Routing in Massively Dense Static Sensor Networks

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Wireless Sensor Networks

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The deployment of wireless sensor networks can be:

- Deterministic,
- Random.

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- Applications in Buildings
  - Monitoring for intruders,
  - Control air conditioning,
  - Smart homes.

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- Consider large number of sensors deployed over an area
- These sensor network has two goals
  1. Sense the environment for events, measurements.
  2. Transport the measurement to a set of collection points.
- Sensors will cooperate over the network.

Each wireless sensor node can:

- Sense the data at the sources of information,
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Questions

- What is the best placement for the wireless nodes ?
- What is the traffic flow it induces ?

Tradeoff between having short routes and avoiding congestion.

Statement Problem

- Study the global and the non-cooperative optimal solution for the routing problem among a large quantity of nodes.
- Find a general optimization framework for handling minimum cost paths in massively dense sensor networks.

Previous Works

- Geometrical Optics
  Jacquet ('04) studied the routing problem as a parallel to an optics problem.
  Drawback: It doesn't consider interaction between each user's decision.
- Electrostatics
  Toumpis ('06) studied the problem of the optimal deployment of wireless sensor networks.
  Drawback: The local cost assumed is very particular ($\mathrm{cost}(f) = |f|^2$ where f is the flow).

Previous Works

- Road Traffic
  Beckmann ('56) studied the system-optimizing pattern.
  Dafermos ('80) studied the user-optimizing and the system-optimizing pattern.

  Drawback: The present mathematical tools from optimal transport theory were not available.

Cost Models

- Minimize the quantity of nodes to carry a given flow.
  Given a flow ϕ assigned through a neighborhood of x, the cost is taken to be
  $$c(x, \phi(x)) = f^{-1}(\phi(x)),$$
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  [Franceschetti et al ('04)] The transport capacity of the network when the nodes are
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- Costs related to energy consumption
  Assuming randomly deployment of nodes where each node has to send a packet to another randomly selected node, the capacity has the form

$$f(\lambda) = \Omega\left(\left(\frac{\lambda}{\log\lambda}\right)^{(q-1)/2}\right),$$

  where q is the path loss [Rodoplu and Meng ('07)].
- Congestion independent costs
  If the queueing delay is negligible with respect to the transmission delay over each hop then the cost depend on local conditions at a given point.

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Let Ω be an open and bounded subset of $\mathbb{R}^2$ with Lipschitz boundary $\Gamma = \partial\Omega$, densely covered by potential routers.
Messages flow from $\Gamma_{\mathcal{S}} \subseteq \Gamma$ to $\Gamma_{\mathcal{R}} \subseteq \Gamma$ (with $\Gamma_{\mathcal{S}} \cap \Gamma_{\mathcal{R}} = \emptyset$).
On the rest $\Gamma_{\mathcal{T}}$ of the boundary, no message should enter nor leave Ω.

Figure: Description of the domain

Assumptions:

- The intensity of message generation $\sigma|_{\Gamma_{\mathcal{S}}} \in L^2(\Gamma_{\mathcal{S}})$ is known.
- The intensity of message reception $\sigma|_{\Gamma_{\mathcal{R}}}$ is unknown.

The total flow of messages emitted and received are equal.

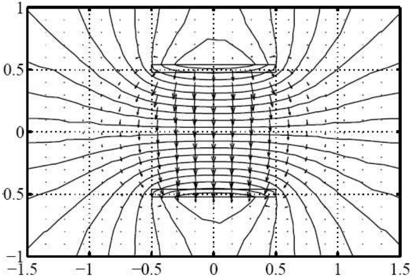

Figure: The function f.

Let the vector field $f = (f_1(x), f_2(x)) \in (H^1(\Omega))^2$ $[\mathrm{bps/m}]$ represents the flow of messages, and $\phi(x) = \|f(x)\|$ be its intensity.

Let $\Gamma_1 = \Gamma_{\mathcal{S}} \cup \Gamma_{\mathcal{T}}$.
Extend the function σ to Γ_1 by $\sigma(x) = 0$ on $\Gamma_{\mathcal{T}}$.
We modelize the conditions on the boundary as

$$\boxed{\forall x \in \Gamma_1 \quad \langle f(x), n(x) \rangle = -\sigma(x)}$$

The Conservation Equation

Suppose there is no source nor sink of messages in Ω.
Over a surface $\Phi_0 \subseteq \Omega$ of arbitrary shape,

$$\oint_{\partial\Phi_0} \langle f(x), n(x) \rangle \mathrm{d}\Phi_0 = 0,$$

where n is the unit normal vector.
Last equation holding for any smooth domain, then

$$\boxed{\forall x \in \Omega \quad \mathrm{div} f(x) = 0.}$$

Let the congestion cost per packet $c = c(x, \phi) \in \mathcal{C}^1(\Omega \times \mathbb{R}_+ \cup \{0\}, \mathbb{R}_+)$ be a strictly positive function, increasing and convex in ϕ for each x.

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Let $e_\theta = (\cos\theta,\ \sin\theta)$ be the direction of travel of a message. Total cost incurred in a path from $x(t_0) = x_0 \in \Gamma_{\mathcal{S}}$ to $x(t_1) = x_1 \in \Gamma_{\mathcal{R}}$ is

$$J(e_\theta(\cdot)) = \int_{x_0}^{x_1} c(x, \|f(x)\|)\sqrt{\mathrm{d}x_1^2 + \mathrm{d}x_2^2} = \int_{t_0}^{t_1} c(x(t), \|f(x(t))\|)\,\mathrm{d}t,$$

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Let $C(x, \phi) := c(x, \phi)\phi$
Total (collective) cost of congestion is

$$G(f(\cdot)) = \int_\Omega c(x, \|f(x)\|)\|f(x)\|\mathrm{d}x = \int_\Omega C(x, \phi(x))\mathrm{d}x.$$

Global Optimum

We seek here the vector field $f^* \in (L^2(\Omega))^2$ minimizing $G(f)$ under the constraints:

$$\forall x \in \Gamma_1 \quad \langle f(x), n(x) \rangle = -\sigma(x)$$
$$\forall x \in \Omega \quad \mathrm{div} f(x) = 0.$$

The function $C(x,\phi) = c(x,\phi)\phi$ is convex in ϕ and coercive (i.e. goes to infinity with ϕ).
Then $f(\cdot) \mapsto G(f(\cdot))$ is continuous, convex and coercive. Moreover, the constraints are linear.

We dualize only the constraint of the divergence with dual variable $p(\cdot) \in L^2(\Omega)$

$$\mathcal{L}(f,p) = \int_\Omega \Big(C(x, \|f(x)\|) + p(x)\mathrm{div} f(x) \Big) \mathrm{d}x \,.$$

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The necessary conditions implies that for $f^*(\cdot)$ to be optimal, there must exist a $p(\cdot) : \Omega \to \mathbb{R}$ such that

$$\begin{array}{ll} \forall x \in \Omega : f^*(x) \neq 0\,, & \nabla p(x) = \mathcal{D}_2 C(x, \|f(x)^\star\|) \frac{1}{\|f^*(x)\|} f^*(x), \\ \forall x \in \Omega : f^*(x) = 0\,, & \|\nabla p(x)\| \leq \mathcal{D}_2 C(x, 0), \\ \forall x \in \Gamma_{\mathcal{R}}\,, & p(x) = 0\,. \end{array}$$

User Optimum

The optimization of the criterion

$$J(e_\theta(\cdot)) = \int_{x_0}^{\mathbf{x}_1} c(x, \|f(x)\|)\sqrt{\mathrm{d}x_1^2 + \mathrm{d}x_2^2} = \int_{t_0}^{t_1} c(x(t), \|f(x(t))\|)\,\mathrm{d}t,$$

via its Hamilton-Jacobi-Bellman equation:
Let $V(x)$ be the return function, it must be a viscosity solution of

$$\begin{array}{ll} \forall x \in \Omega, & \min_\theta \langle e_\theta, \nabla V(x)\rangle + c(x, \|f^*(x)\|) = 0, \\ \forall x \in \Gamma_{\mathcal{R}}, & V(x) = 0. \end{array}$$

The optimal direction of travel is opposite to $\nabla V(x)$, i.e. $e_\theta = -\nabla V(x)/\|\nabla V(x)\|$.

Hence

$$\forall x \in \Omega\,, \quad -\|\nabla V(x)\| + c(x, \|f^*(x)\|) = 0\,,$$
$$\forall x \in \mathcal{R}, \quad V(x) = 0\,.$$

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$$\forall x \in \mathcal{R}, \quad V(x) = 0\,.$$

This is the same system of equations as previously, upon replacing $p(x)$ by $-V(x)$, and $\mathcal{D}_2 C(x,\phi)$ by $c(x,\phi)$.
Conclusion The "Wardrop equilibrium" can be obtained by solving the globally optimal problem in which the cost density is replaced by $\int_0^\phi c(x,\phi)\mathrm{d}\phi$.

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Linear Congestion Cost

If the cost of congestion is linear : $c(x, \phi) = \frac{1}{2}c(x)\phi$, so that

$$C(x, \phi) = \frac{1}{2}c(x)\phi^2 .$$

Then, $\mathcal{L}$ is differentiable everywhere, and the necessary condition of optimality is just that there should exist $p : \Omega \to \mathbb{R}^2$ such that $\nabla p(x) = c(x)f^*(x)$.

Using the divergence equation we obtain:

$$\left.\begin{array}{ll} \forall x \in \Omega,, & \operatorname{div}\left(\frac{1}{c(x)} \nabla p(x)\right) = 0\,, \\ \forall x \in \Gamma_1, & \dfrac{\partial p}{\partial n}(x) = c(x)\sigma(x), \\ \forall x \in \Gamma_{\mathcal{R}}\,, & p(x) = 0\,, \end{array}\right\}$$

for which we get existence and uniqueness of the solution (Lax-Milgram Theorem $p \in H^1_{\Gamma_{\mathcal{R}}}$).
Solution via e.g. finite element method.

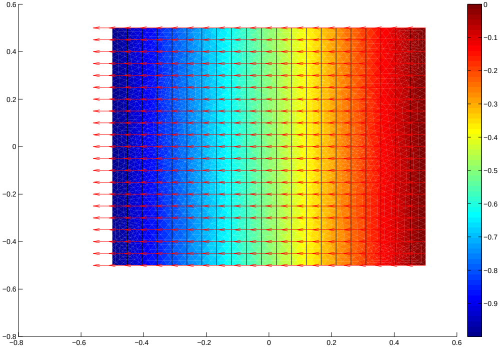
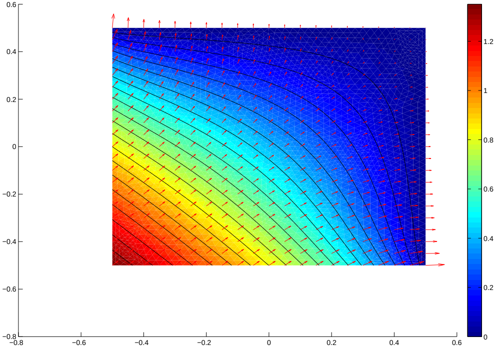


Figure: The function f.



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Conclusions

- We studied a setting to describe the network in terms of macroscopic parameters rather than in terms of microscopic parameters.
- We solved the routing problem for the affine cost per packet obtaining existence and uniqueness of the solution.

Future Works

- Investigate the convergence of the routing problem in a discrete case to this continuous case.

Thank you for your attention!