Weil Representation of the groups $Gl_*^{\varepsilon}(2, A)$ and $Sl_*^{\varepsilon}(2, A)$

José Pantoja

Pontificia Universidad Católica de Valparaíso
Valparaíso
Chile

CMS 2012
Regina, Canada
June 2012

partially supported by Fondecyt Grant 1120578

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Basic Idea

To look at higher rank groups as non commutative analogues of related lower rank groups

- Example: To look at the symplectic similitude group GSp(2n, F) in 2n variables over a field F as a group GL(2) with coefficients in the matrix ring over F, satisfying suitable commutation relations which involve the transpose map*
- To extend to higher rank groups, successful methods used for lower rank groups. In particular, having presentations for higher rank classical groups as non commutative versions of known presentations for the lower rank case. These presentations can be used in constructing linear representations for higher rank groups (generalized Weil representations, for example)

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Non commutative generalizations of classical groups: $Sl_*^{\varepsilon}(2, A)$

Let $(A, *)$ be a unitary ring with involution. Let $\varepsilon = \pm 1$

A left $\varepsilon-$ Hermitian form H on a left A -module M is a function which is biadditive, left linear in the first variable and such that $H(y, x) = \varepsilon H(x, y)^*$, for all $x, y \in M$.

Let us consider the special case where $M = A^2$

We have a matrix description of left $\varepsilon-$ Hermitian forms. Let us put $T^* = (t^*_{ji})_{1 \leq i,j \leq 2}$ for any $T = (t_{ij}))_{1 \leq i,j \leq m} \in M(2, A)$. Now, when a basis $B = \{e_1, e_2\}$ of the free $A-$ module M has been chosen, we define the matrix $[H]$ of H with respect to B by $[H] = (H(e_i, e_j))_{1 \leq i,j \leq 2}$.

Then $H(u, v) = u[H]v^* \quad (u, v \in M)$ and conversely, given a matrix T such that $T^* = \varepsilon T$ we recover an $\varepsilon-$ Hermitian form H_T by $H_T(u, v) = uTv^* \quad (u, v \in M)$.

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The groups $Gl_*^\varepsilon(2, A)$ and $Sl_*^\varepsilon(2, A)$ can be described as follow:

The group $Gl_*^\varepsilon(2, A)$ is the set of all automorphisms g of the $A-$ module $M = A \times A$ such that $H_\varepsilon \circ (g \times g) = \mu_g H_\varepsilon$ ($\mu_g \in A$ central and symmetric with respect to $*$)
or in matrix form as

$$GL_*^\varepsilon(2, A) = \{M \in M(2, A) : MJ_\varepsilon M^* = \mu_g J_\varepsilon\}$$

where $J_\varepsilon = \begin{pmatrix} 0 & 1 \\ \varepsilon & 0 \end{pmatrix}$ and $\varepsilon = \pm 1$
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In other words

$Sl_*^{\varepsilon}(2, A)$ is the group of $g = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ such that

$ab^* = -\varepsilon ba^*$

$cd^* = -\varepsilon dc^*,$

$a^* c = -\varepsilon c^* a,$

$b^* d = -\varepsilon d^* b,$

$ad^* + \varepsilon bc^* = a^* d + \varepsilon c^* b = 1$

Weil representations for $G = Sl_*^{\varepsilon}(2, A)$, G with a Bruhat presentation

We assume that the group $G = Sl_*^{\varepsilon}(2, A)$ has a Bruhat presentation, i.e.,

if $A_s = A^{sym}$ (ε-symmetric elements) is the set of a such that $a^* + \varepsilon a = 0$, G is generated by the following matrices satisfying the minimal relations

$h_t = \begin{pmatrix} t & 0 \\ 0 & t^{*-1} \end{pmatrix}$ $(t \in A^{\times})$, $w = w_{\varepsilon} = \begin{pmatrix} 0 & 1 \\ \varepsilon & 0 \end{pmatrix}$ and

$u_s = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix}$ $(s \in A^{sym})$

$h_t h_{t'} = h_{tt'}, \; u_b u_{b'} = u_{b+b'}$;

$w^2 = h_{\varepsilon}$;

$h_t u_b = u_{tbt^*} h_t$;

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Weakly euclidean rigs

- $(A, *)$ is a weakly euclidean ring if given $a, b \in A$ such that $a^* b = -\varepsilon b^* a$, $Aa + Ab = A$, then there are finite sequences of elements $s_0, s_1, ..., s_{n-1} \in A_s$ and $r_1, r_2, ..., r_n \in A$ with $r_n \in A^\times$ such that
  $a = s_0 b + r_1$
  $b = s_1 r_1 + r_2$
  $\dots$
  $r_{n-2} = s_{n-1} r_{n-1} + r_n$
- We observe that if $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL_*^\varepsilon(2, A)$, then a and b (or c) satisfy above
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  If $(A, *)$ is a weakly euclidean ring, then the elements u_s, h_t and w, generate the group $SL_*^\varepsilon(2, A)$.

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1. A bi-additive function $\chi : M \times M \to \mathbb{C}^\times$ and a character $\alpha \in \widehat{A^\times}$ such that:

   1.1 $\chi(xt, y) = \alpha(tt^*)\chi(x, yt^*)$ for $x, y \in M$ and $t \in A^\times$ (χ is α-balanced).

   1.2 $\chi(y, x) = [\chi(x, y)]^{-\varepsilon}$. We observe that $[\chi(x, y)]^{-\varepsilon} = \chi(-\varepsilon x, y)$ (χ is ε-symmetric).

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   2.1 $\gamma(b + b', x) = \gamma(b, x)\gamma(b', x)$, for all b, b'^{sym} and $x \in M$.

   2.2 $\gamma(b, xt) = \gamma(tbt^*, x)$ or equivalently $\gamma(b, x) = \gamma(tbt^*, xt^{-1})$, for all $b \in A^{sym}$, $t \in A^\times$ and $x \in M$.

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3. We assume moreover that these data are related by the equation:
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   - 2.3 $\gamma(t, x + z) = \gamma(t, x)\gamma(t, z)\chi(x, zt)$ for all $x, z \in M$, $t \in A^{sym}$. where t is $\varepsilon-$ symmetric invertible in A and $c \in \mathbb{C}^\times$ satisfies $c^2 |M| = \alpha(\varepsilon)$.
3. We assume moreover that these data are related by the equation: $c\gamma(-\varepsilon t, x) \sum_{y \in M} \chi(-\varepsilon x, y)\gamma(t^{-1}, y) = \alpha(-t)$.

1. A bi-additive function $\chi : M \times M \to \mathbb{C}^\times$ and a character $\alpha \in \widehat{A^\times}$ such that:

   1.1 $\chi(xt, y) = \alpha(tt^*)\chi(x, yt^*)$ for $x, y \in M$ and $t \in A^\times$ (χ is α-balanced).

   1.2 $\chi(y, x) = [\chi(x, y)]^{-\varepsilon}$. We observe that $[\chi(x, y)]^{-\varepsilon} = \chi(-\varepsilon x, y)$ (χ is ε-symmetric).

   1.3 $\chi(x, y) = 1$ for any $x \in M$, implies $y = 0$ (χ is non-degenerate).

2. A function $\gamma : A^{sym} \times M \to \mathbb{C}^\times$ such that:

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$$c\gamma(-\varepsilon t, x) \sum_{y \in M} \chi(-\varepsilon x, y)\gamma(t^{-1}, y) = \alpha(-t).$$

As a consecuence we have

The following two relations are equivalent:

1. $c\gamma(-\varepsilon t, x) \sum_{y \in M} \chi(-\varepsilon x, y)\gamma(t^{-1}, y) = \alpha(-t)$.
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Weil Representation defined via a presentation

We have the following result:

- Theorem

  There is a representation $(\mathbb{C}^M, \rho)$ of G defined as follows on the basis of Dirac delta functions $\{e_x\}_{x \in M}$ (given by $e_x(y) = 1$ if $y = x$; $e_x(y) = 0$ otherwise):

  $\rho_{u_b}(e_x) = \gamma(b, x)e_x$

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  for $x \in M, b \in A^\times \cap A^\times, t \in A^\times$.

  This representation is called the generalized Weil representation of $SL_*^\varepsilon(2, A)$ associated to the data $(M, \alpha, \gamma, \chi)$.

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Proof: It is enough to verify that $\rho_{u_b}, \rho_{h_t}, \rho_w$ defined as above, satisfy the relations corresponding to the universal relations in the Bruhat presentation of G.
To this end, we observe that

- a) $(\rho_{h_t} \circ \rho_{h_{t'}})(e_x) = \rho_{h_t}(\alpha(t')e_{xt'^{-1}}) = \alpha(t')\alpha(t)e_{xt'^{-1}t^{-1}} = \alpha(t')\alpha(t)e_{x(tt')^{-1}}$
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We mention an example:

We take A to be the full matrix ring $M_n(k)$, where k denotes the finite field $\mathbb{F}_q$ with q elements, endowed with the transpose involution $*$.
It has been proved that the group $SL_*^{-1}(2, A)$ has a Bruhat presentation
Let M be the $k-$vector space $\bigoplus_{1 \leq i \leq m} E_i$,where $E_i = k^n$, endowed with a non-degenerate $k-$quadratic form Q_0 with associated $k-$bilinear form B_0, defined by

$$B_0(u, v) = Q_0(u + v) - Q_0(u) - Q_0(v)$$

$u, v \in k^n$
This induces canonically a non degenerate $A-$valued quadratic form $\mathbb{Q}$ on M

given by

1. $\mathbb{Q}(x)_{ii} = Q_0(x_i)$
2. $\mathbb{Q}(x)_{ij} = B_0(x_i, x_j)$
3. $Q(x)_{ji} = 0$
   for all $x = (x_1, \ldots, x_m) \in M, 1 \leq i, j \leq n$.

and a k bilinear form $\mathbb{B}$ on M with values in A given by $\mathbb{B}(x, y)_{ij} = B_0(x_i, y_j)$

If we pass now to the quotient modulo "anti-traces", i.e. if we define $\overline{\mathbb{Q}} = pr \circ \mathbb{Q} : M \to \bar{A}$, and $\overline{\mathbb{B}} = pr \circ \mathbb{B} : M \times M \to \bar{A}$ where pr denotes the canonical projection of A onto A/A^0, with $A^0 = \{a - a^* | a \in A\}$, We fix moreover a non trivial character ψ of k^+ and we denote by tr the usual matrix trace from A onto k.

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Then $\underline{\psi} = \psi \circ tr$ is a non trivial character of A^+ such that $\underline{\psi}(ab) = \underline{\psi}(ba)$ and $\underline{\psi}(a^*) = \underline{\psi}(a)$ for all $a, b \in A$. On the other hand, we have

$$\underline{\psi} \circ \mathbb{Q} = \psi \circ Q$$

where Q denotes the $k-$valued non-degenerate quadratic form $tr \circ \mathbb{Q}$ over k, whose associated $k-$bilinear form will be denote by B.

We put here

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- A G-Hilbert vector bundle is a tuple (E, p, B, τ) where E is the total space, B is the base, $p : E \to B$ is the projection, and $\tau = (\tau, \tau^B)$ is the action of G on E and B, respectively, such that $p \circ \tau_g^E = \tau_g^B \circ p$ for all $g \in G$. The fiber $p^{-1}(b)$ $(b \in B)$ is denoted by E_b, and is a (finite dimensional) Hilbert space with inner product $<,>$.
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Weil representation of G by contraction of Lagrangian fiber bundles

- We assume that A is a finite $k-$algebra with an involution that fixes the finite base field.
- Let S be a left $A-$module (which is right $A-$module via $s.a = a^*.s$ for $a \in A, s \in S$), finite dimensional as $k-$vector space
- Let $\eta : S \times S \to k$. be a non-degenerate, $k-$bilinear, symmetric $A-$balanced pairing (i.e. $\eta(s.a, t) = \eta(s, a.t)$ for $a \in A, \quad s, t \in S$
- We set $W = S \oplus S$ and we define a symplectic form B on W by
$$B((s,t),(s',t')) = \eta(s,t') - \eta(t,s')$$
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- A Lagrangian L in W is a $A-$submodule which is maximal totally isotropic for B, i.e., $L = L^{\perp}$
- The group G acts on W by matrix multiplication, and we have $B(gw, gw') = B(w, w')$
- Then the Lagrangian bundle of G associated to S is given by

  i $\mathcal{B}$ is the set of all Lagrangians of $W = S \oplus S$

  ii E is the disjoint union of the spaces

  $$E_L = \{ f : W \to \mathbb{C} \mid f(w + \varsigma) = \chi(w, \varsigma) f(w); \;\; w \in W, \; \varsigma \in L \},$$

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$G-$equivariant connections

Assume that $2 \in A$ is invertible. Then the family

$$\Gamma = \left\{\gamma_{L',L} \mid \gamma_{L',L} : E_L \to E_{L'}\right\}_{L,L' \in \mathcal{B}}$$

of linear isomorphisms

$$\gamma_{L',L}(f)(w) = \frac{1}{\sqrt{|L|\,|L \cap L'|}} \sum_{\zeta' \in L'} \overline{\chi(w,\zeta')} f(w+\zeta'), \;\; (f \in \; E_L, w \in W)$$

is a G-equivariant connection with multiplier

$$\mu_\Gamma(L'', L', L) = \sqrt{\frac{|L'' \cap L'|}{|L \cap L''|\,|L' \cap L|\,|L|}} S_W(L; L', L'') \quad (L, L', L'' \in \mathcal{B}),$$

where the geometric Gauss sum $S_W(L; L', L'')$ is given by

$$S_W(L; L', L'') = \sum_{\zeta \in L \cap (L', L'')} \chi(\zeta' + \zeta'')$$

where $\zeta \in L \cap (L' + L'')$ is written as $\zeta' + \zeta''$ with $\zeta' \in L', \zeta'' \in L''$.

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