bases, we may conjoin the original knowledge bases, i.e., take the union of the original knowledge bases as the result of merging. But when the original knowledge bases are prioritized, i.e., formulas are ordered according to their priorities, it is not advisable to conjoin them. For example, suppose there are two prioritized knowledge bases where one of them is inconsistent $B_1 = \{(\neg\phi, 0.7), (\phi, 0.6), (\gamma, 0.8)\}$ and $B_2 = \{(\phi, 0.6), (\gamma, 0.8)\}$, where $\phi$, $\neg\phi$ and $\gamma$ are classical propositions and the weights assigned to the formulas denote certainty degrees of the formulas. Then by conjoining them we obtain a knowledge base $B = \{(\neg\phi, 0.7), (\phi, 0.6), (\gamma, 0.8)\}$. Clearly, information provided by $B_2$ is ignored. Since $\gamma$ is strongly supported by both sources, its certainty degree should increase, i.e., there is a reinforcement between $B_1$ and $B_2$ for $\gamma$. For formulas $\phi$ and $\neg\phi$, they are involved in the inconsistency of $B_1 \cup B_2$, so their necessity degrees in general should not increase.

The importance of priorities in belief revision and information fusion has been addressed by many researchers in recent years (Benferhat et al. 1998; Gärdenfors 1988; Lin & Mendelzon 1998). Possibilistic logic (Dubois et al. 1994) provides a good framework to express priorities. In possibilistic logic, each classical first order formula is attached with a number or weight, denoting the necessity degree of the formula. The necessity degrees can be interpreted as the priorities of formulas. A possibilistic knowledge base is a set of possibilistic formulas. Possibilistic logic also provides a good framework to deal with inconsistency. From a (partial) inconsistent possibilistic knowledge base we can infer some nontrivial consequences using the possibilistic consequence relation (also called $\pi$-consequence relation).

In (Benferhat & Kaci 2003), a method to merge possibilistic knowledge bases was introduced. The result of merging may be an inconsistent possibilistic knowledge base, although the original possibilistic knowledge bases were assumed to be individually consistent. This method could also be used to merge individually inconsistent knowledge bases. However, the merging method was constrained by the

$\pi$-consequence relation, which had been criticized for the "drowning problem". Namely, the $\pi$-consequence relation only uses those formulas whose necessities are greater than the *inconsistency degree*, so some useful information may be lost. In this paper, we extend the $\pi$-consequence relation based merging method with an argument-based method. When the original knowledge bases are individually consistent, our revised merging method is reduced to the original method (Benferhat & Kaci 2003). However, if the original knowledge bases are individually inconsistent, it contains more useful information than the original one. A deficiency of the revised method is that it cannot differentiate between consistent and conflict information. To overcome this, we also propose a multiple-operator based method, which combines consistent and conflict information using different operators.

This paper is organized as follows. Section 2 introduces some basic definitions in possibilistic logic. In Section 3, we review a merging method introduced in (Benferhat & Kaci 2003) and compare it with related methods. In Section 4, we present two methods to combine possibilistic knowledge bases that may be individually inconsistent. Finally, we give the conclusion in Section 5.

## Some Basic Definitions in Possibilistic Logic

In this section, we introduce some basic definitions in possibilistic logic (Dubois et al. 1994). We only consider a finite propositional language denoted by $\mathcal{L}$. The classical consequence relation is denoted as $\models$. $\phi, \psi, \gamma,...$ represent classical formulas.

In possibilistic logic, at the semantic level, the basic notion is a *possibility distribution*, denoted by $\pi$, which is a mapping from a set of interpretations $\Omega$ to the interval [0,1]. $\pi(\omega)$ represents the possibility degree of the interpretation $\omega$ with the available beliefs. From a *possibility distribution* $\pi$, two measures defined on a set of propositional or first order formulas can be determined. One is the possibility degree of formula $\phi$, denoted as $\Pi(\phi) = max\{\pi(\omega) : \omega \models \phi\}$. The other is the necessity degree of formula $\phi$, and is defined as $N(\phi) = 1 - \Pi(\neg\phi)$.

At the syntactic level, a formula, called a *possibilistic formula*, is represented by a pair $(\phi, \alpha)$ where $\phi$ is a classical first-order, closed formula and $\alpha \in [0,1]$. Uncertain pieces of information can then be represented by a *possibilistic knowledge base* which is a finite set of *possibilistic formulas* of the form $B = \{(\phi_i, \alpha_i) : i = 1,...,n\}$. A *possibilistic formula* $(\phi_i, \alpha_i)$ means that the necessity degree of $\phi_i$ is at least equal to $\alpha_i$, i.e. $N(\phi_i) \geq \alpha_i$. The classical base associated with $\mathcal{B}$ is denoted as $\mathcal{B}^*$, namely $\mathcal{B}^* = \{\phi_i|(\phi_i, \alpha_i) \in \mathcal{B}\}$. The formulas in $B$ can be rearranged by setting their weights such that $a_1 = 1 > a_2 > ... > a_n > 0$. Then $B$ can be equivalently expressed as a layered belief base $\Sigma = S_1 \cup ... \cup S_n$, where $S_i = \{\phi : (\phi, a_i) \in B\}$, that is, each $S_i$ is associated with a weight $a_i$. $\Sigma$ is called the stratification of $B$.

**Definition 1 (Dubois et al. 1994)** *Let $B$ be a possibilistic base, and $\alpha \in [0, 1]$. We call the $\alpha$-cut (respectively strict $\alpha$-cut) of $B$, denoted by $B_{\geq\alpha}$ (respectively $B_{>\alpha}$), the set of classical formulas in $B$ having a necessity degree at least equal to $\alpha$ (respectively strictly greater than $\alpha$).*

The *inconsistency degree* of $B$, which defines the level of inconsistency of $B$, is defined as (Dubois et al. 1994):

$$Inc(B) = max\{\alpha_i | B_{\geq\alpha_i} \text{ is inconsistent}\}.$$

**Definition 2 (Dubois et al. 1994)** *Let $B$ and $B'$ be two possibilistic knowledge bases. $B$ and $B'$ are said to be equivalent, denoted by $B \equiv_s B'$, iff*

$$\forall a \in [0,1], \ B_{\geq a} \equiv B'_{\geq a}.$$

In (Benferhat et al. 1993b), some consequence relations in possibilistic logic are defined to deal with inconsistency.

**Definition 3** *Let $\Sigma = S_1 \cup ... \cup S_n$ be a layered belief base stratified from a possibilistic knowledge base. A formula $\phi$ is said to be a $\pi$-consequence of $\Sigma$ with weight $a_i$, denoted by $\Sigma \vdash_\pi (\phi, a_i)$, if and only if:*

1. *$S_1 \cup ... \cup S_i$ is consistent,*
2. *$S_1 \cup ... \cup S_i \vdash \phi$, and*
3. *$\forall j < i, S_1 \cup ... \cup S_j \nvdash \phi$.*

**Definition 4** *A subbase $\Sigma_i$ of $\Sigma$ is said to be an argument for a formula $\phi$ with weight $a$, denoted by $\Sigma_i \vdash_A (\phi, a)$ if it satisfies the following conditions.*

1. *$\Sigma_i \nvdash \perp$ (consistency)*
2. *$\Sigma_i \vdash_\pi (\phi, a)$ (relevance)*
3. *$\forall (\psi, b) \in \Sigma_i, \ \Sigma_i - \{(\psi, b)\} \nvdash_\pi (\phi, a)$ (economy)*

Another consequence relation which is "stronger" than the $\pi$-consequence was defined as follows (Benferhat et al. 1993b).

**Definition 5** *A formula $\phi$ is said to be an argumentative consequence[1] of $\Sigma$, denoted by $\Sigma \vdash_A (\phi, a)$, if and only if:*

1. *there exists an argument for $(\phi, a)$ in $\Sigma$, and*
2. *for each argument of $(\neg\phi, b)$ in $\Sigma$, we have $b < a$.*

Given a *possibilistic base B*, a unique *possibility distribution*, denoted by $\pi_B$, can be obtained by the principle of minimum specificity. For all $\omega \in \Omega$,

$$\pi_\mathcal{B}(\omega) = \begin{cases} 1 & \text{if } \forall(\phi_i, \alpha_i) \in \mathcal{B}, \omega \models \phi_i, \\ 1 - max\{\alpha_i | \omega \nvDash \phi_i, (\phi_i, \alpha_i) \in \mathcal{B}\} & \text{otherwise.} \end{cases}$$

(1)

## $\pi$-consequence Relation based Merging Method

### Definition of the $\pi$-consequence relation based merging method

In (Benferhat & Kaci 2003), the authors introduced a syntactic method to merge a set of *n consistent* possibilistic knowledge bases $B_1, ..., B_n$, where the result of merging can be inconsistent. A possibilistic merging operator, denoted by $\oplus$, which is a function from $[0,1]^n$ to $[0,1]$, is used to merge the certainty degrees associated with pieces of information

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[1]The argumentative consequence defined here is identical to the argued consequence in (Benferhat et al. 1998).

provided by different sources. The result of the combination of $B_i$ is $\mathcal{B}_\oplus$ such that (see also Fig.1),

$$\mathcal{B}_\oplus = \{(\phi, \oplus(a_1, ..., a_n)) : B_i \vdash_\pi (\phi, a_i)\}. \qquad (2)$$

Since the merging method defined by Equation (2) is restricted by the $\pi$-consequence relation, for convenience, we call it a $\pi$-consequence relation based method in this paper.

The operator $\oplus$ should satisfy the following properties.
(Mer1) $\oplus(0, ..., 0) = 0$.
(Mer2) If $\forall\ i = 1, ..., n,\ a_i \geq b_i$ then $\oplus(a_1, ..., a_n) \geq \oplus(b_1, ..., b_n)$. (unanimity property)



$$B_1 \cdots\cdots\cdots\cdots\cdots\cdots\cdots B_n$$

$$(\phi, a_1) \cdots\cdots\cdots\cdots\cdots (\phi, a_n)$$
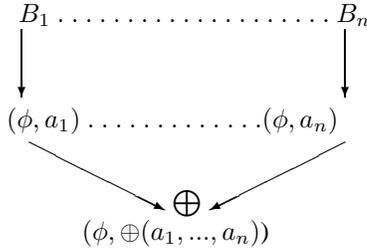
$$\oplus$$

$$(\phi, \oplus(a_1, ..., a_n))$$

Fig. 1. Merging possibilistic bases.

Given two possibilistic knowledge bases, a simplification of the computation of $\mathcal{B}_\oplus$ can be obtained by the following lemma (Benferhat & Kaci 2003).

**Lemma 1** *Let* $B_1 = \{(\phi_i, a_i) : i = 1, ..., n\}$ *and* $B_2 = \{(\psi_j, b_j) : j = 1, ..., m\}$ *be two possibilistic bases. Let* $\mathcal{B}_\oplus$ *be the result of merging* $B_1$ *and* $B_2$ *using* $\oplus$ *and* $\mathcal{B}_\oplus$ *follows Equation (2). Then,* $\mathcal{B}_\oplus$ *is equivalent to:*

$$\mathcal{B}_\oplus^\pi = \{(\phi_i, \oplus(a_i, 0)) : (\phi_i, a_i) \in B_1\} \cup \{(\psi_j, \oplus(0, b_j)) : (\psi_j, b_j) \in B_2\} \cup \{(\phi_i \vee \psi_j, \oplus(a_i, b_j)) : (\phi_i, a_i) \in B_1 \ and \ (\psi_j, b_j) \in B_2\}.$$

The $\mathcal{B}_\oplus^\pi$ in Lemma 1 provides an easy way to compute $\mathcal{B}_\oplus$ defined by Equation (2).

As criticized in (Benferhat et al. 1993a), the $\pi$-consequence relation will ignore the formulas whose necessity degrees are lower than the inconsistency degree. So when $B_1$ or $B_2$ are individually inconsistent, the $\pi$-consequence relation based method has to delete some formulae from the original knowledge base. More precisely, suppose the weights of formulas in $B_1$ and $B_2$ have been ordered such that $a_1 > a_2 > ... > a_n$ and $b_1 > b_2 > ... > b_m$. Before applying Lemma 1, $B_i = \{(\phi_k, a_k) : k = 1, ..., n\}$ will be replaced by $B_i' = \{(\phi_k, a_k) : k = 1, ..., j-1\} \cup \{(\bot, a_j)\}$, where $a_j = Inc(B_i)$.

The following corollary follows from Lemma 1 and the discussion above.

**Corollary 1** *Let* $B_1 = \{(\phi_i, a_i) : i = 1, ..., n\}$ *and* $B_2 = \{(\psi_j, b_j) : j = 1, ..., m\}$ *be two individually inconsistent possibilistic bases, where* $a_1 > a_2 > ... > a_n$ *and* $b_1 > b_2 > ... > b_m$. *Suppose* $a_k = Inc(B_1)$ *and* $b_l = Inc(B_2)$. *Let* $B_1' = \{(\phi_i, a_i) : i = 1, ..., k-1\} \cup \{(\bot, a_k)\}$ *and* $B_2' = \{(\psi_j, b_j) : j = 1, ..., l-1\} \cup \{(\bot, b_l)\}$. *Then,* $\mathcal{B}_\oplus^\pi$ *in Lemma 1 is revised to:*

$$\mathcal{B}_\oplus'^\pi = \{(\phi_i, \oplus(a_i, 0)) : i = 1, ..., k-1\} \cup \{(\psi_j, \oplus(0, b_j)) : j = 1, ..., l-1\} \cup \{(\phi_i \vee \psi_j, \oplus(a_i, b_j)) : i = 1, ..., k-1 \ and \ j = 1, ..., l-1\} \cup \{(\bot, \oplus(a_k, b_l))\}. \quad (3)$$

Corollary 1 shows that the formulas with weights less than the inconsistency degree of each knowledge base do not appear in the result of merging using Equation (2) when the original knowledge bases are individually inconsistent.

## Comparison with other merging methods

Since each possibilistic knowledge base $B$ can be associated with a unique possibilistic distribution through Equation (1), merging $n$ possibilistic knowledge bases could also be performed at the semantic level by merging the corresponding $\pi_{B_i}$. An approach to merging $n$ consistent possibilistic knowledge bases is to apply the *minimum* operator to $\pi_{B_i}$ as $\pi = min(\pi_{B_i})$ where $\pi$ is the possibility distribution associated with the merged knowledge base $B$. The syntactic equivalence of $\pi = min(\pi_{B_i})$ is $B = \cup(B_i)$ (Benferhat et al. 1997a).

More generally, the following proposition reveals the relationship between the semantic combination and its syntactic counterpart (Benferhat et al. 2001).

**Proposition 1** *Let* $B_1$ *and* $B_2$ *be two possibilistic knowledge bases. Let* $\pi_{\oplus'}$ *be the combination of* $\pi_{B_1}$ *and* $\pi_{B_2}$ *based on the operator* $\oplus'$. *Then* $\pi_{\oplus'}$ *is associated with the following belief base:*

$$B_1 \oplus' B_2 = \{(\phi_i, 1 - ((1 - a_i) \oplus' 1)) : (\phi_i, a_i) \in B_1\} \cup \{(\psi_j, 1 - (1 \oplus' (1 - b_j))) : (\psi_j, b_j) \in B_2\} \cup \{(\phi_i \vee \psi_j, \oplus'(a_i, b_j)) | (\phi_i, a_i) \in B_1, and \ (\psi_j, b_j) \in B_2\}, \qquad (4)$$

When $\oplus' = min$, it is easy to check that $B_1 \oplus' B_2 = B_1 \cup B_2$.

**Proposition 2** *Let* $B_1$ *and* $B_2$ *be two possibilistic knowledge bases. If the operator* $\oplus$ *in Equation (2) is the maximum operator and the operator* $\oplus'$ *in Equation (4) is the minimum operator, then we have*

$$\mathcal{B}_\oplus^\pi \equiv_s B_1 \oplus' B_2, \qquad (5)$$

*Proof.*
If $\oplus = max$, then by Lemma 1, $\mathcal{B}_\oplus^\pi \equiv_s B_1 \cup B_2$. Therefore, by Proposition 1, $\mathcal{B}_\oplus^\pi \equiv_s B_1 \oplus' B_2$.

**Definition 6** *Let* $\oplus_1$ *and* $\oplus_2$ *be two merging operators satisfying (Mer1) and (Mer2).* $\oplus_1$ *and* $\oplus_2$ *are said to be dual if and only if* $\oplus_1(a, b) = 1 - (1-a) \oplus_2 (1-b)$.

The typical dual merging operators are T-norm and T-conorm in (Klement et al. 2000).

The following proposition follows from Proposition 1 and Definition 6.

**Proposition 3** *Let* $B_1$ *and* $B_2$ *be two possibilistic knowledge bases. Let* $\oplus_1$ *and* $\oplus_2$ *be two dual operators, then we have*

$$\mathcal{B}_{\oplus_1}^\pi \equiv_s B_1 \oplus_2 B_2,$$

Proposition 3 shows that the $\pi$-consequence relation based method is equivalent to the corresponding syntactic method in (Benferhat et al. 2001) when the merging operators used are dual.

## Combining Individually Inconsistent Prioritized Knowledge Bases

Most merging methods assume that the original knowledge bases are individually consistent. But in practice, we may confront the problem of combining individually inconsistent knowledge bases. Inconsistency can appear in given knowledge bases as well as resulting from combination or revision (Benferhat et al. 1995; Priest 2001). In this section, we will propose two different methods for combining individually inconsistent prioritized knowledge bases, where priorities between formulas are handled in the framework of possibilistic logic. The first method is called an argument-based merging method which extends the $\pi$-consequence relation based method introduced in Benferhat & Kaci 2003. The other method is called a multiple-operator based method, which combines consistent and conflict information using different operators.

### Argument-based merging method
### Definition of the argument-based merging method

The merging method introduced in the last section is not advisable for combining individually inconsistent knowledge bases. Because this method is constrained by the $\pi$-consequence relation. When it is applied to inconsistent knowledge bases, some information will have to be deleted before merging. In this section, we will define an argument-based merging method by replacing the $\pi$-consequence relation with the argument-based consequence relation in Equation (2). By Definition 4, we know that the argument-based consequence relation will keep all the information in the knowledge base $\Sigma$ because every formula in $\Sigma$ has an argument for it. Therefore, the revised merging method will not delete any information in the original knowledge bases. This is reasonable, because all the formulas, including formulas in conflict, are viewed to be useful.

**Definition 7** *Let $\mathcal{B} = \{B_1, ..., B_n\}$ be a set of n individually inconsistent possibilistic knowledge bases, then the result of merging the bases in $\mathcal{B}$ by a merging operator $\oplus$ is*

$$\mathcal{B}_\oplus = \{(\phi, \oplus(a_1, ..., a_n)) : \Sigma_{B_i} \vdash_A (\phi, a_i)\} \quad (6)$$

*where $\Sigma_{B_i}$ is the layered belief base associated with $B_i$.*

This method will not ignore any formula in $B_i$ even if $B_i$ is inconsistent.

The following lemma provides a method to compute $\mathcal{B}_\oplus$ defined by Equation (6).

**Lemma 2** *Let $B_1 = \{(\phi_i, a_i) : i = 1, ..., n\}$ and $B_2 = \{(\psi_j, b_j) : j = 1, ..., m\}$ be two inconsistent possibilistic bases. The merging result $\mathcal{B}_\oplus$ of $B_1$ and $B_2$ following Equation (6) is equivalent to*

$$\mathcal{B}_\oplus^A = \{(\phi_i, \oplus(a_i, 0)) : (\phi_i, a_i) \in B_1 \text{ and } \phi_i \notin B_2{}^*\} \cup$$
$$\{(\psi_j, \oplus(0, b_j)) : (\psi_j, b_j) \in B_2 \text{ and } \psi_j \notin B_1{}^*\} \cup$$

$$\{(\phi_i \vee \psi_j, \oplus(a_i, b_j)) : (\phi_i, a_i) \in B_1 \text{ and } (\psi_j, b_j) \in B_2\}. \quad (7)$$

*Proof.*

*The proof of Lemma 2 is similar to that of Lemma 1 in Benferhat & Kaci 2003. We first show $\mathcal{B}_\oplus^A \subseteq \mathcal{B}_\oplus$.*

*(1) Let $(\phi_i, \oplus(a_i, 0))$ (resp. $(\psi_j, \oplus(0, b_j))$) be a formula in $\mathcal{B}_\oplus^A$, where $(\phi_i, a_i) \in B_1$ and $\phi_i \notin B_2{}^*$. Since $(\phi_i, a_i) \in B_1$, there exists an argument for $(\phi_i, a_i)$ in $\Sigma_{B_1}$. Although $\phi \notin B_2{}^*$, we can add $(\phi_j, 0)$ to $B_2$, the revised possibilistic knowledge base is equivalent to $B_2$. So we have found an argument $(\phi_i, 0)$ for $\phi_i$ in $\Sigma_{B_2}$. Therefore, by Equation (6), $(\phi_i, \oplus(a_i, 0)) \in \mathcal{B}_\oplus$.*

*(2) Let $(\phi_i \vee \psi_j, \oplus(a_i, b_j))$ be a formula in $\mathcal{B}_\oplus^A$, where $(\phi_i, a_i) \in B_1$ and $(\psi_j, b_j) \in B_2$. Since $(\phi_i, a_i) \in B_1$, there must exist an argument for $(\phi_i \vee \psi_j, a)$ s.t. $a \geq a_i$ in $\Sigma_{B_1}$. Similarly, we can show that there exists an argument for $(\phi_i \vee \psi_j, b)$ s.t. $b \geq b_j$ in $\Sigma_{B_2}$. Then, by Equation (6), we have $(\phi_i \vee \psi_j, \oplus(a, b)) \in \mathcal{B}_\oplus$. Following the unanimity property of $\oplus$, we have $\oplus(a, b) \geq \oplus(a_i, b_j)$. So we can equivalently add $\oplus(a, b) \geq \oplus(a_i, b_j)$ to $\mathcal{B}_\oplus$.*

*Next, we will show that $\forall (\phi, b)$, if $(\phi, b) \in \mathcal{B}_\oplus$ (and we suppose $b > Inc(\mathcal{B}_\oplus)$, for otherwise, $(\phi, b)$ must be subsumed by $\mathcal{B}_\oplus$) and $(\phi, b) \notin \mathcal{B}_\oplus^A$, then $(\phi, b)$ is subsumed by $\mathcal{B}_\oplus$. Since $(\phi, b) \in \mathcal{B}_\oplus$, there must exist an argument for $(\phi, a_1)$ in $\Sigma_{B_1}$ and an argument for $(\phi, a_2)$ in $\Sigma_{B_2}$ s.t. $b = \oplus(a_1, a_2)$, with $a_1 \geq 0$ and $a_2 \geq 0$.*

*(1) Suppose $a_1 = 0$ and $a_2 = 0$, and since $\oplus(0, 0) = 0$, then $b = 0$. So $(\phi, b)$ can be deleted from $\mathcal{B}_\oplus$.*

*(2) Suppose $a_1 > 0$ and $a_2 = 0$ (resp. $a_1 = 0$ and $a_2 > 0$), then $b = \oplus(a_1, 0)$. Suppose $S$ is an argument for $(\phi, a_1)$ in $\Sigma_{B_1}$, then by Definition 3 and Definition 4, there exists a subset of $S$, denoted by $S_1$, which is also an argument for $(\phi, a_1)$ and the necessity degrees of all the formulas in $S_1$ are greater than $a_1$. Since $\mathcal{B}_\oplus^A \subseteq \mathcal{B}_\oplus$, each formula $\phi_i$ with necessity degree $a_i$ in $S_1$ belongs to $\mathcal{B}_\oplus$ with the necessity degree at least $\oplus(a_i, 0)$. By the unanimity property of $\oplus$, $\oplus(a_i, 0) \geq \oplus(a_1, 0)$, for each $(\phi_i, a_i) \in S_1$. We have assumed that $b = \oplus(a_1, 0) > Inc(\mathcal{B}_\oplus)$, so $\oplus(a_i, 0) > Inc(\mathcal{B}_\oplus)$, for each $(\phi_i, a_i) \in S_1$. Therefore, there is a formula $(\phi, a)$ s.t $a \geq \oplus(a_1, 0)$ in $\mathcal{B}_\oplus$, and so the formula $(\phi, \oplus(a_1, 0))$ is subsumed in $\mathcal{B}_\oplus$.*

*(3) Suppose $a_1 > 0$ and $a_2 > 0$. Let $S_1$ is an argument for $(\phi, a_1)$ in $\Sigma_{B_1}$ and $S_2$ be an argument for $(\phi, a_2)$ in $\Sigma_{B_2}$ such that the necessity degrees of formulas in $S_1$ and $S_2$ are greater than $a_1$ and $a_2$ respectively. Since $\mathcal{B}_\oplus^A \subseteq \mathcal{B}_\oplus$, the disjunctions between formulas of $S_1$ and $S_2$, which entail $\phi$, belong to $\mathcal{B}_\oplus$ with the weight $\oplus(a_i, a_j)$, where $(\phi_i, a_i) \in S_1$ and $(\psi_j, a_j) \in S_2$. By the unanimity property, $\oplus(a_i, a_j) \geq \oplus(a_1, a_2)$, for any $(\phi_i, a_i) \in S_1$ and $(\psi_j, a_j) \in S_2$. Therefore, $\phi$ is subsumed in $\mathcal{B}_\oplus$.*

In Equation (7), the formula $(\phi_i, \oplus(a_i, 0))$ belongs to $\mathcal{B}_\oplus^A$ because there is an argument $(\phi_i, a_i)$ for $\phi_i$ in $B_1$ and no argument exists for $\psi_j$ in $B_2$. The same explanation applies to the formula $(\psi_j, b_j)$. Moreover, the formula $(\phi_i \vee \psi_j, \oplus(a_i, b_j))$ is in $\mathcal{B}_\oplus^A$ because it has an argument $(\phi_i, a_i)$ in $B_1$ and an argument $(\psi_j, b_j)$ in $B_2$.

$\mathcal{B}_{\oplus}^A$ is different from $B_1 \oplus' B_2$ defined by Equation (4), where $\oplus'$ is the dual operator of $\oplus$, this can be seen in terms of two aspects.

1. For each formula $\phi$, if $(\phi, a) \in B_1$ and $(\phi, b) \in B_2$, where $a, b > 0$, it belongs to $\mathcal{B}_{\oplus}^A$ with the weight $\oplus(a, b)$. By contrast, it will appear in $B_1 \oplus' B_2$ with three different forms, i.e., $(\phi, \oplus(a, 0))$, $(\phi, \oplus(0, b))$ and $(\phi, \oplus(a, b))$. It is clear that $(\phi, \oplus(a, 0))$ and $(\phi, \oplus(0, b))$ are redundant information and we can delete them to make the knowledge base simpler.

**Example 1** *Let* $B_1 = \{(\phi, 0.4), (\psi, 0.5), (\neg\phi, 0.7), (\gamma, 0.7)\}$ *and* $B_2 = \{(\phi, 0.2), (\psi \vee \gamma, 0.6)\}$ *be two possibilistic knowledge bases. If we take the "bounded sum", which is defined as* $\oplus_b(a, b) = min(1, a + b)$, *as the merging operator, then by Lemma 2, the result of merging* $B_1$ *and* $B_2$ *is* $\mathcal{B}_{\oplus_b}^A = \{(\phi, 0.6), (\psi, 0.5), (\neg\phi, 0.7), (\gamma, 0.7), (\psi \vee \gamma, 0.6), (\phi \vee \psi \vee \gamma, 1), (\psi \vee \gamma, 1), (\phi \vee \psi, 0.7), (\neg\phi \vee \psi \vee \gamma, 1), (\gamma \vee \phi, 0.9), (\gamma \vee \psi, 1)\}$. *By contrast, if we combine* $B_1$ *and* $B_2$ *using Equation (4) with the "Łukasiewicz t-norm"* $(\oplus_L(a, b) = max(0, a + b - 1))$, *the result of merging is* $B_1 \oplus_L B_2 = \{(\phi, 0.6), (\phi, 0.4), (\phi, 0.2), (\psi, 0.5), (\neg\phi, 0.7), (\gamma, 0.7), (\psi \vee \gamma, 0.6), (\phi \vee \psi \vee \gamma, 1), (\psi \vee \gamma, 1), (\phi \vee \psi, 0.7), (\neg\phi \vee \psi \vee \gamma, 1), (\gamma \vee \phi, 0.9), (\gamma \vee \psi, 1)\}$.

In Example 1, $\phi$ appears in $\mathcal{B}_{\oplus_b}^A$ with weight 0.6, however, it appears in $B_1 \oplus_L B_2$ with three different weights 0.6, 0.4, and 0.2 respectively. The formulas $(\phi, 0.4)$ and $(\phi, 0.2)$ are redundant information, because we have combined $(\phi, 0.4)$ in $B_1$ and $(\phi, 0.2)$ in $B_2$ into $(\phi, 0.6)$ and we have no reason to keep $(\phi, 0.4)$ and $(\phi, 0.2)$ in the result of merging.

2. In (Benferhat et al. 2001), some merging operators were introduced to combine two possibilistic knowledge bases using Equation (4). It has been pointed out that the *maximum* operator is appropriate when the sources are highly conflicting with each other and the *minimum* operator is meaningful when the sources are consistent. When the *maximum* operator is chosen, the result of merging is $B_1 \oplus_{max} B_2 = \{(\phi_i \vee \psi_j, min(a_i, b_j)) : (\phi_i, a_i) \in B_1 \text{ and } (\psi_j, b_j) \in B_2\}$. Clearly $B_1 \oplus_{max} B_2$ is too *weak* for the result of merging, i.e., a lot of information is lost. The reason that the *maximum* operator is chosen is because the inconsistency is viewed as a bad thing and needs to be avoided in (Benferhat et al. 2001). However, if we believe that the inconsistency may contain some important information and keep it, we will not choose the *maximum* operator. Therefore, we think it is implicitly assumed that the original knowledge bases are self-consistent or at least the inconsistency need to be avoided by choosing appropriate merging operators in (Benferhat et al. 2001). By contrast, our method is applied to merge inconsistent knowledge bases explicitly and we want to keep all the information in the original knowledge bases after merging.

## Properties of the argument-based method

In this subsection, we discuss the relationship between the argument-based method and the original method in (Benferhat & Kaci 2003). The following two propositions show that the argument-based method is a generalization of the $\pi$-consequence relation based merging method.

**Proposition 4** *Let* $\mathcal{B} = \{B_1, ..., B_n\}$ *be a set of n individually consistent possibilistic knowledge bases, then the result of merging* $B_i$ *satisfying Equation (6) is the same as that satisfying Equation (2).*

*Proof.*

When $B_i$ $(i = 1, ..., n)$ are consistent, for any possibilistic formula $(\phi, a_i)$, $B_i \vdash_\pi (\phi, a_i)$ iff there exists an argument for $(\phi, a_i)$ in $\Sigma_{B_i}$. Therefore, the result of $B_i$ satisfying Equation (6) is the same as that satisfying Equation (2).

**Proposition 5** *Let* $B_1$ *and* $B_2$ *be two possibilistic knowledge bases, if* $\mathcal{B}_{\oplus}^A$ *is the the result of merging* $B_i$ *satisfying Equation (7) and* $\mathcal{B}_{\oplus}^{\prime\pi}$ *is the result of merging* $B_i$ *satisfying Equation (3), then* $\mathcal{B}_{\oplus}^{\prime\pi} \equiv_s \mathcal{B}_{\oplus}^A$ *and* $(\mathcal{B}_{\oplus}^{\prime\pi})^* \subseteq (\mathcal{B}_{\oplus}^A)^*$.

*Proof.*

By Lemma 1, Lemma 2 and Corollary 1, it is easy to check that $\mathcal{B}_{\oplus}^{\prime\pi} \equiv_s \mathcal{B}_{\oplus}^A$. By Equation (3) and Equation (7), it is clear $(\mathcal{B}_{\oplus}^{\prime\pi})^* \subseteq (\mathcal{B}_{\oplus}^A)^*$. To show the converse is not true, let us consider the following counter-example. Let $B_1 = \{(\phi, 0.7), (\neg\phi, 0.5), (\gamma, 0.5)\}$ and $B_2 = \{(\psi, 0.6)\}$. Since $Inc(B_1) = 0.5$, $\gamma$ cannot appear in $\mathcal{B}_{\oplus}^{\prime\pi}$. However, we have $(\gamma, \oplus(0.5, 0)) \in \mathcal{B}_{\oplus}^A$.

Proposition 5 shows that the merging result obtained by the argument-based method contains some information that is ignored by the $\pi$-consequence relation based method. In the counter-example, the formula $\gamma$ is not in conflict with other formulas, so it is not advisable to delete it. In fact, $\gamma$ may be important information and can be recovered from $\mathcal{B}_{\oplus}^{\prime A}$ by some inconsistency-tolerant consequence relations in (Benferhat et al. 1993b; 1998). For example, it is easy to show that $\gamma$ is an argumentative consequence of $\mathcal{B}_{\oplus}^A$.

The following proposition compare $\mathcal{B}_{\oplus}^A$ and $B_1 \oplus' B_2$.

**Proposition 6** *Let* $B_1$ *and* $B_2$ *be two possibilistic knowledge bases. If* $\mathcal{B}_{\oplus}^A$ *is the the result of merging* $B_1$ *and* $B_2$ *satisfying Equation (7) and* $B_1 \oplus' B_2$ *is the result of merging* $B_1$ *and* $B_2$ *satisfying Equation (4), where* $\oplus'$ *is the dual operator of* $\oplus$, *then* $\mathcal{B}_{\oplus}^A \equiv_s B_1 \oplus' B_2$ *and* $\mathcal{B}_{\oplus}^A \subseteq B_1 \oplus' B_2$.

The proof of Proposition 6 is obvious, so we will not provide it here.

## Multiple-operator based method

The merging methods introduced above use only a single operator to define the combination of possibilistic knowledge bases even if some information in it is in conflict. Let us use the following example to see the problem of a single operator based merging methods.

**Example 2** *Let* $B_1 = \{(\phi, 0.7), (\psi, 0.7), (\xi, 0.8)\}$ *and* $B_2 = \{(\neg\phi, 0.8), (\phi, 0.5), (\psi, 0.8), (\xi, 0.6), (\gamma, 0.4)\}$ *be two independent knowledge bases.* $\psi$ *and* $\xi$ *are supported by both* $B_1$ *and* $B_2$ *with high degrees and they are not involved in inconsistency of* $B_1 \cup B_2$, *so there should be a reinforcement effect for them. Suppose the merging operator is the* probabilistic sum *defined as* $\oplus(a, b) = a + b - ab$, *which is a common used operator with reinforcement effect. By Lemma 2, the result of combination of* $B_1$ *and* $B_2$ *is* $\mathcal{B}_{\oplus}^A = \{(\neg\phi, 0.8), (\gamma, 0.4), (\phi, 0.85), (\phi \vee \psi, 0.94), (\phi \vee \xi, 0.88), (\phi \vee \gamma, 0.82), (\neg\phi \vee \psi, 0.94), (\phi \vee$

$\psi, 0.85), (\psi, 0.94), (\psi \vee \xi, 0.88), (\psi \vee \gamma, 0.82), (\neg\phi \vee \xi, 0.96), (\phi \vee \xi, 0.9), (\psi \vee \xi, 0.94), (\xi, 0.92), (\xi \vee \gamma, 0.88)\}$.

In this example, the necessity degrees of $\psi$ and $\xi$ increase because the *probabilistic sum* has reinforcement effect. However, formulas $\phi$ and $\neg\phi$ are strongly in conflict and so they should counteract with each other. Therefore, the necessity degrees of both $\phi$ and $\neg\phi$ should be lower than the original ones. On the contrary, the necessity degree of $\phi$ increases to 0.85 and the necessity degree of $\neg\phi$ remains high (0.8) after the combination, which is unreasonable. This problem is caused by using only a single operator to combine both the consistent and conflict formulas.

Let $B_1$ and $B_2$ be two possibilistic knowledge bases from two different sources. For those formulas that are involved in the conflict in $B_1 \cup B_2$, their necessity degrees should decrease after combination because they will counteract with each other. By contrast, the necessity degree should increase for those formulas that are supported by both sources.

Before giving the definition of the multiple-operator based merging method, let us introduce a merging operator in Benferhat & Kaci 2003.

**Definition 8** *An operator $\oplus$ is said to be strongly conjunctive on [0,1] if for all $(a_1, ..., a_n)$*

$$\oplus(a_1, ..., a_n) \geq max(a_1, ..., a_n).$$

A strongly conjunctive operator has many favourable properties because it satisfies most postulates[2] that are introduced in (Benferhat & Kaci 2003) to characterize a merging operator. If a strongly conjunctive operator satisfies $\oplus(a_1, ..., a_n) > max(a_1, ..., a_n)$ when $\forall a_i \neq 1$, and $\oplus(a_1, ..., a_n) = 1$ when $\exists i$ such that $a_i = 1$, it is called a *reinforcement* operator. A strongly conjunctive operator is suitable to merge formulas that are not involved in conflict, especially those supported by both sources.

We propose another operator as follows.

**Definition 9** *An operator $\oplus$ is said to be an up-averaging operator if for all $(a_1, ..., a_n)$*

$$\oplus(a_1, ..., a_n) \leq max(a_1, ..., a_n).$$

This operator reflects that a merging result cannot be greater than the greatest of all. An example of an up-averaging operator is a *weighted average*, which is defined as $\oplus(a, b) = xa + yb$, where $x, y \in [0, 1]$ and $x + y = 1$. When $x = y = 1/2$, this operator is the standard average operator and when $x > y$ (or $x < y$), the source associated with $x$ is given more credit than the other source (or vice versa). If an up-averaging operator satisfies $\oplus(a_1, ..., a_n) < max(a_1, ..., a_n)$ when $\exists i, a_i \neq 0$, it is called a *counteract* operator. An up-averaging operator is suitable to merge formulas that are involved in conflict.

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[2]A strongly conjunctive operator satisfies the postulates $\mathbf{A}_i$, i=1,...,5. Only the postulate $\mathbf{A}_6$ which refers to a decomposition of one group into two groups are satisfied by a strongly conjunctive operator is not satisfied by a strongly conjunctive operator.

**Definition 10 (Benferhat et al. 1997b)** *A subbase B of a classical knowledge base $\Sigma$ is said to be minimally inconsistent (mI-subbase for short) if and only if it satisfies the following two requirements:*

- $B \models false$, *and*
- $\forall\phi \in B, B - \{\phi\} \not\models false$.

**Definition 11** *A formula $\phi$ is said to be in conflict in a classical knowledge base $\Sigma$ iff it belongs to some minimally inconsistent subbase of $\Sigma$. The set of formulas in conflict in $\Sigma$ is denoted as $Conflict(\Sigma)$.*

Now we provide our multiple-operator based merging method. We always assume that if a formula $\phi$ does not appear in a possibilistic knowledge base *B*, then $(\phi, 0)$ has been added to *B*.

**Definition 12** *Let $B_1 = \{(\phi_i, a_i) : i = 1, ..., n\}$ and $B_2 = \{(\psi_j, b_j) : j = 1, ..., m\}$ be two possibilistic knowledge bases. Let $\oplus_s$ and $\oplus_a$ be a strong conjunctive operator and an up-averaging operator respectively. The combination of $B_1$ and $B_2$ is defined as $\Delta_{\oplus_s, \oplus_a}(B_1, B_2) = C \cup D$, where*

$$
\begin{aligned}
C &= \{\phi, \oplus_a(a, b) | \phi \in (Conflict(B_1 \cup B_2))^*, (\phi, a) \in B_1 \\
&\quad and\ (\phi, b) \in B_2\},
\end{aligned}
$$

$$
\begin{aligned}
D &= \{\phi, \oplus_s(a, b) | \phi \notin (Conflict(B_1 \cup B_2))^*, (\phi, a) \in B_1 \\
&\quad and\ (\phi, b) \in B_2\}
\end{aligned}
$$

In Definition 12, we use two operators, one is a strongly conjunctive operator and the other is an up-averaging operator, to merge the possibilistic knowledge bases. For those formulas that are not in conflict in $B_1 \cup B_2$, we choose the strongly conjunctive operator to combine them. But for those formulas that are in conflict, we use the up-averaging operator to combine them.

Another important point in favour of Definition 12 is that $\Delta_{\oplus_s, \oplus_a}(B_1, B_2)$ only contains the formulas in $B_1$ and $B_2$ and does not consider the formulas that can be inferred from $B_1$ or $B_2$. In practice, when an agent needs to combine the information given by some different agents, he or she will not always consider the implicit information, i.e., the information that can be inferred from a source, because sometimes the amount of information is vast, it may not be feasible to spend that much time on inferring all the consequences. Moreover, considering only the formulas in each knowledge base makes the computation easy because we do not need to compute the formulas inferred from the original knowledge bases.

**Example 3** *(Continue Example 2) Suppose the merging operators are $\oplus_s(a, b) = a + b - ab$ and $\oplus_a(a, b) = (a + b)/2$. By Definition 12, the result of the combination of $B_1$ and $B_2$ is $\Delta_{\oplus_s, \oplus_a}(B_1, B_2) = \{(\phi, 0.6), (\neg\phi, 0.4), (\psi, 0.82), (\xi, 0.5), (\gamma, 0.4)\}$.*

In Example 3, the necessity degrees of both $\phi$ and $\neg\phi$ decrease and the necessity degree of $\phi$ is greater than $\neg\phi$ after combination. The necessity degrees of other formulas in Example 3 are the same as those in Example 2. However, those formulas appearing in disjunctive form in Example 2 do not

exist in Example 3. Although we can only infer such formulas from $\Delta_{\oplus_s,\oplus_a}(B_1,B_2)$ with necessity degrees lower than those in Example 2, $\Delta_{\oplus_s,\oplus_a}(B_1,B_2)$ is much simpler than $\mathcal{B}_{\oplus}^A$ in Example 2.

In Definition 12, all the conflict formulas are weakened to have lower necessity degrees after combination. However, in some cases, it may be more reasonable to have the necessity degrees of some formulas in conflict increased. For example, suppose we have two possibilistic knowledge bases $B_1 = \{(\phi,0.7),(\psi,0.7)\}$ and $B_2 = \{(\neg\phi,0.4),(\phi,0.7),$ $(\psi,0.4),(\xi,0.5),(\gamma,0.4)\}$ from two sources of information. Clearly, $\phi$ is supported by $B_1$. Although $\phi$ is involved in conflict in $B_2$, the necessity degree of $\phi$ is greater than that of $\neg\phi$, so $\phi$ can be considered to be supported by $B_2$ as a whole. Therefore, both sources *support* $\phi$ and then the necessity degree of $\phi$ should increase.

**Definition 13** *Let $B$ be an inconsistent knowledge base. A formula $\phi$ that is in conflict in $B$ is said to be weakly supported by $B$ if and only if $\exists(\phi,a)\in B$ such that $a>b$ for all $(\neg\phi,b)\in B$.*

**Definition 14** *Let $B_1$ and $B_2$ be two possibilistic knowledge bases. A formula $\phi$ is said to be in weak conflict with regard to $B_1$ and $B_2$ if and only if $\phi$ is weakly supported by $B_1$ and $B_2$ separately. The set of formulas in weak conflict with regard to $B_1\cup B_2$ is denoted as $Weak(B_1\cup B_2)$.*

Now we define another multiple-operator based method.

**Definition 15** *Let $B_1 = \{(\phi_i,a_i) : i = 1,...,n\}$ and $B_2 = \{(\psi_j,b_j) : j = 1,...,m\}$ be two possibilistic knowledge bases. Let $\oplus_s$ and $\oplus_a$ be a strong conjunctive operator and an up-averaging operator respectively. The combination of $B_1$ and $B_2$ is defined as $\Delta'_{\oplus_s,\oplus_a}(B_1,B_2) = C\cup D$, where*

$$C = \{(\phi,\oplus_a(a,b))|\phi\in(Conflict(B_1\cup B_2)\backslash Weak$$
$$(B_1\cup B_2))^*, (\phi,a)\in B_1 \ and \ (\phi,b)\in B_2\},$$

$$D = \{(\phi,\oplus_s(a,b))|\phi\notin(Conflict(B_1\cup B_2))^* \ or \ \phi\in$$
$$(Weak(B_1\cup B_2))^*, (\phi,a)\in B_1 \ and \ (\phi,b)\in B_2\}$$

In $\Delta'_{\oplus_s,\oplus_a}(B_1,B_2)$, necessity degrees of those formulas that are in conflict and are not weakly supported by both sources will decrease. By contrast, the necessity degrees of the formulas that are not involved in conflict or weakly supported by both sources will increase.

**Example 4** *Let $B_1 = \{(\phi,0.6),(\psi,0.7)\}$ and $B_2 = \{(\neg\phi,0.4),(\phi,0.7),(\psi,0.4),(\xi,0.5),(\gamma,0.4)\}$. Suppose the merging operators are $\oplus_s(a,b) = a + b - ab$ and $\oplus_a(a,b) = (a + b)/2$. Since $\phi$ is in conflict weak with regard to $B_1\cup B_2$, by Definition 15, the result of merging is $\Delta'_{\oplus_s,\oplus_a}(B_1,B_2) = \{(\phi,0.88),(\neg\phi,0.2),(\psi,0.82),(\xi,0.5),(\gamma,0.4)\}$.*

Although the result of the second multiple-operator based merging method is more reasonable than that of the first one, it is computationally more expensive because it needs to check whether a formula is weakly supported by two sources.

## Conclusions

In this paper, we proposed two different methods to merge several possibilistic knowledge bases. The first method, called an argument based method, is a revision of the merging method in Benferhat & Kaci 2003. This method is proved to be more advisable than the method in Benferhat & Kaci 2003 to merge individually inconsistent knowledge bases. The single operator based methods combines all the formulas using one operator. Therefore, we can not differentiate the consistent formulas and conflict formulas. Moreover, the combination is applied to the belief set, i.e., the set of formulas closed under some consequence relation. This is computationally too expensive. The second method, called a multiple-operator based method, has two different versions, both of them deploy two operators for consistent and conflict formulas respectively. In the first version, one operator decreases the degree of belief of a formula in the conflict set and another increases the degree of belief of a formula belong to the consistent set. In the second version, the operator that deals with the conflict formulas is revised to decrease those formulae that are *indeed* conflict, whilst the rest of the so-called conflict formulas are merged using the second operator, since both sources show the support for them.

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