Some indecomposable representations in physics

Dr. Hubert de Guise

Department of Physics
Lakehead University
Thunder Bay, Ontario, Canada

Indecomposable representations

- In the paper *The Future of Atomic Physics* (Int. J. Theor. Phys. **23** (1984) 677), Dirac call matrices of the form

$$T = \begin{pmatrix} A & L \\ 0 & B \end{pmatrix}$$

pathological representations.

Indecomposable representations

- In the paper *The Future of Atomic Physics* (Int. J. Theor. Phys. **23** (1984) 677), Dirac call matrices of the form

$$T = \begin{pmatrix} A & L \\ 0 & B \end{pmatrix}$$

  pathological representations.
- A and B are representation of the Lorentz group $\sim SO(3,1)$.

Indecomposable representations

- In the paper *The Future of Atomic Physics* (Int. J. Theor. Phys. **23** (1984) 677), Dirac call matrices of the form

$$T = \begin{pmatrix} A & L \\ 0 & B \end{pmatrix}$$

pathological representations.
- A and B are representation of the Lorentz group $\sim SO(3,1)$.
  - The Lorentz group is the group of transformations that leave the inner product $-(ct)^2 + x^2 + y^2 + z^2$ invariant.

Indecomposable representations

- In the paper *The Future of Atomic Physics* (Int. J. Theor. Phys. **23** (1984) 677), Dirac call matrices of the form

$$T = \begin{pmatrix} A & L \\ 0 & B \end{pmatrix}$$

pathological representations.

- A and B are representation of the Lorentz group $\sim SO(3,1)$.
  - The Lorentz group is the group of transformations that leave the inner product $-(ct)^2 + x^2 + y^2 + z^2$ invariant.
  - This is the basic invariant of the theory of special relativity

Indecomposable representations in physics

Quoting Dirac:

“We have a theory in which infinite factors appear when we try to solve the equations. These infinite factors are swept into a renormalization procedure. The result is a theory which is not based on strict mathematics, but is rather a set of working rules.

Indecomposable representations in physics

Quoting Dirac:

“We have a theory in which infinite factors appear when we try to solve the equations. These infinite factors are swept into a renormalization procedure. The result is a theory which is not based on strict mathematics, but is rather a set of working rules.

Many people are happy with this situation because it has a limited amount of success. But this is not good enough.

Indecomposable representations in physics

Quoting Dirac:

"We have a theory in which infinite factors appear when we try to solve the equations. These infinite factors are swept into a renormalization procedure. The result is a theory which is not based on strict mathematics, but is rather a set of working rules.

Many people are happy with this situation because it has a limited amount of success. But this is not good enough. *Physics must be based on strict mathematics.*

Indecomposable representations in physics

Quoting Dirac:

"We have a theory in which infinite factors appear when we try to solve the equations. These infinite factors are swept into a renormalization procedure. The result is a theory which is not based on strict mathematics, but is rather a set of working rules.

Many people are happy with this situation because it has a limited amount of success. But this is not good enough. *Physics must be based on strict mathematics.* One can conclude that the fundamental ideas of the existing theory are wrong. A new mathematical basis is needed."

Indecomposable representations

- $T = \begin{pmatrix} A & L \\ 0 & B \end{pmatrix}$ acts on the vector $\psi = \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix}$ with

$$T\psi \to \psi' : \quad \begin{array}{rcl} \psi'_A & = & A\psi_A + L\psi_B \\ \psi'_B & = & B\psi_B \end{array}$$

Indecomposable representations

- $T = \begin{pmatrix} A & L \\ 0 & B \end{pmatrix}$ acts on the vector $\psi = \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix}$ with

$$T\psi \to \psi' : \quad \begin{array}{rcl} \psi'_A & = & A\psi_A + L\psi_B \\ \psi'_B & = & B\psi_B \end{array}$$

- Models the disintegration of an atom represented by ψ_A into a new state of this atom $A\psi_A$ plus another emitted particle represented by ψ_B.

In this talk:

Indecomposable representations

- $T = \begin{pmatrix} A & L \\ 0 & B \end{pmatrix}$ acts on the vector $\psi = \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix}$ with

$$T\psi \to \psi' : \quad \begin{array}{rcl} \psi'_A & = & A\psi_A + L\psi_B \\ \psi'_B & = & B\psi_B \end{array}$$

- Models the disintegration of an atom represented by ψ_A into a new state of this atom $A\psi_A$ plus another emitted particle represented by ψ_B.

In this talk:

Indecomposable representations

- $T = \begin{pmatrix} A & L \\ 0 & B \end{pmatrix}$ acts on the vector $\psi = \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix}$ with

$$T\psi \to \psi' : \quad \begin{array}{rcl} \psi'_A & = & A\psi_A + L\psi_B \\ \psi'_B & = & B\psi_B \end{array}$$

- Models the disintegration of an atom represented by ψ_A into a new state of this atom $A\psi_A$ plus another emitted particle represented by ψ_B.

In this talk:

- Eventually discuss (finite dimensional) indecomposable representations of Poincaré.
  - it contains Lorentz $\sim SO(3,1)$ as a subgroup
  - it also contains 4 space-time translation
  - representations are “naturally” of the indecomposable type

Indecomposable representations

- $T = \begin{pmatrix} A & L \\ 0 & B \end{pmatrix}$ acts on the vector $\psi = \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix}$ with

$$T\psi \to \psi' : \quad \begin{array}{rcl} \psi'_A & = & A\psi_A + L\psi_B \\ \psi'_B & = & B\psi_B \end{array}$$

- Models the disintegration of an atom represented by ψ_A into a new state of this atom $A\psi_A$ plus another emitted particle represented by ψ_B.

In this talk:

- Eventually discuss (finite dimensional) indecomposable representations of Poincaré.
  - it contains Lorentz $\sim SO(3,1)$ as a subgroup
  - it also contains 4 space-time translation
  - representations are "naturally" of the indecomposable type
- Start by discussing $E(2)$: group of Euclidean motion in 2d
  - it contains rotations in the plane $\sim SO(2)$ as a subgroup
  - it also contains translations in the plane

Why pathological?

According to *Wigner's theorem*:

Why pathological?

According to *Wigner's theorem*:

- Transformations of quantum mechanical states should be unitary or antiunitary transformations.

Why pathological?

According to *Wigner's theorem*:

- Transformations of quantum mechanical states should be unitary or antiunitary transformations.
- This is required to preserve probabilities:
    - The probability of finding a system in a state $|\chi\rangle$ given it is in a state $|\xi\rangle$ is $|\langle\,\chi\,|\,\xi\,\rangle|^2$.
    - If we transform the system - translate or rotate the axes etc. - by some operation T, then

$$|\chi\rangle \to |\chi'\rangle = |T\chi\rangle\,, \qquad |\xi\rangle \to |\xi'\rangle = |T\zeta\rangle$$

    - This should not change the probabilities: $|\langle\,T\chi\,|\,T\xi\,\rangle|^2 = |\langle\,\chi\,|\,\xi\,\rangle|^2$.

E(2)

The group $E(2)$ is the group of Euclidean motion in the plane.

E(2)

The group $E(2)$ is the group of Euclidean motion in the plane.

- One rotation in the plane.

$$(x, y) \to (x \cos\theta - y \sin\theta, y \cos\theta + x \sin\theta)$$

E(2)

The group $E(2)$ is the group of Euclidean motion in the plane.

- One rotation in the plane.

$$(x, y) \to (x\cos\theta - y\sin\theta, y\cos\theta + x\sin\theta)$$

- Two commuting translations

$$(x, y) \to (x + a, y + b)$$

E(2)

The group $E(2)$ is the group of Euclidean motion in the plane.

- One rotation in the plane.

$$(x, y) \to (x\cos\theta - y\sin\theta, y\cos\theta + x\sin\theta)$$

- Two commuting translations

$$(x, y) \to (x + a, y + b)$$

- "Natural" representation

$$\pi : T(\theta; a, b) \mapsto \left(\begin{array}{cc|c} \cos\theta & -\sin\theta & a \\ \sin\theta & \cos\theta & b \\ \hline 0 & 0 & 1 \end{array}\right)$$

E(2)

The group $E(2)$ is the group of Euclidean motion in the plane.

- One rotation in the plane.

$$(x, y) \to (x\cos\theta - y\sin\theta, y\cos\theta + x\sin\theta)$$

- Two commuting translations

$$(x, y) \to (x + a, y + b)$$

- "Natural" representation

$$\pi : T(\theta; a, b) \mapsto \left(\begin{array}{cc|c} \cos\theta & -\sin\theta & a \\ \sin\theta & \cos\theta & b \\ \hline 0 & 0 & 1 \end{array}\right)$$

- This is an example of an indecomposable representation:
  - non-unitary

Basic examples

- Go to complex coordinates $z = x + iy$

$$\pi : T(\theta; x, y) \mapsto \begin{pmatrix} \mathrm{e}^{i\theta} & z \\ 0 & 1 \end{pmatrix}$$

$$\tilde{\pi} : T(\theta; x, y) \mapsto \begin{pmatrix} 1 & 0 \\ \bar{z} & \mathrm{e}^{-i\theta} \end{pmatrix} \qquad \bar{z} = x - iy$$

Basic examples

- Go to complex coordinates $z = x + iy$

$$\pi : T(\theta; x, y) \mapsto \begin{pmatrix} \mathrm{e}^{i\theta} & z \\ 0 & 1 \end{pmatrix}$$

$$\tilde{\pi} : T(\theta; x, y) \mapsto \begin{pmatrix} 1 & 0 \\ \bar{z} & \mathrm{e}^{-i\theta} \end{pmatrix} \qquad \bar{z} = x - iy$$

- Composition rule

$$T(\theta_1; z_1) \cdot T(\theta_2; z_2) = T(\theta_1 + \theta_2; z_1 + z_2 \mathrm{e}^{i\theta})$$

Basic setup

$\pi(T(\theta, z))$ a rep. of $E(2)$ on V (finite dim.) ($\bar{z}$ implicit).

Basic setup

$\pi(T(\theta, z))$ a rep. of $E(2)$ on V (finite dim.) ($\bar{z}$ implicit).

- Decompose $V = \oplus W_k$ into a finite sum of "weight" subspaces W_k.

Basic setup

$\pi(T(\theta,z))$ a rep. of $E(2)$ on V (finite dim.) ($\bar{z}$ implicit).

- Decompose $V = \oplus W_k$ into a finite sum of "weight" subspaces W_k.
- W_k is an $SO(2)$ subspace: if $v \in W_k$ then

$$\pi(T(\theta,0))\,v = \mathrm{e}^{-ik\theta}v$$

Basic setup

$\pi(T(\theta, z))$ a rep. of $E(2)$ on V (finite dim.) ($\bar{z}$ implicit).

- Decompose $V = \oplus W_k$ into a finite sum of “weight” subspaces W_k.
- W_k is an $SO(2)$ subspace: if $v \in W_k$ then

$$\pi(T(\theta, 0))\, v = \mathrm{e}^{-ik\theta} v$$

- Since $T(\theta + 2\pi; 0) = T(\theta; 0) \Rightarrow k \in \mathbb{Z}$

Basic setup

$\pi(T(\theta, z))$ a rep. of $E(2)$ on V (finite dim.) ($\bar{z}$ implicit).

- Decompose $V = \oplus W_k$ into a finite sum of "weight" subspaces W_k.
- W_k is an $SO(2)$ subspace: if $v \in W_k$ then

$$\pi(T(\theta, 0))\, v = \mathrm{e}^{-ik\theta} v$$

- Since $T(\theta + 2\pi; 0) = T(\theta; 0) \Rightarrow k \in \mathbb{Z}$
- Generators realized as

$$\ell_0 = i\frac{\partial}{\partial\theta}\pi(T(\theta, z)) \quad p_+ = \frac{\partial}{\partial z}\pi(T(\theta, z)) \quad p_- = \frac{\partial}{\partial \bar{z}}\pi(T(\theta, z))\,.$$

Basic setup

$\pi(T(\theta, z))$ a rep. of $E(2)$ on V (finite dim.) ($\bar{z}$ implicit).

- Decompose $V = \oplus W_k$ into a finite sum of "weight" subspaces W_k.
- W_k is an $SO(2)$ subspace: if $v \in W_k$ then

$$\pi(T(\theta, 0))\, v = \mathrm{e}^{-ik\theta} v$$

- Since $T(\theta + 2\pi; 0) = T(\theta; 0) \Rightarrow k \in \mathbb{Z}$
- Generators realized as

$$\ell_0 = i\frac{\partial}{\partial\theta}\pi(T(\theta, z)) \quad p_+ = \frac{\partial}{\partial z}\pi(T(\theta, z)) \quad p_- = \frac{\partial}{\partial \bar{z}}\pi(T(\theta, z))\,.$$

- Commutators: $[p_+, p_-] = 0\,, \quad [\ell_0, p_\pm] = \pm p\pm$.

Basic setup

$\pi(T(\theta, z))$ a rep. of $E(2)$ on V (finite dim.) ($\bar{z}$ implicit).

- Decompose $V = \oplus W_k$ into a finite sum of "weight" subspaces W_k.
- W_k is an $SO(2)$ subspace: if $v \in W_k$ then

$$\pi(T(\theta, 0))\, v = \mathrm{e}^{-ik\theta} v$$

- Since $T(\theta + 2\pi; 0) = T(\theta; 0) \Rightarrow k \in \mathbb{Z}$
- Generators realized as

$$\ell_0 = i\frac{\partial}{\partial\theta}\pi(T(\theta, z)) \quad p_+ = \frac{\partial}{\partial z}\pi(T(\theta, z)) \quad p_- = \frac{\partial}{\partial \bar{z}}\pi(T(\theta, z))\,.$$

- Commutators: $[p_+, p_-] = 0\,, \quad [\ell_0, p_\pm] = \pm p\pm$.
- Ladder action: $\ell_0 W_k \subseteq W_k; \quad p_\pm W_k \subseteq W_{k\pm1}$

Basic setup

$\pi(T(\theta, z))$ a rep. of $E(2)$ on V (finite dim.) ($\bar{z}$ implicit).

- Decompose $V = \oplus W_k$ into a finite sum of "weight" subspaces W_k.
- W_k is an $SO(2)$ subspace: if $v \in W_k$ then

$$\pi(T(\theta, 0))\, v = \mathrm{e}^{-ik\theta} v$$

- Since $T(\theta + 2\pi; 0) = T(\theta; 0) \Rightarrow k \in \mathbb{Z}$
- Generators realized as

$$\ell_0 = i\frac{\partial}{\partial\theta}\pi(T(\theta, z)) \quad p_+ = \frac{\partial}{\partial z}\pi(T(\theta, z)) \quad p_- = \frac{\partial}{\partial\bar{z}}\pi(T(\theta, z))\,.$$

- Commutators: $[p_+, p_-] = 0\,, \quad [\ell_0, p_\pm] = \pm p \pm$.
- Ladder action: $\ell_0 W_k \subseteq W_k; \quad p_\pm W_k \subseteq W_{k\pm1}$
- Finite dimensional: $\exists M$ s.t. $p_+ W_M = 0$ and $\exists N$ s.t. $p_- W_N = 0$.

String representations

Assume all W_k are of dim=1.

String representations

Assume all W_k are of dim=1.

- Every 1d rep of $E(2)$ is: $\chi_k : T(\theta; z) \mapsto \mathrm{e}^{-ik\theta} \quad k \in \mathbb{Z}$.

String representations

Assume all W_k are of dim=1.

- Every 1d rep of $E(2)$ is: $\chi_k : T(\theta; z) \mapsto \mathrm{e}^{-ik\theta} \quad k \in \mathbb{Z}$.
- $p_+ p_- = 0$
  - Choose $\varphi_k \in W_k$ arbitrary $\neq 0$ vector

$$
\begin{aligned}
p_+ p_- \varphi_k &= \alpha_k \varphi_k && W_k \text{ is 1-dim.} \\
(p_+ p_-)^n \varphi_k &= \alpha_k^n \varphi_k && \\
&= (p_+)^n (p_-)^n \varphi_k && p_+, p_- \text{ commute.} \\
&= 0 && \text{for } n > N \text{ s.t. } p_-^N \varphi_k = 0.
\end{aligned}
$$

  - Thus $\alpha_k^N = 0 \Rightarrow \alpha_k = 0$.

This means:

- Some graphs are allowed:

$$p_-\varphi_{m+1} = 0 \qquad p_+\varphi_m \neq 0$$

$$p_-\varphi_{m+1} \neq 0 \qquad p_+\varphi_m = 0$$

$$p_-\varphi_{m+1} = 0 \qquad p_+\varphi_m = 0$$

$p_+p_- = 0$

This means:

- Some graphs are allowed:

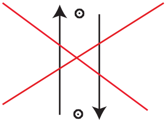
$$p_-\varphi_{m+1} = 0 \qquad p_+\varphi_m \neq 0$$

$$p_-\varphi_{m+1} \neq 0 \qquad p_+\varphi_m = 0$$

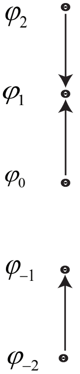
$$p_-\varphi_{m+1} = 0 \qquad p_+\varphi_m = 0$$

- Graphs cannot contain

$$p_-\varphi_{m+1} \neq 0 \qquad p_+\varphi_m \neq 0$$

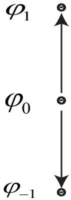


Example: a reducible representation



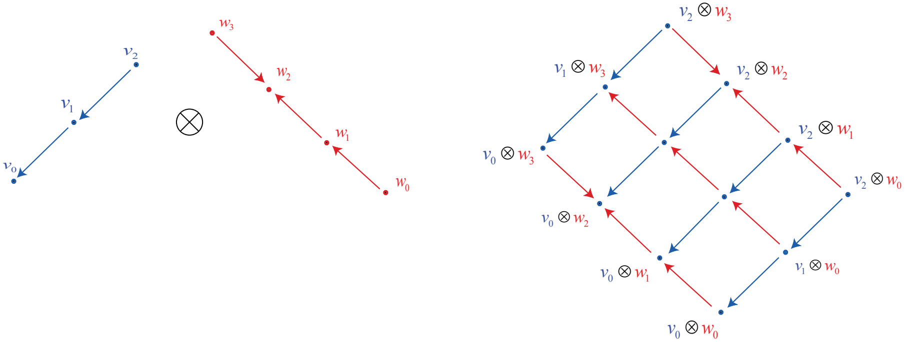
$$T(\theta, z) \mapsto \left(\begin{array}{ccc|cc} \mathrm{e}^{2i\theta} & 0 & 0 & & \\ \mathrm{e}^{2i\theta}\,\bar{z} & \mathrm{e}^{i\theta} & z & & \\ 0 & 0 & 1 & & \\ \hline & & & \mathrm{e}^{-i\theta} & \mathrm{e}^{-2i\theta}\,z \\ & & & 0 & \mathrm{e}^{-2i\theta} \end{array}\right)$$

Example: the "natural" representation



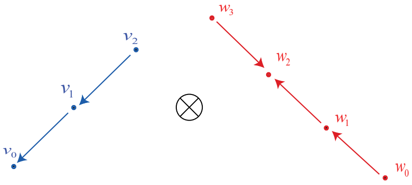
$$T(\theta, z) \mapsto \begin{pmatrix} e^{i\theta} & z & 0 \\ 0 & 1 & 0 \\ & \bar{z} & e^{-i\theta} \end{pmatrix}$$

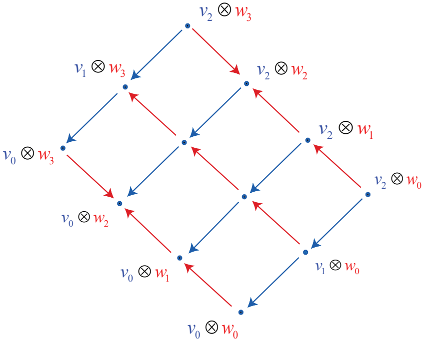
Tensoring strings



$$P_+ P_- \, v_2 \otimes w_0 = v_1 \otimes w_1 \neq 0$$

Tensoring strings

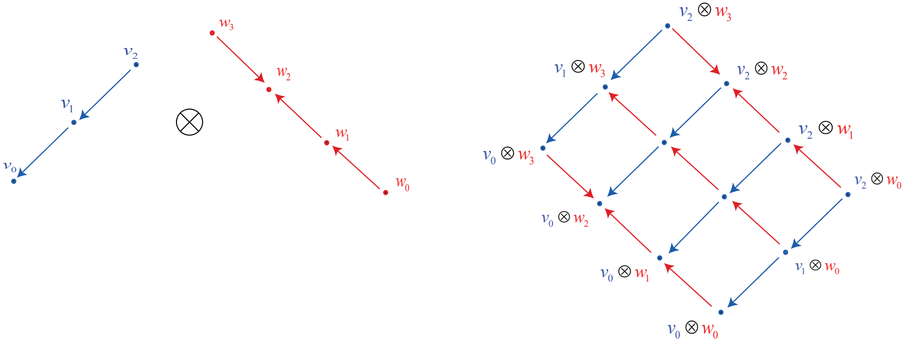




$$P_+ P_- \, v_2 \otimes w_0 = v_1 \otimes w_1 \neq 0$$

- There are weight multiplicities

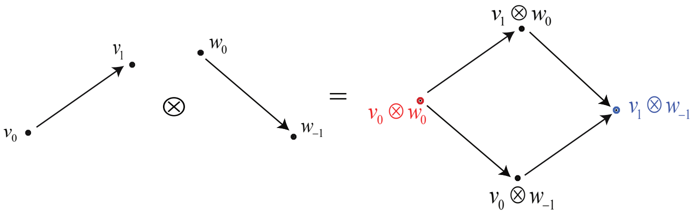
Tensoring strings



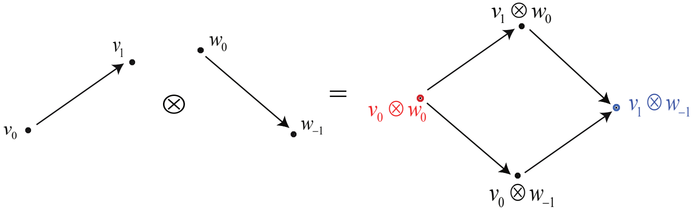
$$P_+ P_- \; v_2 \otimes w_0 = v_1 \otimes w_1 \neq 0$$

- There are weight multiplicities
- $p_+ p_-$ nilpotent.

Parallelograms

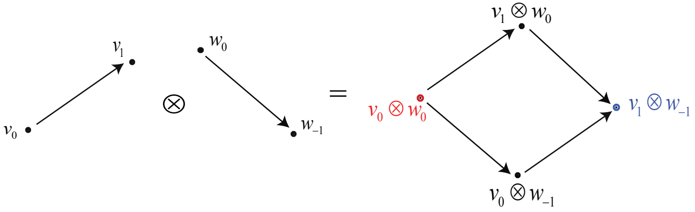


Parallelograms



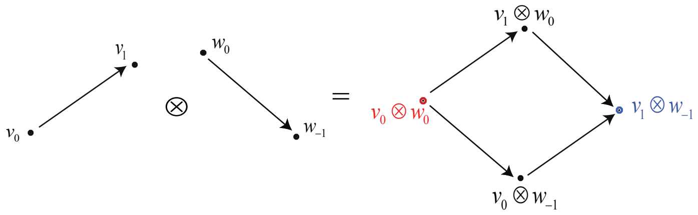
Claim: it is indecomposable.

Parallelograms



Claim: it is indecomposable.
Proof: Assume it does: $V = V_1 \oplus V_2$.

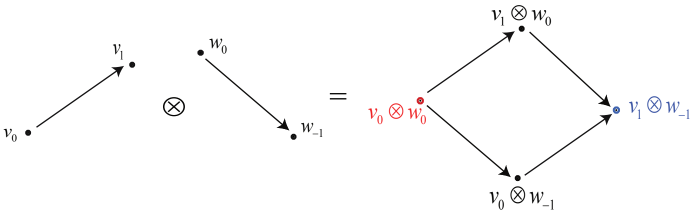
Parallelograms



Claim: it is indecomposable.
Proof: Assume it does: $V = V_1 \oplus V_2$.

- Pick $v = a\, v_0 \otimes w_0 + b\, v_1 \otimes w_{-1} \in V_1$ with $a \neq 0$.

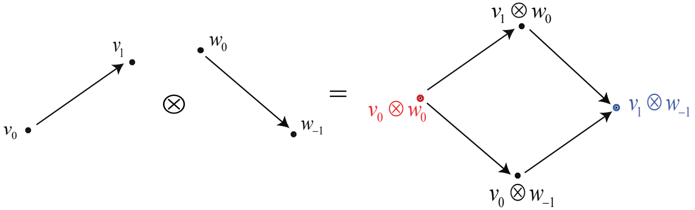
Parallelograms



Claim: it is indecomposable.
Proof: Assume it does: $V = V_1 \oplus V_2$.

- Pick $v = a\, v_0 \otimes w_0 + b\, v_1 \otimes w_{-1} \in V_1$ with $a \neq 0$.
- $p_+ p_- v = a\, v_1 \otimes w_{-1} \in V_1 \Rightarrow v_1 \otimes w_{-1} \in V_1$

Parallelograms

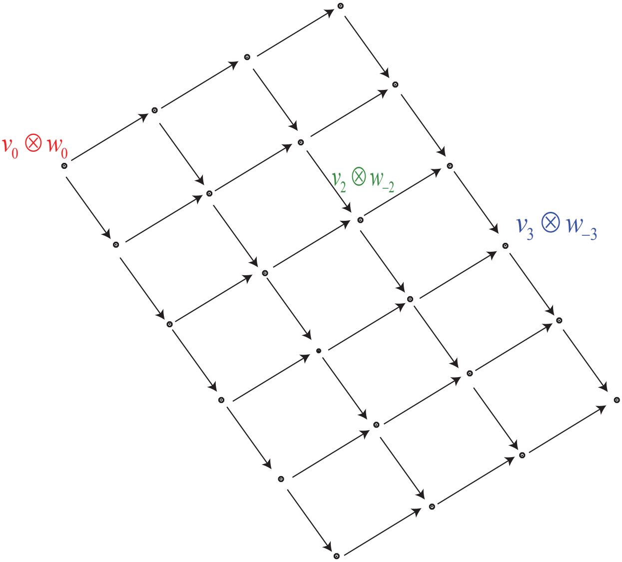


Claim: it is indecomposable.
Proof: Assume it does: $V = V_1 \oplus V_2$.

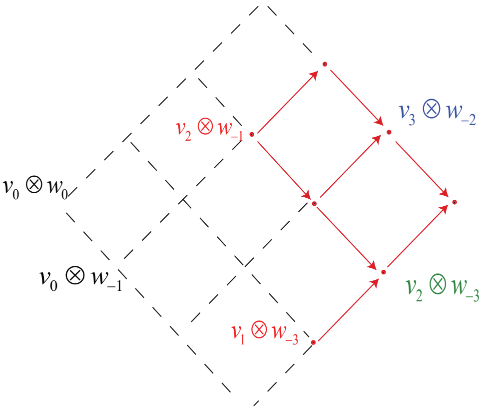
- Pick $v = a\, v_0 \otimes w_0 + b\, v_1 \otimes w_{-1} \in V_1$ with $a \neq 0$.
- $p_+ p_- v = a\, v_1 \otimes w_{-1} \in V_1 \Rightarrow v_1 \otimes w_{-1} \in V_1$
- Thus $v - b\, v_1 \otimes w_{-1} = a\, v_0 \otimes w_0 \in V_1 \Rightarrow v_0 \otimes w_0 \in V_1$.

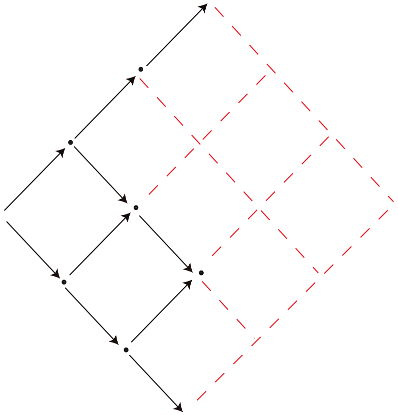
Parallelograms

Likewise, this parallelogram is indecomposable:



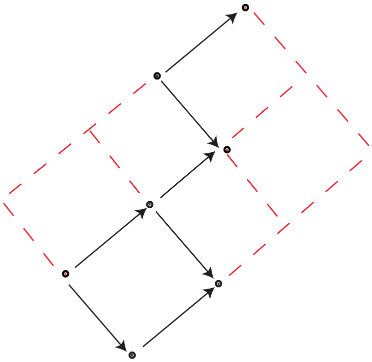
Subrepresentations and Quotients

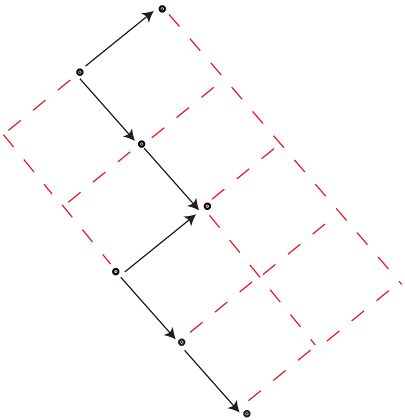




are also indecomposable.

Combinations and general strings





are also indecomposable. Note this last is a string.

Extension to Poincaré

Can this be adapted to Poincaré ?

$e(2) \sim [\mathbb{R}^2]so(2)$	$\mathfrak{p}_{3,1} \sim [\mathbb{R}^4]so(3,1)$
Rotations $so(2)$ irreps are 1d	spacetime rotations $so(3,1)$ irreps are finite and ∞ dim.
translations $\mathbb{R}^2$ $\{p_x p_y\}$ $p_\pm$ transform by 1d irreps invariant: $P \cdot P = p_+ p_- = 0$	spacetime translations $\mathbb{R}^4$ $\{p_x, p_y, p_z, E/c\}$ P_i transform by 4-dim irrep invariant: $P \cdot P = -(mc)^2$ $= p_x^2 + p_y^2 + p_z^2 - (E/c)^2.$

$so(3,1)_{\mathbb{C}} \to sl(2,\mathbb{C}) \oplus sl(2,\mathbb{C})$

- *Finite dimensional* reps of $so(3,1)$ are reps of $sl(2,\mathbb{C}) \oplus sl(2,\mathbb{C})$.

$so(3,1)_{\mathbb{C}} \to sl(2,\mathbb{C}) \oplus sl(2,\mathbb{C})$

- *Finite dimensional* reps of $so(3,1)$ are reps of $sl(2,\mathbb{C}) \oplus sl(2,\mathbb{C})$.
- denoted by two integers (λ, μ)

$so(3,1)_{\mathbb{C}} \to sl(2,\mathbb{C}) \oplus sl(2,\mathbb{C})$

- *Finite dimensional* reps of $so(3,1)$ are reps of $sl(2,\mathbb{C}) \oplus sl(2,\mathbb{C})$.
- denoted by two integers (λ, μ)
- dim $V(\lambda, \mu) = (\lambda + 1)(\mu + 1)$

$so(3,1)_{\mathbb{C}} \to sl(2,\mathbb{C}) \oplus sl(2,\mathbb{C})$

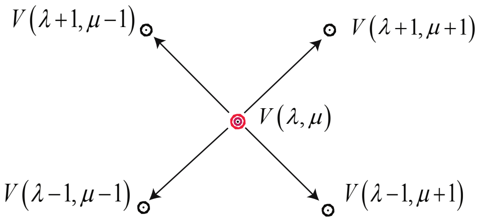
- *Finite dimensional* reps of $so(3,1)$ are reps of $sl(2,\mathbb{C}) \oplus sl(2,\mathbb{C})$.
- denoted by two integers (λ, μ)
- dim $V(\lambda,\mu) = (\lambda+1)(\mu+1)$
- The action of P_i on $\varphi \in V(\lambda,\mu)$:

$$
\begin{aligned}
P_i\varphi \quad \subset \quad & V(\lambda+1,\mu-1) \oplus V(\lambda+1,\mu+1) \\
& \oplus V(\lambda-1,\mu-1) \oplus V(\lambda-1,\mu+1)
\end{aligned}
$$

$so(3,1)_{\mathbb{C}} \to sl(2,\mathbb{C}) \oplus sl(2,\mathbb{C})$

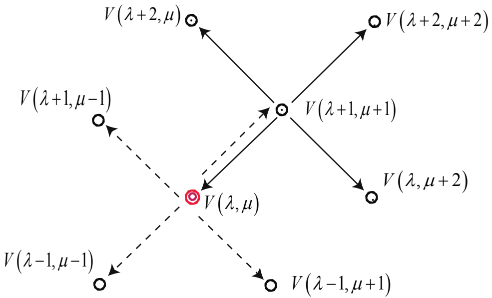
- *Finite dimensional* reps of $so(3,1)$ are reps of $sl(2,\mathbb{C}) \oplus sl(2,\mathbb{C})$.
- denoted by two integers (λ, μ)
- dim $V(\lambda,\mu) = (\lambda+1)(\mu+1)$
- The action of P_i on $\varphi \in V(\lambda,\mu)$:

$$\begin{aligned} P_i\varphi \quad \subset \quad & V(\lambda+1,\mu-1) \oplus V(\lambda+1,\mu+1) \\ & \oplus V(\lambda-1,\mu-1) \oplus V(\lambda-1,\mu+1) \end{aligned}$$



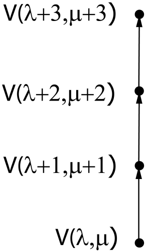
- In general, for $\varphi, \in V(\lambda,\mu)$:

$$\begin{aligned} P_i P_k\, \varphi &= \varphi' \; + \text{pieces in other } V(\lambda',\mu') \\ P\cdot P\, \varphi &= \alpha\varphi \end{aligned}$$



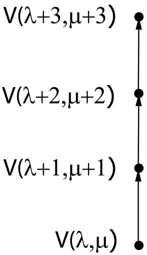
String representations in $\mathfrak{p}(3,1)$

- Naive generalization of (raising) string representations:
  - Suppose rep has a lowest weight $\psi \in V(\lambda, \mu)$.
  - P_i maps states in $V(\lambda', \mu') \to V(\lambda' + 1, \mu' + 1)$.
  - V is indecomposable



String representations in $\mathfrak{p}(3,1)$

- Naive generalization of (raising) string representations:
  - Suppose rep has a lowest weight $\psi \in V(\lambda, \mu)$.
  - P_i maps states in $V(\lambda', \mu') \to V(\lambda'+1, \mu'+1)$.
  - V is indecomposable



- Slightly less naive generalization:
  - P_i maps vectors $\in V(\lambda', \mu')$ to one of

$$V(\lambda', \mu') \to \begin{cases} V(\lambda'+1, \mu'-1) & \text{or} \\ V(\lambda'-1, \mu'+1) & \text{or} \\ V(\lambda'-1, \mu'-1) & \end{cases}$$

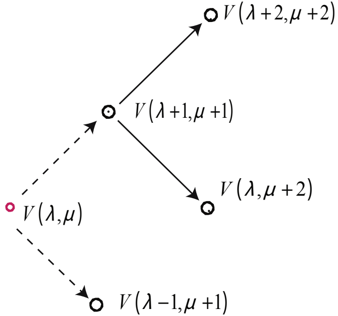
  - $V(\lambda_1, \mu_1) \xrightarrow{P_i} V(\lambda_2, \mu_2) \xrightarrow{P_j} V(\lambda_3, \mu_3)$ with

    $V(\lambda_3, \mu_3) \neq V(\lambda_1, \mu_1)$
  - Still indecomposable.

String representations in $\mathfrak{p}(3,1)$

- Even less naive generalization:

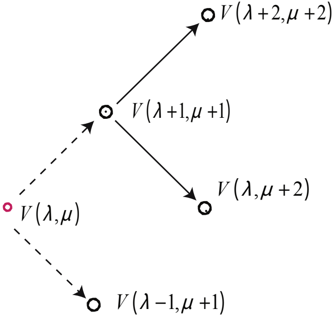
$$V(\lambda', \mu') \xrightarrow{P_i} V(\lambda'+1, \mu'+1)$$
$$\oplus V(\lambda'-1, \mu'+1)$$



String representations in $\mathfrak{p}(3,1)$

- Even less naive generalization:

$$V(\lambda', \mu') \xrightarrow{P_i} V(\lambda'+1, \mu'+1)$$
$$\oplus V(\lambda'-1, \mu'+1)$$

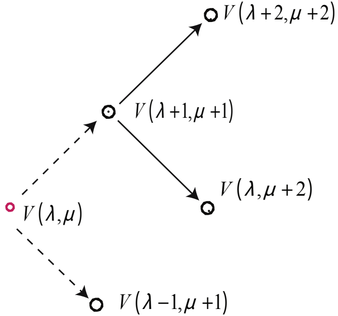


- The notion of “raising” or “lowering” action is formally lost, although the map still defines a direction.

String representations in $\mathfrak{p}(3,1)$

- Even less naive generalization:

$$V(\lambda', \mu') \xrightarrow{P_i} V(\lambda' + 1, \mu' + 1)$$
$$\oplus V(\lambda' - 1, \mu' + 1)$$



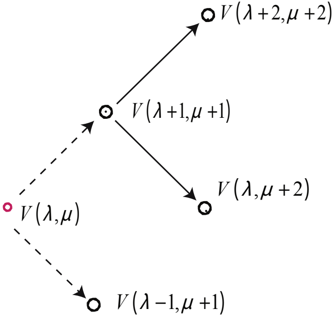
- The notion of “raising” or “lowering” action is formally lost, although the map still defines a direction.
- A string should not contains the sequence of maps $V(\lambda, \mu) \to V(\lambda', \mu') \to V(\lambda, \mu)$.

String representations in $\mathfrak{p}(3,1)$

- Even less naive generalization:

$$V(\lambda', \mu') \xrightarrow{P_i} V(\lambda'+1, \mu'+1)$$

$$\oplus V(\lambda'-1, \mu'+1)$$

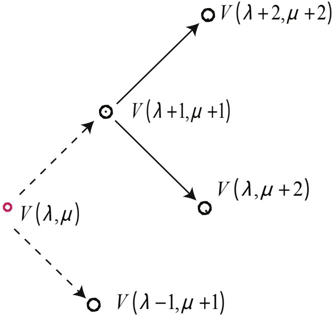


- The notion of “raising” or “lowering” action is formally lost, although the map still defines a direction.
- A string should not contains the sequence of maps $V(\lambda, \mu) \to V(\lambda', \mu') \to V(\lambda, \mu)$.
- *Probably* indecomposable.

String representations in $\mathfrak{p}(3,1)$

- Even less naive generalization:

$$V(\lambda', \mu') \xrightarrow{P_i} V(\lambda'+1, \mu'+1) \oplus V(\lambda'-1, \mu'+1)$$



- The notion of "raising" or "lowering" action is formally lost, although the map still defines a direction.
- A string should not contains the sequence of maps $V(\lambda, \mu) \to V(\lambda', \mu') \to V(\lambda, \mu)$.
- *Probably* indecomposable.
- Note that, for such strings, $P \cdot P = 0$ i.e. they would describe "decay" of 0-mass particles.

Parallelograms in $\mathfrak{p}(3,1)$

- The tensor product of two (generalized) strings will yield a (generalized) parallelogram.

Parallelograms in $\mathfrak{p}(3, 1)$

- The tensor product of two (generalized) strings will yield a (generalized) parallelogram.
- On such parallelograms we have $P \cdot P$ nilpotent.

Parallelograms in $\mathfrak{p}(3, 1)$

- The tensor product of two (generalized) strings will yield a (generalized) parallelogram.
- On such parallelograms we have $P \cdot P$ nilpotent.
- Properties?
  - Indecomposable?
  - Quotients and subrepresentations?

Parallelograms in $\mathfrak{p}(3,1)$

- The tensor product of two (generalized) strings will yield a (generalized) parallelogram.
- On such parallelograms we have $P \cdot P$ nilpotent.
- Properties?
  - Indecomposable?
  - Quotients and subrepresentations?
- Interpretation?

Other work

- Andrew Douglas & Joe Repka, on various aspects,

Other work

- Andrew Douglas & Joe Repka, on various aspects,
- David Ridout (Australian National University) on indecomposable representations of Virasoro, applications in logarithmic conformal field theory (with St-Aubin, with Kytölä)

Other work

- Andrew Douglas & Joe Repka, on various aspects,
- David Ridout (Australian National University) on indecomposable representations of Virasoro, applications in logarithmic conformal field theory (with St-Aubin, with Kytölä)
- Lots of early work on indecomposable of Poincaré (60s, 70s, 80s):
  - Gruber, J.Phys. A **19** (1986) 1
  - Raczka, Ann. Inst. Henri Poincaré **XIX** (1973) 341 (discusses a theory of relativistic unstable particles with 0 mass using induced representations)
  - George and Lévy-Nahas, JMP **7** (1966) 980

Other work

- Andrew Douglas & Joe Repka, on various aspects,
- David Ridout (Australian National University) on indecomposable representations of Virasoro, applications in logarithmic conformal field theory (with St-Aubin, with Kytölä)
- Lots of early work on indecomposable of Poincaré (60s, 70s, 80s):
  - Gruber, J.Phys. A **19** (1986) 1
  - Raczka, Ann. Inst. Henri Poincaré **XIX** (1973) 341 (discusses a theory of relativistic unstable particles with 0 mass using induced representations)
  - George and Lévy-Nahas, JMP **7** (1966) 980
- Thank you.