CRM-PIMS Summer School in Probability
Montreal June 15-July 10 2015

Short Talks

Yoshihiro Abe, Kyoto University

Maximum and minimum of local times for two-dimensional random walk

In this talk, I will consider local times of the simple random walk on the two-dimensional torus at times proportional to the cover time. I will describe the leading orders of the maximum and the minimum of the local times. I will also describe the number of points with large (or small) values of the local times. These are closely related to estimates on the two-dimensional Gaussian free field due to Bolthausen, Deuschel and Giacomin (2001) and Daviaud (2006).

Shuyang Bai, Boston University

Universality of homogeneous polynomial forms and critical limit under strong dependence

There is the following universality result: if a sequence of o-diagonal homogeneous off-diagonal polynomial forms in i.i.d. standard normal random variables converges in distribution to a normal, then the convergence also holds if one replaces these i.i.d. standard normal random variables in the polynomial forms by any independent standardized random variables with uniformly bounded third absolute moment. The result, which was stated for polynomial forms with finite number of terms, can be extended to allow an infinite number of terms in the polynomial forms. Based on a criterion derived from this extended universality result, we prove a central limit theorem for a strongly dependent nonlinear stationary process, whose memory parameter lies at the boundary between short and long memory.

Gianmarco Bet, Eindhoven University of Technology

Depletion-of-points effects in critical queues

Stochastic processes consisting of a random component and a deterministic drift are a key signature of criticality for a wide range of discrete models. We consider a queueing model having only a finite (but large) number of potential customers. We show that for appropriate critical conditions and suitably rescaled, the queue length process converges to a Brownian motion with negative parabolic drift. The same scaling limit arises in describing the component sizes in critical Erdos-Rényi random graphs [1]. The appearance of the negative parabolic drift is related to the so-called depletion-of-points effect (with time, the number of vertices or potential customers decreases). We make this connection explicit and present a more general result in which the Brownian motion is replaced by a stable motion. References: [1] D. Aldous, Brownian excursions, critical random graphs and the multiplicative coalescent, Ann. Probab. 25 (1997), no. 2, 812-854.

José Javier Cerda Hernández, University of Campinas--UNICAMP

Ising model on causal triangulations

Linxiao Chen, Université Paris-Sud

Local limit of cFK-maps via the hamburger-cheeseburger bijection

A critical Fortuin-Kasteleyn random map (or cFK-map for short) of size n is a map M randomly chosen among all the rooted planar maps with n edges, with a probability which is proportional to the partition function of the critical FK-percolation model on M. The cFK-maps form a family of random maps depending on a parameter q>0. When q=1 we recover the now well-known model of uniform planar maps. In 2013 Sheffield constructed a bijection between cFK-maps and a model of random words. In this presentation I will explain how this bijection allows one to construct the local limit of cFK-maps.Work of

master thesis under the supervision of Jérémie Bouttier and Nicolas Curien.
http://arxiv.org/abs/1502.01013

Angus Davidson, Bristol University

Steiner trees in the stochastic mean field model of distance

Consider the complete graph on n nodes with i.i.d. exponential weights of unit mean on the edges. A number of properties of this model have been investigated including first passage percolation, diameter, minimum spanning trees, etc. In particular, Janson showed that the typical distance between two nodes scales as (log n)/n, whereas the diameter (maximum distance between any two nodes) scales as 3(log n)/n. Bollobas et al. showed that, for any fixed k, the weight of the Steiner tree connecting k typical nodes scales as (k-1)(log n)/n, which recovers Janson's result for k=2. We extend this result to show that the worst case k-Steiner tree, over all choices of k nodes, has weight scaling as (2k-1)(log n)/n.

Souvik Dhara, Eindhoven University of Technology

Critical window for Configuration Model and Critical percolation

We investigate the component sizes of the configuration model with degree sequence $mathbf{d}_n$ within the critical scaling window, as well as the related problem of critical percolation on a supercritical configuration model. For the latter, we use that critical percolation on a super-critical configuration model yields a critical configuration model. The results are obtained under the finite third moment assumption of the asymptotic degree distribution, which we believe to be close to necessary. We also have preliminary results describing the dynamics of the component sizes within the percolation critical window. In the critical window, the scaling limit of the suitably scaled ordered component sizes are shown to be equal to the multiplicative coalescent process introduced by Aldous to describe the critical clusters of the Erdös-Renyi random graph.

Laura Eslava, McGill University

High degrees in random recursive trees

An increasing tree is a labelled rooted tree in which labels along any branch from the root are in increasing order. A simple example of random increasing trees are Random recursive trees (RRT's). The classical construction of RRT's involves the sequential addition of vertices, starting from the root: The root is labelled 1 and for i >1, vertex i is connected to a uniformly chosen vertex in [i-1]={1,2, ..., i-1}. In this talk we present a different construction which builds an RRT's on n vertices starting from the leaves (our procedure is essentially Kingman's coalescent). This approach allows us to obtain the limiting distribution of the degrees close to the maximum degree D_n; and to tighten approximations of the limiting distribution for D_n (the latter previously studied by Devroye, and Goh and Schmutz). Future work includes the extension of such bottom-to-top construction to study other distributions of increasing trees (e.g. uniform plane trees). This is joint work with Louigi Addario-Berry.

Laura Florescu, Courant Institute NYU

Spectral thresholds in the bipartite stochastic block model

We consider a bipartite stochastic block model on vertex sets V_1 and V_2 of size n_1 and n_2 respectively, with planted partitions in each, and ask at what densities can spectral algorithms recover the partition of the smaller vertex set. The model was used in cite{feldman2014algorithm} to give a unified algorithm for random planted hypergraph partitioning and planted random k-SAT. When $n_2 gg n_1$, multiple thresholds emerge. We show that the singular vectors of the rectangular adjacency matrix exhibit a localization / delocalization phase transition at edge density $p = tilde Theta(n_1^{-2/3} n_2^{-1/3})$, giving recovery below the threshold and no recovery above. Nevertheless, we propose two simple spectral algorithms, the Diagonal Deletion SVD and Essential SVD, which both recover the partition at density $p = tilde Theta(n_1^{-1/2} n_2^{-1/2})$. Finally, we locate a sharp threshold for detection of the partition, in the sense of the results of Mossel, Neeman, Sly and Massouli'e for the stochastic block model. This gives the best known bounds for planted k-SAT and hypergraph partitioning as well as showing a barrier to further improvement via the reduction to the bipartite block model. Joint work with Will Perkins.

Eric Foxall, University of Victoria

Critical behaviour of the partner model

We consider a stochastic model of infection spread incorporating monogamous partnership dynamics. In previous work, a basic reproduction number R_0 is defined with the property that if $R_0<1$ the infection dies out within $O(log N)$ units of time, while if $R_0>1$ the infection survives for at least $e^{gamma N}$ units of time, for some $gamma>0$. Here we consider the critical case $R_0=1$ and show that the infection dies out within $O(sqrt{N/log log N})$ units of time, and moreover that this estimate is sharp.

Franck Gabriel, LPMA, Jussieu, Paris

Partitions, cumulants and Schur-Weyl duality

The introduction of a geometry on the partition algebra allows to define a new class of cumulants which generalize the classical and free cumulants. This allows to work with random matrices which are invariant by conjugation by the symmetric group. We will explain also how the use of dualities like the Schur-Weyl duality allows to recover the new cumulants and allows to prove general theorems of convergence of matricial multiplicative and additive Lévy processes.

Reza Gheissari, Courant Institute NYU

Conformal Invariance of Spin-Pattern Probabilities on the Planar Ising Model

We study the 2-dimensional Ising model at critical temperature on a simply connected subset Omega_{delta} of the square grid deltamathbb{Z}^{2}. The scaling limit of the critical Ising model is conjectured to be described by Conformal Field Theory. We refine this conjecture into a correspondence between lattice fields of the Ising model and the local fields of CFT. As a central tool towards the proof of this conjecture, we introduce spin pattern probabilities (probabilities of local spin configurations), compute their infinite-volume limits, and prove their conformal covariance at first order. We formulate these probabilities in terms of fermionic observables, which enables the study of their scaling limits.

Ewa Infeld, Dartmouth College

Probabilistic combinatorics and online privacy

The issues of privacy and anonymity on the Internet have led me to a few interesting problems in probabilistic combinatorics, such as maximizing the number of possible ways users of a system can be matched to activities or building the most efficient network that hides a user's identity. In this talk, I will try to convince you that it's not just cryptographers who can change the world through mathematics.

Aukosh Jagannath, Courant Institute NYU

Approximate ultrametricity for random measures and applications to spin glasses

We introduce a notion called “Approximate Ultrametricity” which encapsulates the phenomenology of a sequence of random probability measures having supports that behave like ultrametric spaces insofar as they decompose into nested balls. We provide a sufficient condition for a sequence of random probability measures on the unit ball of a separable Hilbert space to admit such a a decomposition. We also characterize the laws of the measures of the sets in this regime by showing that they converge in law to the weights of a Ruelle Probability Cascade. These results apply to a large class of classical models in mean field spin glasses. In particular, they show that for this class of models, the Gibbs measure admits an approximate decomposition into “pure states” and their “combinations” at large but finite volume, as predicted in the physics literature. We further illustrate the notion of approximate ultrametricity by proving two important conjectures related to mixed p-spin glasses.

Damir Kinzebulatov, University of Toronto

Strong Feller processes with weakly form-bounded vector fields, and L^p-theory of $-Delta + b cdot nabla$

We construct a strong Feller process associated with $-Delta +b cdot nabla$, for a wide class of vector fields $b:mathbb R^d rightarrow mathbb R^d$, $$ | |b|^frac{1}{2} (lambda - Delta)^{-frac{1}{4}} |_{2 to 2} leqslant sqrt{delta} quad text{ for some } lambda=lambda_delta>0. $$ (weakly form-bounded vector fields). The class of weakly form-bounded vector fields is closed with respect to addition (up to the corresponding change in $delta$), and contains as proper subclasses the class of form-bounded vector fields ($supset L^{d,infty}(mathbb R^d,mathbb R^d)$) and the Kato class $mathbf{K}^{d+1}$; thus, weakly form-bounded vector fields can combine different kinds of singularities, e.g.~$|x|^{-1}$ and $(|x|-1)^{-beta}$, $beta<1$. Our construction of the process is a consequence of our L^p-theory of $-Delta +b cdot nabla$ (for $p>d-1$ close to $d-1$). Our starting object is an operator-valued function, which, we prove, coincides with the resolvent of an operator realization of $-Delta + bcdot nabla$, the generator of a holomorphic C_0-semigroup on $L^p(mathbb R^d)$. Then the very form of the operator-valued function yields crucial information about smoothness of the domain of the generator (as a function of the weak form-bound $delta$) needed to construct the process. The proof appeals to the L^p-inequalities between operator $(lambda-Delta)^{frac{1}{2}}$ and ``potential'' $|b|$. {sf http://arxiv.org/abs/1502.07286}

Daniel Kious, EPFL

Local trapping for elliptic random walks in random environments in Z^d

We consider elliptic random walks in i.i.d. random environments on Z^d. The main goal of this paper is to study under which ellipticity conditions local trapping occurs. Our main result is to exhibit an ellipticity criterion for ballistic behavior which extends previously known results. We also show that if the annealed expected exit time of a unit hypercube is infinite then the walk has zero asymptotic velocity.This is a joint work with Alexander Fribergh.

Hanbaek Lyu, Ohio State University

Synchronization of pulse-coupled oscillators on grids and cyclic cellular automata

We propose a simple cellular automata model for discrete-time pulse-coupled oscillators and study its network behavior. We show that our CA model synchronizes arbitrary initial configurations on paths, trees, and with random perturbation, on any connected graphs. In particular, our main result is the following local-global principle for tree networks: for any n ∈ {3,4,5,6}, any n-periodic network on a tree synchronizes arbitrary initial configuration if and only if the maximum degree of the tree is less than the period n. For n=3, we establish a connection between our network model with the 3-color cyclic cellular automata model, invented by David Griffeath and studied by Robert Fisch. We borrow their techniques and show that 3-periodic network on Z^1 clusters with high probability.

Mihai Nica, Courant Institute NYU

TBA

Izumi Okada, Tokyo Institute of Technology

Geometry structures of favorite sites of random walk range

We consider the question: how many times does a simple random walk revisit the most frequently visited site among the inner boundary points? It is known that the number of visits to a most frequently visited site among all of the points has the phase transition between two dimension and higher dimensions. On the other hand, we prove that the corresponding number among the inner boundary does not has it.
In addition, we can state that the most frequently visited site among all of the points does not appear in the inner boundary from some time on with probability one.

Harishchandra Ramadas, University of Washington, Seattle

Mixing of the Noisy Voter Model

The noisy voter model is a simple `interacting particle system' that can serve as a crude stochastic model for the spread of opinions in a human population, or the competition for territory between two species. We study the convergence to equilibrium (`mixing') of this model, and show that this happens extremely fast -- more precisely, on an arbitrary graph with n vertices, the model mixes in time of O(log n), for arbitrarily small values of the `noise parameter'.

Subhabrata Sen, Stanford University

Extremal Cuts of Sparse Random Graphs

We will discuss how ideas from statistical mechanics shed light on properties of extremal cuts of sparse random graphs. We establish that for Erdős-Rényi random graphs with average degree γ, and uniformly random γ-regular graphs on n vertices, with high probability the size of both the Max-Cut and maximum bisection are $n(\frac{\gamma}{4} + \mathsf{P}_*\sqrt{\frac{\gamma}{4}} + o(\sqrt{\gamma})) + o(n)$ while the size of the minimum bisection is $n(\frac{\gamma}{4} - \mathsf{P}_*\sqrt{\frac{\gamma}{4}} + o(\sqrt{\gamma})) + o(n)$. Here P_* is the ground state energy of the SK model.

Erik Thornblad, Uppsala University

Eventual leadership in an infinite Pólya urn

We consider a Pòlya-type urn model with an infinite number of colours. We show that, with probability 1, there is a colour that eventually (and always thereafter) contains more balls than any other colour. In the process we are lead to consider another extension of the original Pólya urn model.

Viktoria Vadon, Budapest University of Technology and Economics

First passage percolation on the Newman-Watts small world model

Small talk is based on my BSc thesis, scientific student paper and article (in preparation) with Dr. Júlia Komjáthy. Newman-Watts small world model is a random graph model, designed to copy some features of real-life networks, such as clustering and small diameter. The graph is constructed by taking a cycle on n vertices, then adding edges randomly in a fashion similar to the Erdős-Rényi random graph model. We studied the case when we add i.i.d. exponential edge weights. Then the typical distances (i.e. shortest weight path between two uniformly chosen vertices) scale as log n, and the smaller order terms are random variables, arising from the branching process we use for approximating the neighborhood of a vertex. The hopcount along this shortest weight path satisfies a CLT. From the typical distances, we can also derive the so-called epidemic curve function for a susceptible-infected-removed epidemic spread on the graph.

Ran Wei, National University of Singapore

Characterization of Long-range Directed Polymer

Directed polymer is a well-built probabilistic model that was first introduced in statistical mechanics. To describe the interaction between polymers and solvents, we consider a random walk on lattices with identically independent increment in some random environment. So far, most study of directed polymer is restricted on simple random walk. In this talk, we will consider a class of more general random walks. To be specific, we study those random walks whose one-step distribution is in the domain of attraction of some α stable law, where $0<\alpha<2$. We will revisit free energy, weak disorder, and strong disorder, which have been well studied in simple random walk case. And then, we will present the dichotomy of weak and strong disorder, central limit theorem under the polymer measures, and some upper bounds for free energy in our long-range version.

Sarah Wolff, Dartmouth College

Random Walks on the BMW Monoid: An Algebraic Approach

We consider the problem of generating random basis elements of a semisimple algebra; namely, the Birman Murakami Wenzl (BMW) monoid basis of the BMW algebra. We present a Metropolis scan algorithm that translates to a natural random walk on the BMW monoid. Interpreting this walk as a left multiplication operator in the BMW algebra allows for analysis using tools from representation theory and Fourier analysis.

Kazuo Yamazaki, Washington State University

3-D stochastic micropolar and magneto-micropolar fluid systems with non-Lipschitz multiplicative noise

Micropolar and magneto-micropolar fluid systems are systems of PDEs that resemble the Navier-Stokes equations and magnetohydrodynamics system but also have distinctive features in particular from the Boussinesq system that makes its mathematical analysis of much interest and great challenge. The results to be discussed include the following: global regularity result for in 2D space with zero angular viscosity in the deterministic case, the existence of a weak martingale solution in 3D space and the unique strong solution in 2D space under a suitable condition on the noise for the stochastic case. If time permits, we extend our discussion onto the stochastic density-dependent Navier-Stokes equations.

Jinjiong Yu, National University of Singapore

Edwards-Wilkinson Fluctuations in the Howitt-Warren flows

We study current fluctuations in a one-dimensional interacting particle system known as the dual smoothing process that is dual to random motions in a Howitt-Warren flow. The Howitt-Warren flow can be regarded as the transition kernels of a random motion in a continuous space-time random environment. It turns out that the current fluctuations of the dual smoothing process fall in the Edwards-Wilkinson universality class, where the fluctuations occur on the scale $t^{1/4}$ and the limit is a universal Gaussian process. Along the way, we prove a quenched invariance principle for a random motion in the Howitt-Warren flow. Meanwhile, the centered quenched mean process of the random motion also converges on the scale $t^{1/4}$, where the limit is another universal Gaussian process.

Serena Yuan, Courant Institute NYU

The Exponential Intersection Tail Property of Lattices and Other Graphs

The exponential intersection tail is a method for proving that the critical probability of a graph $p_c(G) <1$, based on an idea of Kesten. There are certain rooted graphs that admit the exponential intersection tail property which include the d-dimensional lattice for $d >= 4$, the binary tree, nonamenable graphs, and more. We will look at these examples and we will look at how certain graphs admit this property. We will additionally look at how this method is used in percolation of graphs.