Unitarity of Nonspherical Minimal Principal Series

The purpose of this talk is to give an exposition of some recent progress on the computation of the unitary dual of reductive groups.

Let $G(\mathbb{R})$ be the set of real points of a linear connected reductive group defined over $\mathbb{R}$. Assume that $G(\mathbb{R})$ is split. Every *minimal principal series* $X(\delta, \nu)$ of $G(\mathbb{R})$ has a canonical completely reducible subquotient denoted by $L(\delta, \nu)$. We want to determine when the constituents of $L(\delta, \nu)$ are unitary.

This talk has two main parts. In the first half, we recall the general theory of intertwining operators for principal series of real split groups. These operators are used to construct the invariant Hermitian form on $L(\delta, \nu)$. In the second half, we introduce the technical notion of *petite* K*-types* for real groups, and use this notiion to relate the unitarizability of $L(\delta, \nu)$ with the spherical unitary dual for certain groups $G^0(\delta)$ attached to δ. (This is joint work with Dan M. Barbasch.)

In the example of $Sp(4, \mathbb{R})$, $L(\delta, \nu)$ is unitary if and only if ν parameterizes a unitary spherical representation for the split group $G(\delta)$. The "only if" statement in this proposition remains true for all nonspherical principal series of *classical* split simple groups. Some exceptions occur for exceptional groups. For instance, there is at least one minimal principal series of F_4 for which the unitary set for $L(\delta, \nu)$ is larger than the spherical unitary dual for the corresponding $G^0(\delta)$.