Lie racks of type D
Unipotent conjugacy classes in finite groups of Lie type

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Joint work with N. Andruskiewitsch and G. Carnovale.

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We deal concretely with the case when G is a finite group of Lie type.

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We say that a rack is *simple* if $|X| > 1$ and any rack epimorphism $X \twoheadrightarrow Y$ is bijective or $|Y| = 1$.

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Determine all simple racks of type D

Finite simple racks where classified in [AG] [1] and [J][2].

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If L is a finite group of Lie type, we call a (twisted) conjugacy class of type (L^t, u) a Lie rack.

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Thus, non-trivial (twisted) conjugacy classes in simple groups of Lie type are Lie racks.

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Since $x_u \in K$, $\mathcal{O}_{x_u}^K$ is a subrack of $\mathcal{O}_x^G$ and we can reduce our study to the case when x is either unipotent or semisimple.

Theorem

Assume q is odd and let $u \in SL(n, q)$ be a nontrivial unipotent element. Then $\mathcal{O}_u$ is of type D if and only if $n \neq 2$ or $n = 2$ and q is a square different from 9.

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Note: Even if the conjugacy classes $\mathcal{O}_u^{SL(2,q)}$ are not of type D for q not a square, by [FGV][4], $\mathcal{O}_u^{SL(2,q)}$ collapses for q odd.

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If $x \in GL(n,q)$ is of type (n), we call it *regular*.

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Sketch of the proof:

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Let H be the image of the natural embedding of $SL(\lambda_1, q) \times \cdots \times SL(\lambda_k, q)$ into $SL(n,q)$ containing u.

Lemma 1

Let $u \in SL(n, q)$ be a unipotent element of type $(\lambda_1, \dots, \lambda_k)$ with $u_i \in \mathbb{F}_q^{\lambda_i \times \lambda_i}$. If $\mathcal{O}_{u_i}^{SL(\lambda_i, q)}$ is of type D, then $\mathcal{O}_u$ is of type D.

Sketch of the proof: Assume $\mathcal{O}_{u_1}^{SL(\lambda_1, q)}$ is of type D.

Let H be the image of the natural embedding of $SL(\lambda_1, q) \times \cdots \times SL(\lambda_k, q)$ into $SL(n, q)$ containing u.

Then $\mathcal{O}_u^H$ is a subrack of $\mathcal{O}_u^{SL(n,q)}$ and the projection of H onto its first factor induces a rack epimorphism of $\mathcal{O}_u^H$ onto $\mathcal{O}_{u_1}^{SL(\lambda_1, q)}$.

Lemma 2

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Corollary

Assume q is odd and let $u \in SL(n, q)$ be a unipotent element of type $(\lambda_1, \dots, \lambda_k)$.If $\lambda_1 > 2$, then the conjugacy class $\mathcal{O}_u$ is of type D.

Lemma 3

Assume q is odd. Let $u \in SL(n, q)$ be a unipotent element of type $(2, 2)$ or $(2, 1)$. Then the conjugacy class $\mathcal{O}_u$ is of type D.

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Case $(2, 2)$ in $SL(4, 3)$ done with GAP.

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Lemma 4

Assume $q = p^m$ is odd. Then any non-trivial unipotent conjugacy class in $SL(2, q)$ is of type D if and only if q is a square different from 9.

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If the Dynkin diagram of $\mathbb{G}$ has at least a component of rank greater than 1, then every regular unipotent class of $\mathbb{G}^F$ is of type D.

THANK YOU

MERCI

GRACIAS

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