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Oil prices and depletion path

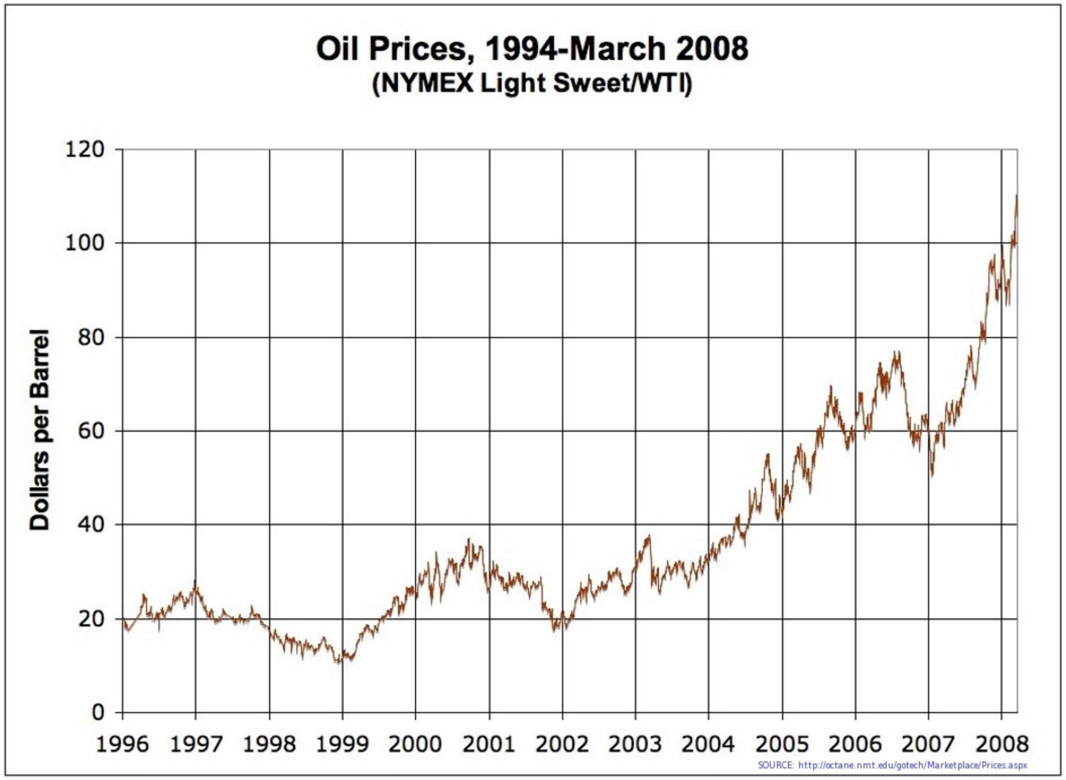
Hubbert oil peak and Hotelling rent through a combined Simulation and Optimisation approach

Presentation overview

- **EDF** is interested in **oil** as a key driving force **within the energy commodities market**

- ➔ **a pedagogic tool** to get a grasp on oil prices and depletion

1. Origination and Principle
2. Optimisation model
3. Simulation model



Origination and Principle

Ricardo, Hubbert and Hotelling

Introduction

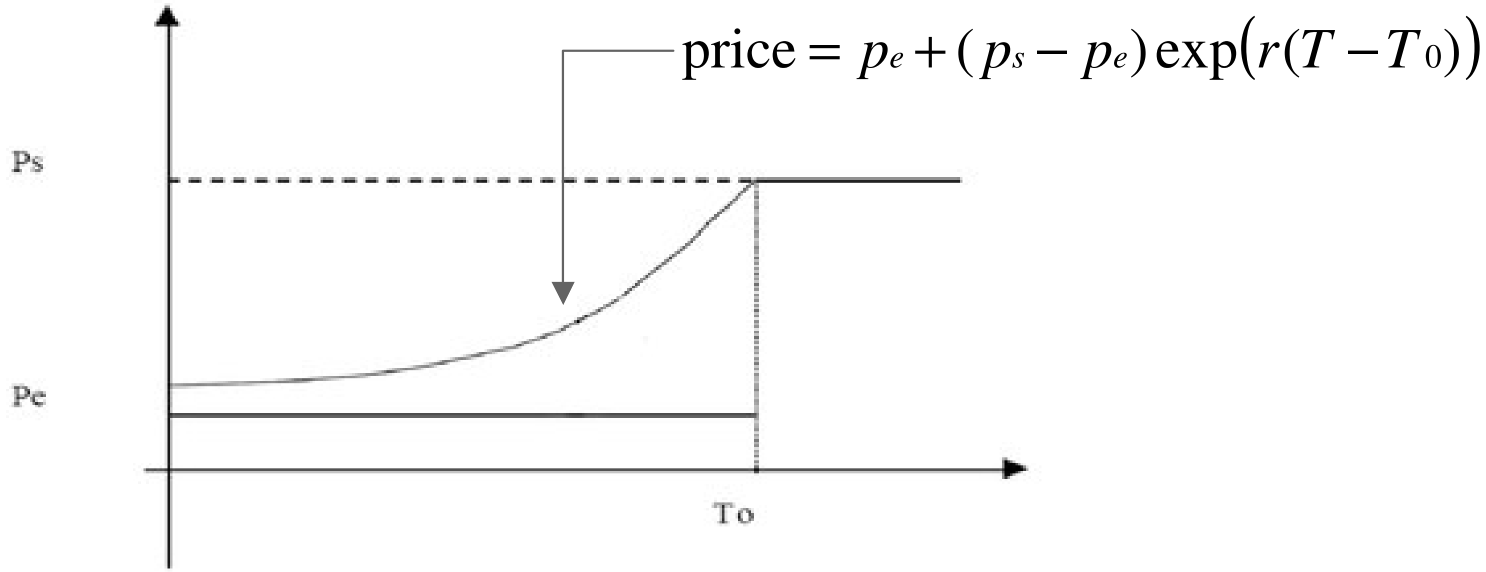
- a **joint** CERNA (Commodities Economics Lab, Ecole des Mines de Paris) and EDF R&D **research project**
- **revisit the long term oil market fundamental** driving forces
- still **work in progress** … few results, much methodology
- starting from the classical approaches of the field
  - **Ricardo** differential rents
  - **Hotelling** rent
  - **Hubbert** oil peak
- a double modelling approach (which one is best?)
  1. an **optimisation model** to determine the exploration strategy
  2. a **simulation model** based on rational agents behaviour

Hotelling model

- an **exhaustible resource** with a given stock and extraction cost
- a given demand
- presence of an **abundant backstop technology** with a given extraction cost (e.g. hydrogen via fuel cells)
- **perfect competition** between deposits owners
- no limit on production capacity
- presence of a market with a **rate of return *r***

no arbitrage opportunity → production of resource is optimal any time → unchanged discounted rent over time

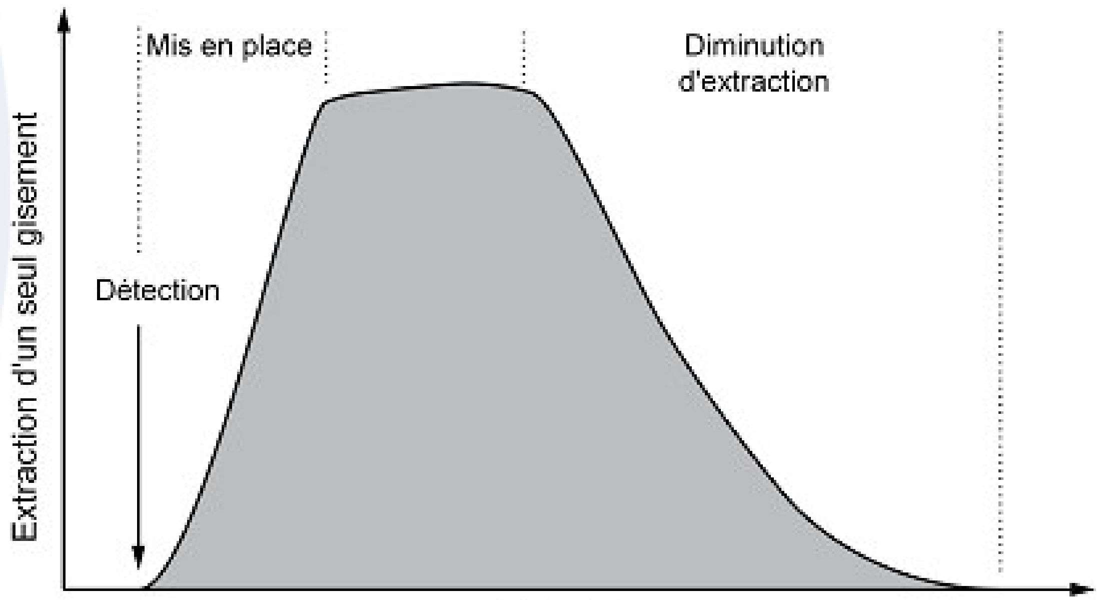
Hotelling model (2)



price of a substitutable exhaustible resource through time

Hubbert peak

- oil production is bound to **reach a peak** ➔ how? when?…
- oil production is **geologically constrained**:
  - production level cannot increase too fast
  - there exists a **production cap** above which global recovery rate is not optimal

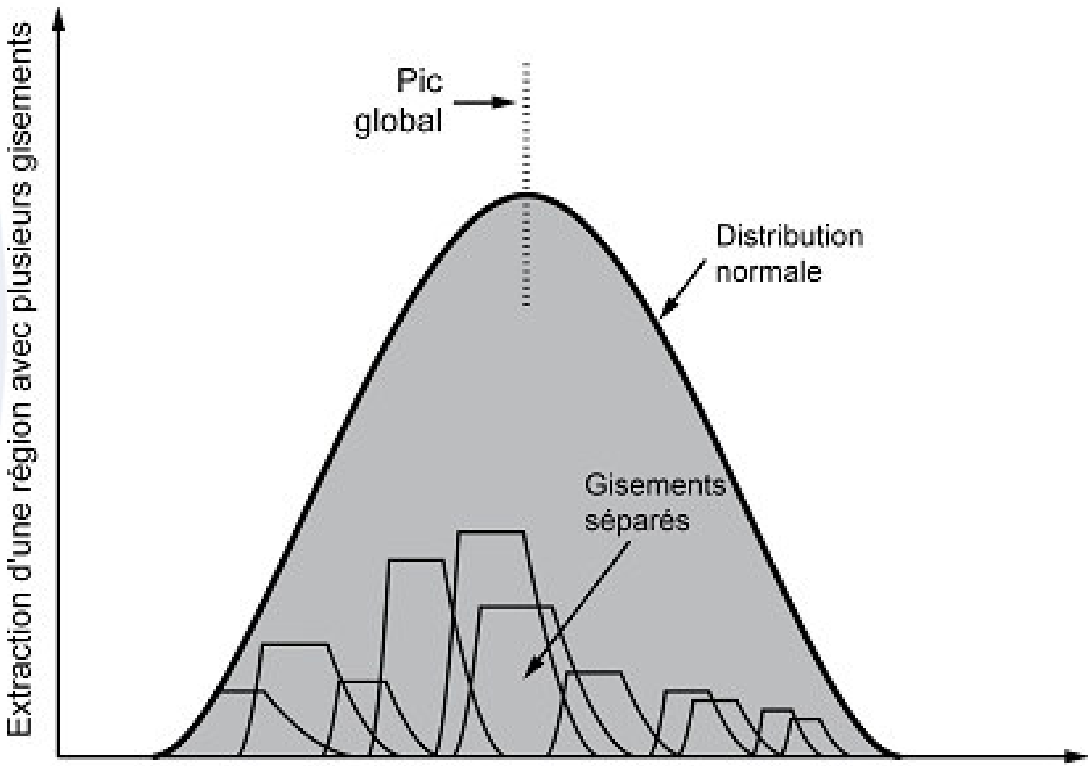


production path of an oil well through time

Hubbert peak (2)

- empirical result

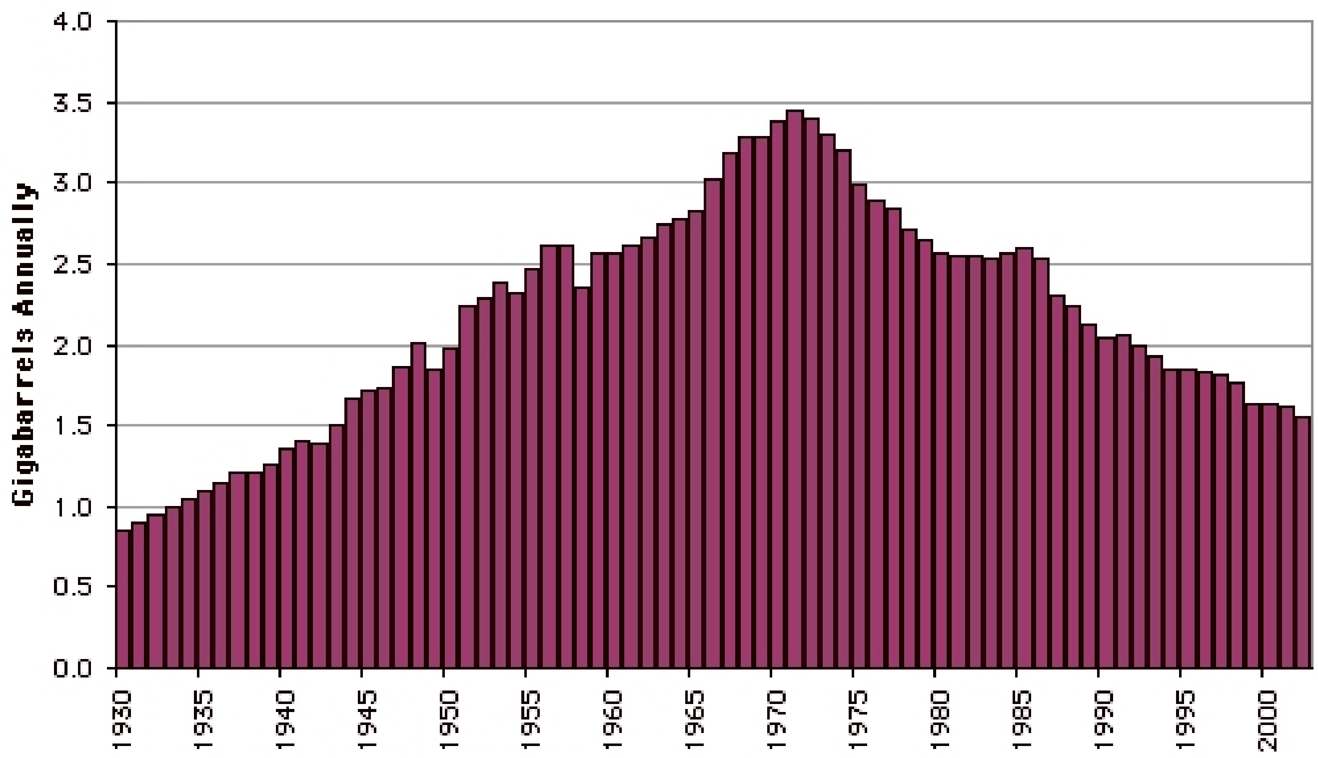
total production of a multi-deposit region also **shows a peak** (when half of total reserves are depleted)



production path of several oil wells through time

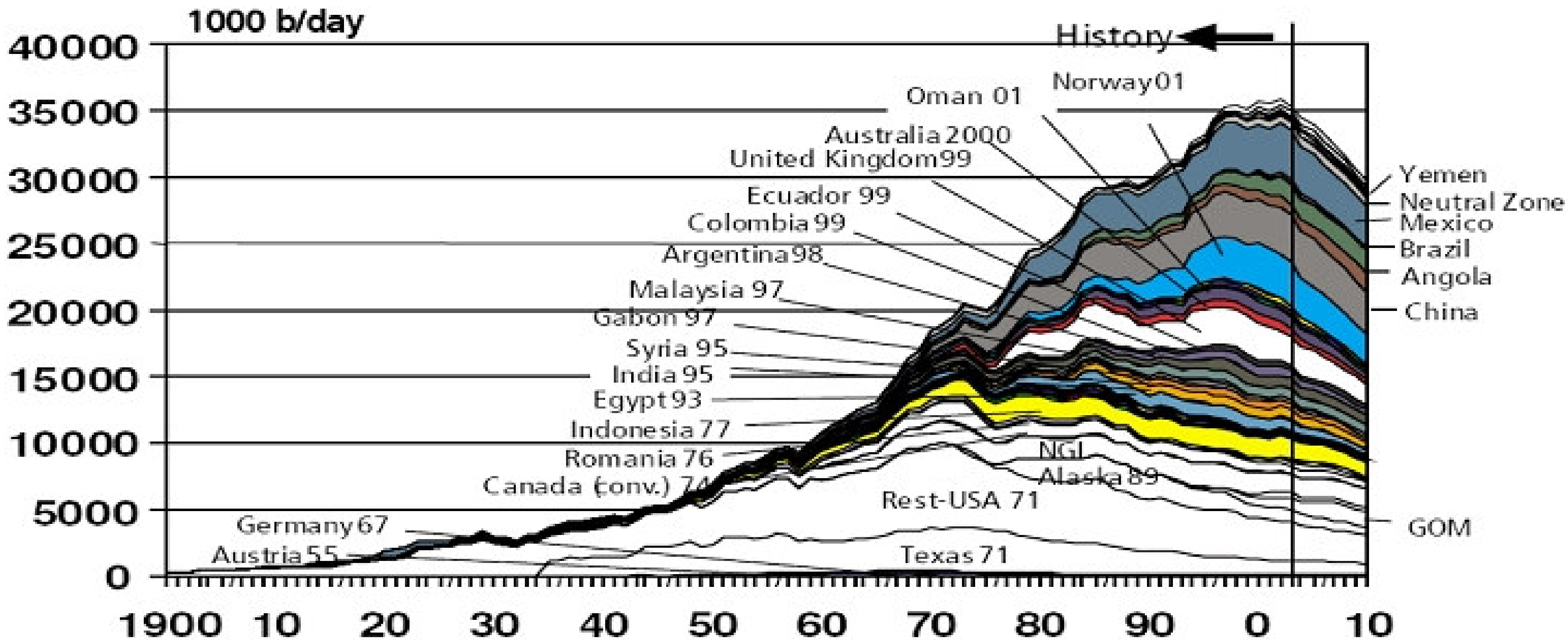
Hubbert peak (3)

- 48-US oil production peak had been **forecasted by Hubbert 15 years before** (with a 1 year error!)



Hubbert peak (3)

- other oil producing countries production peak?



Source: Industry database, 2003 (IHS 2003)
OGJ, 9 Feb 2004 (Jan-Nov 2003)

Optimisation model

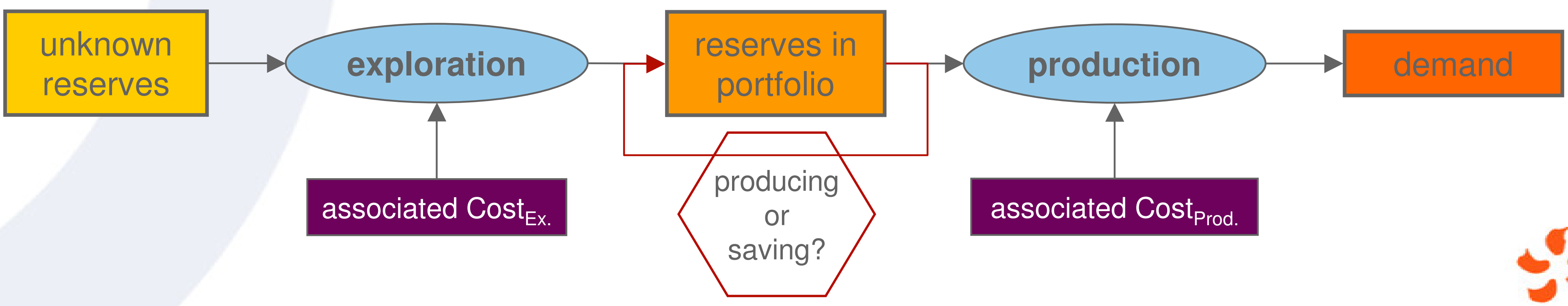
Objectives
Principle
Method

Model objectives

- enrich original Hotelling model
- get a grasp on the **long-term dynamic of oil price setting**, by accounting for
  - the **need to explore** to be able to produce oil
  - the (random) **discovery of new reserves** through exploration
  - reserves **exhaustion**
  - oil production **technical constraints**
  - 2 different production costs
- be able to find out the **optimal exploration strategy**

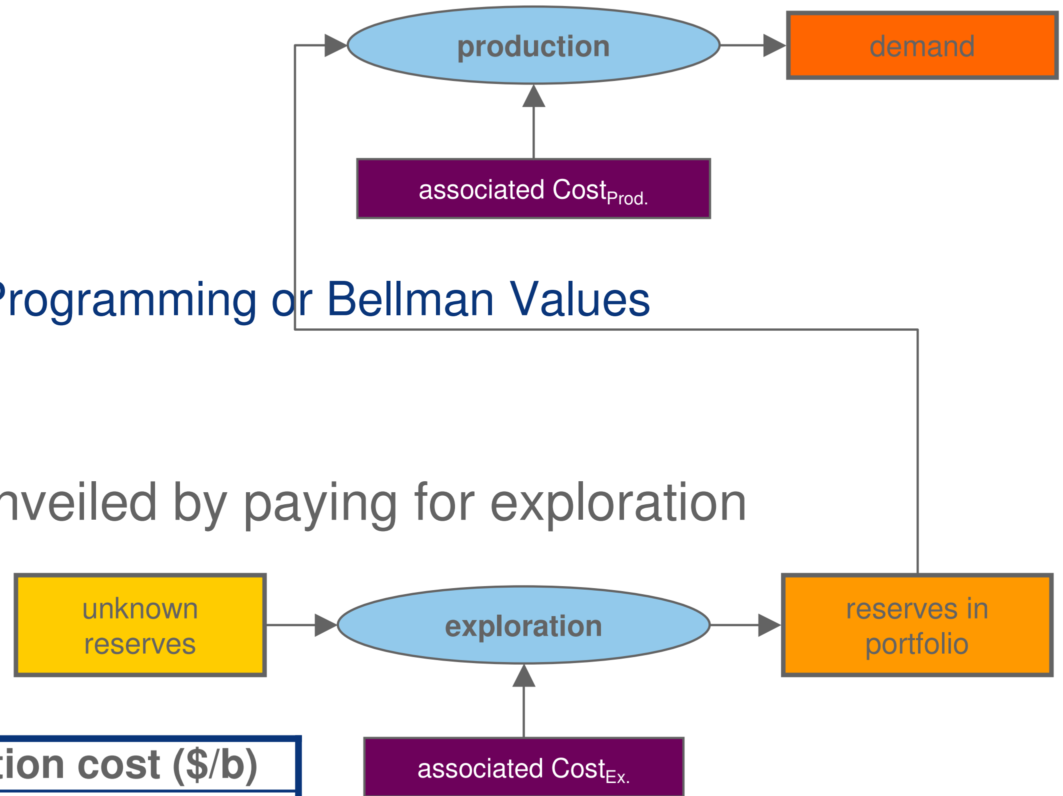
Model principles

- if reserves were known to actors, we would expect
  - Ricardo ➔ oil production starting in the **ascending cost order**
  - Hotelling ➔ presence of a **scarcity rent**
  - **oil production** subject to technical constraint
- but reserves first have to be discovered, therefore
  - need for modelling how they get into the **agent production portfolio**
  - this can be viewed as an **information “production”** about reserves



Model principles (2)

- meeting known Supply and Demand under minimum cost through time
  - optimisation fundamental model
  - deterministic or stochastic
  - numerically tractable via Linear Programming or Bellman Values



- here Supply characteristics are unveiled by paying for exploration
  - strategy arbitrage :

time step t	**Exploration cost ($/b)**	**Production cost ($/b)**
"carpe diem" strategy	Expectation: low Risk : low	Expectation: medium Risk : high
"exploring" strategy	Expectation: medium Risk : medium	Expectation: low Risk : medium

→ dual effect

Model principles (3)

- **dual effect** or **one-armed bandit problem**
  ➔ methodological core of the problem…

  - paying in the hope of decreasing total cost?

	Exploration cost ($/b)	Production cost ($/b)
"carpe diem" strategy	Expectation: low Risk : low	Expectation: medium Risk : high
"explore all" strategy	Expectation: high! (cf. discount rate) Risk : low	Expectation: low Risk : none

  - classical optimisation techniques **very hard** to use
    - **linear programming** ➔ exponential explosion of number of nodes to explore
    - Bellman **stochastic dynamic programming** ➔ limited number of states possible only which does not allow to consider a rich set of hypothesis

Model method

- **2 states** Bellman stochastic dynamic programming
- **minimizing total cost** under supply/demand constraint
- constant and inelastic demand
- **2 types of oil** available (cheap and expensive), spread into 150 unknown reserves (unique reserve size)
  - randomness on future production cost of discoverable reserves
- infinitely and immediately available backstop technology
- knowledge of the agent
  - statistical knowledge of the reserves to be discovered
  - able to calculate *ex ante* the exhaustion date
- discovery cost per reserve increases linearly with portfolio reserves

Model method (2)

- 2 states Bellman stochastic dynamic programming
  - Bellman states: 2 types of oil reserves in portfolio

$$V_t\left(\left\{R_{1,t}^N, R_{2,t}^N\right\}\right) = \min_{r_t^D}\left(e^{-r\cdot t\cdot\Delta}\left(I_t + C_t^P\right) + \sum_{\omega\in\{0,\dots,q^D\}} C_{\omega}^{q^D} p^{\omega}(1-p)^{q^D-\omega}\cdot V_{t+1}\left(\left\{R_{1,t+1}^N, R_{2,t+1}^N\right\}\right)\right)$$

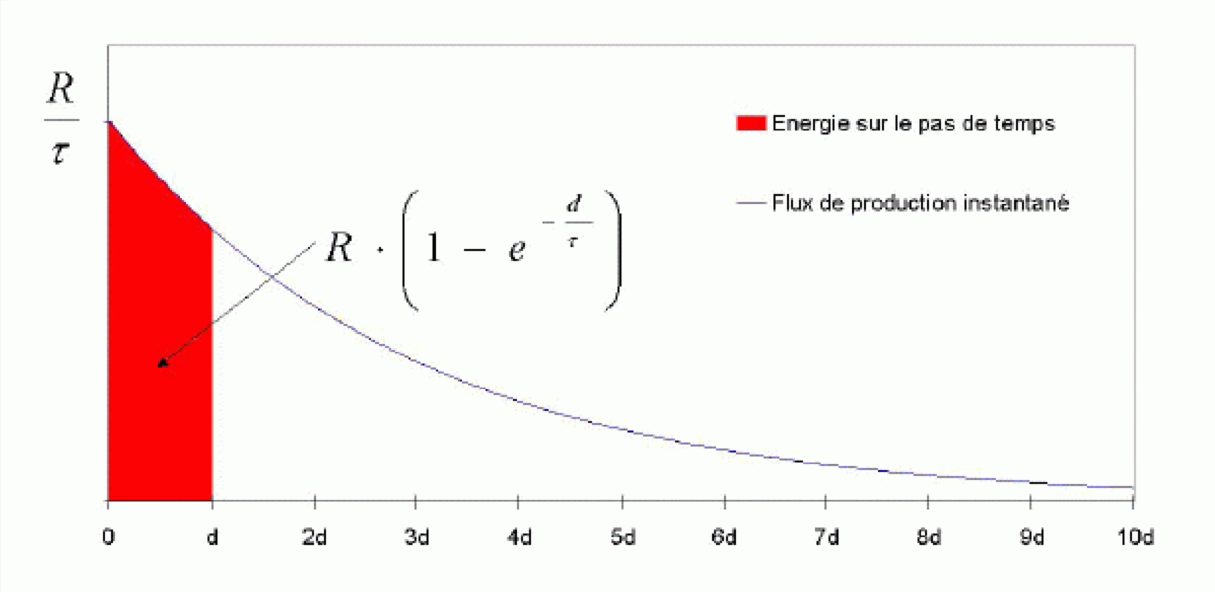
  - where: r_t^D is the control variable : the amount of oil to be discovered for each t
  - Bellman Values are then used to simulate any scenario

Model method (3)

- demand is satisfied by putting new reserves into production, in the **ascending cost order**

  - ➔ under a technical constraint: **production exponential decrease**
    production rate is proportional to current level of reserve with half-life time τ

  - less realistic but only way **to allow for a tractable solvency** (see next slide)

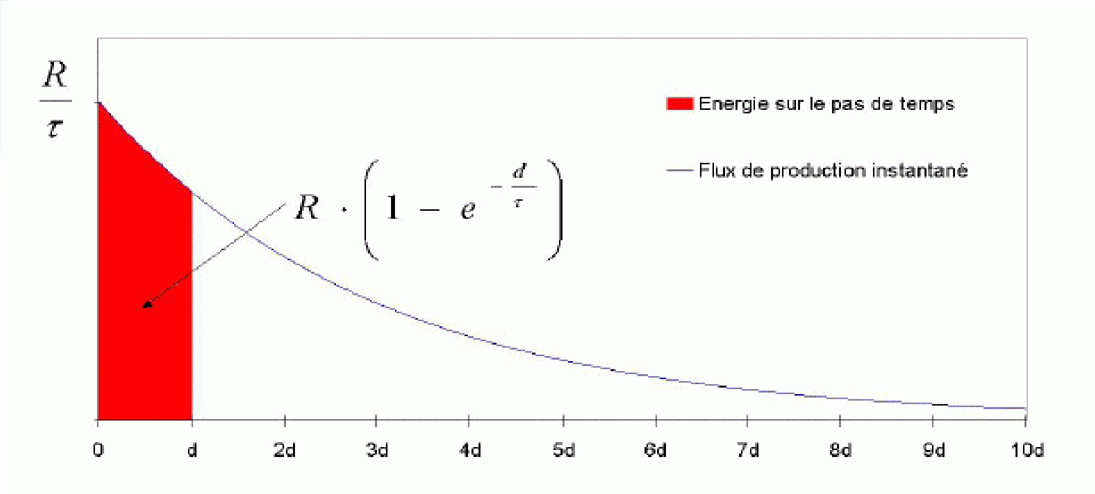


production flow of an oil reserve

Model method (4)

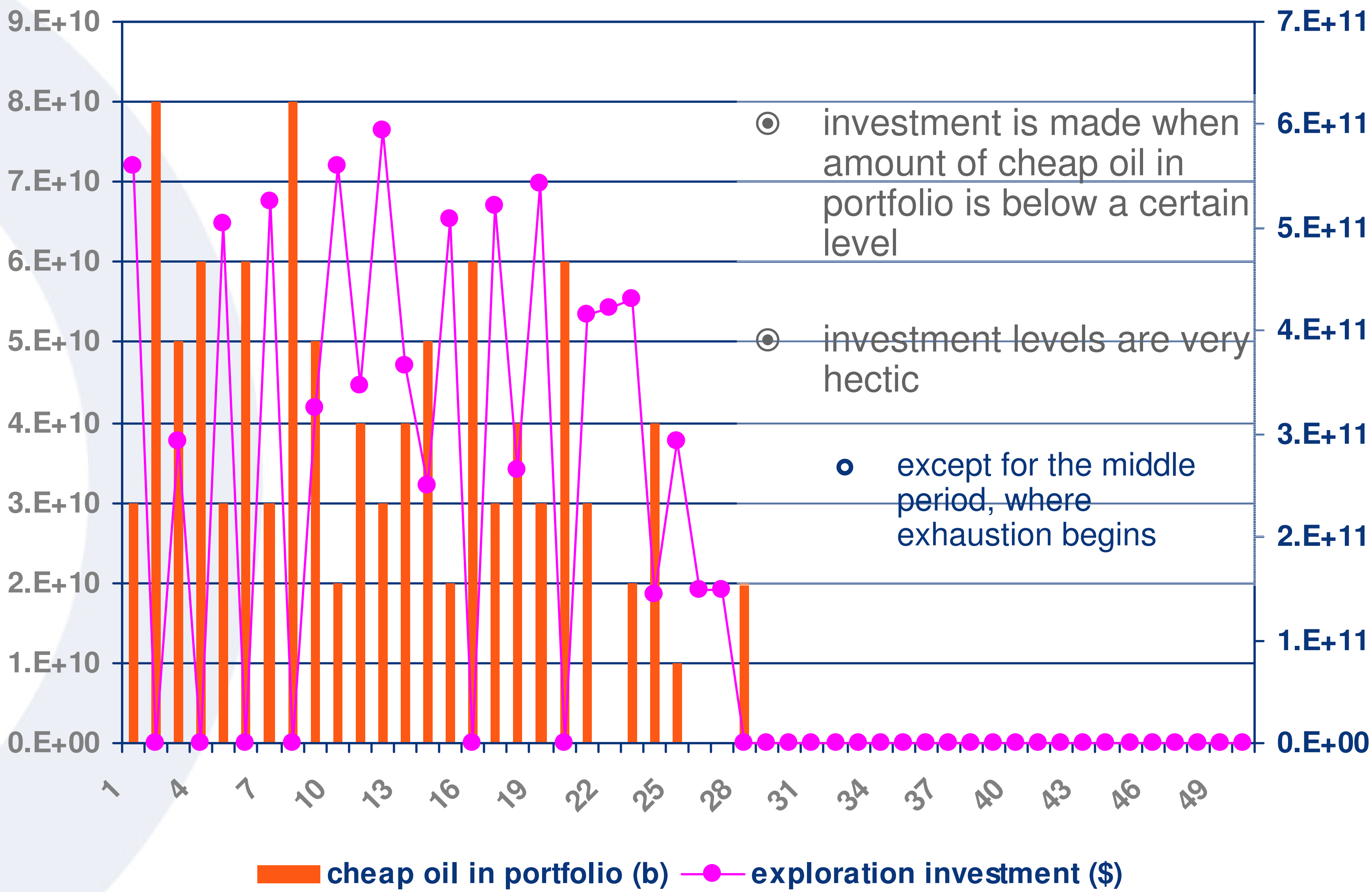
- **production exponential decrease** hypothesis has several modelling consequences

1. **production rate is proportional to current level** of reserve with half-life time τ ➔ reserves in production are fungible through time
2. since demand is non decreasing, one can calculate *ex ante* the exhaustion date
3. producing reserves need not be considered into Bellman Values since they can be deduced from the state of portfolio
4. 2 states Bellman stochastic dynamic programming



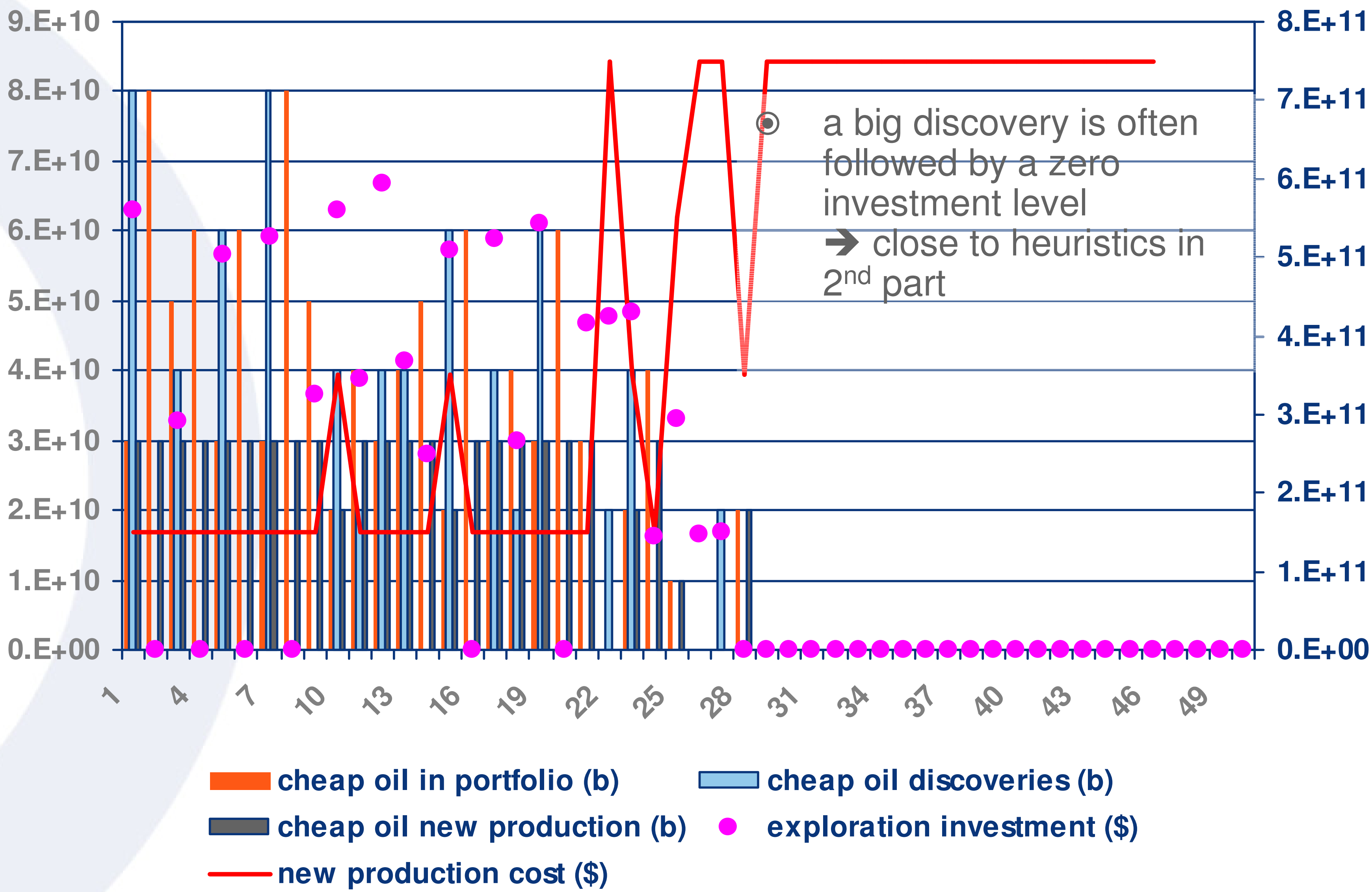
production flow of an oil reserve

Model results examples



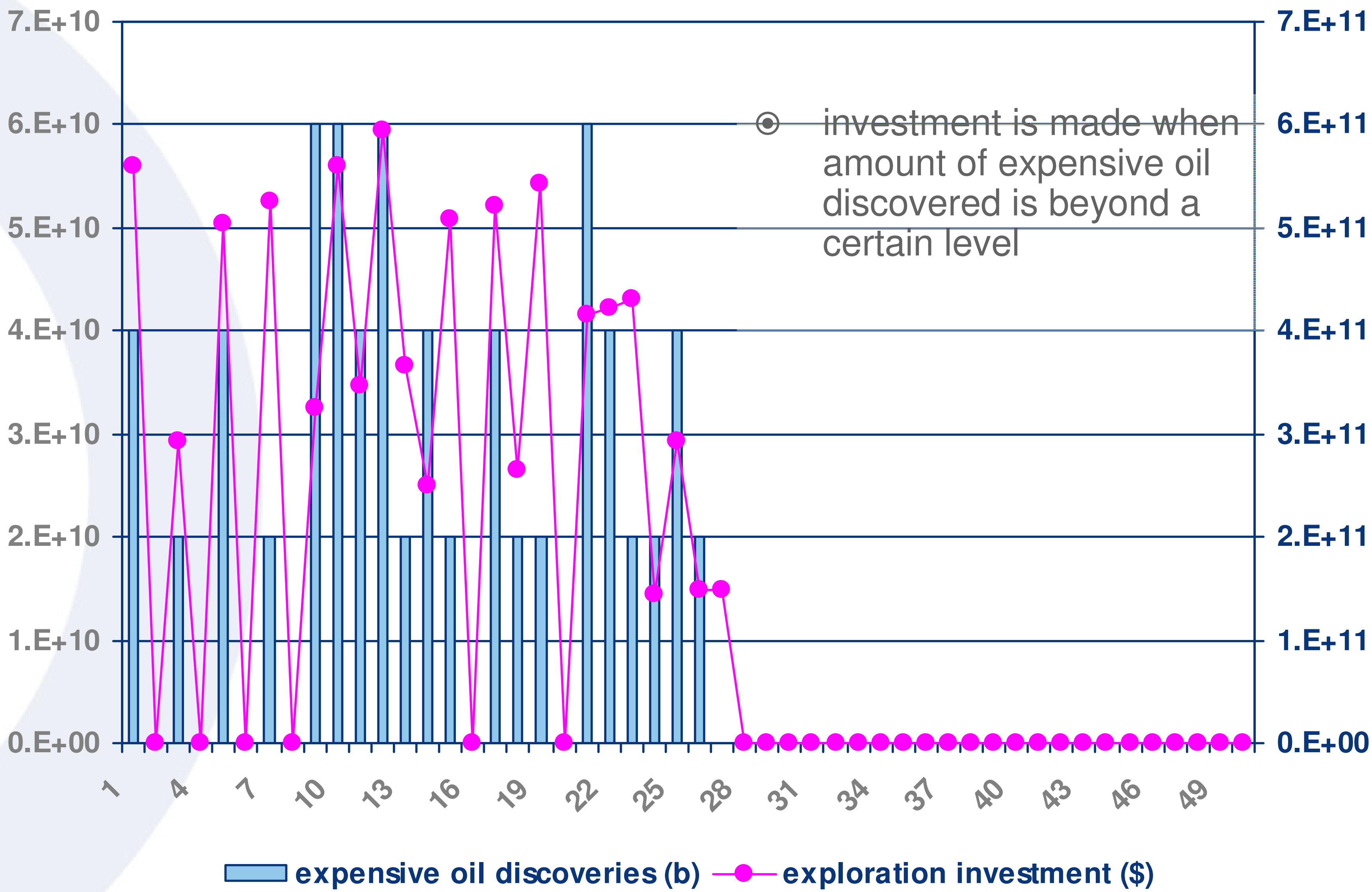
- investment is made when amount of cheap oil in portfolio is below a certain level
- investment levels are very hectic
  - except for the middle period, where exhaustion begins

Model results examples (2)



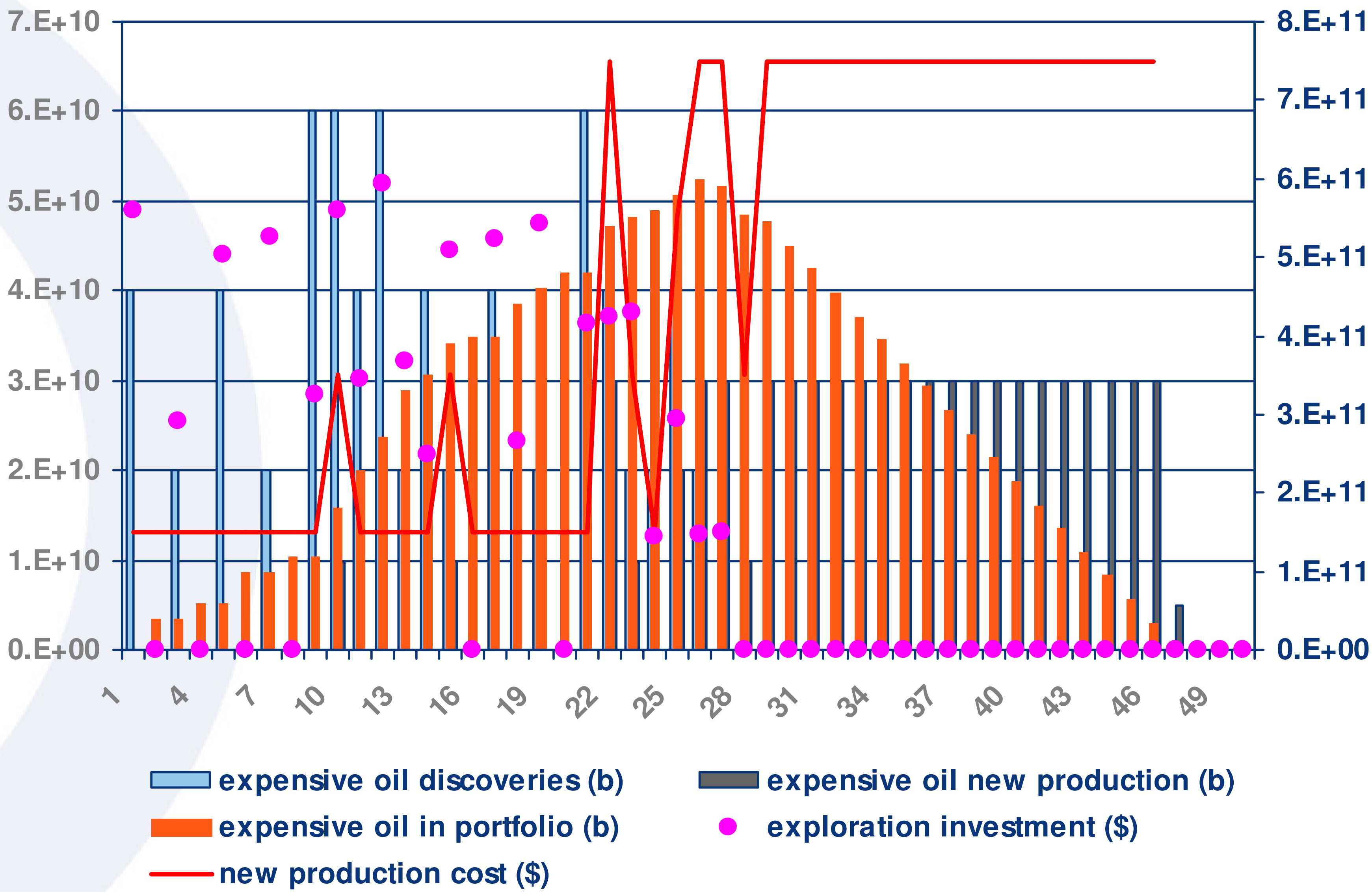
- a big discovery is often followed by a zero investment level
  ➔ close to heuristics in 2nd part

Model results examples (3)



- investment is made when amount of expensive oil discovered is beyond a certain level

Model results examples (4)



Model extension attempt

- enrich hypothesis set
  - to account for a **more realistic randomness**
  - to better fit the **reality of oil market**
- stay close from optimal strategy while approximating
- sliding random tree
  - use a **small depth random tree** to account for exploration randomness
  - use Linear Programming to find the **optimal time-local exploration strategy**
  - make it slide along the time axis

Model extension attempt (2)

- sliding random tree

  - size of tree can be huge…

    $$K\left(n+1, r_t^E\right) = \sum_{s=0}^{r_t^E} K(n, s)$$

    where $K(n,s)$ is the number of allocation of s reserves of n different categories

  - numeric example :

    - n=9 (3 oil production cost and 3 reserve sizes) ; r^E=3 and p=3
    - size of tree: 4,519,515 variables and number of constraints is the same order of magnitude
    - numerical limit for Linear Programming…

➔ does not allow for an interesting set of hypothesis

Possible model extensions

- sliding deterministic tree
  - using **expectation** of discoverable reserves
- sliding random non branching tree
  - defining **several deterministic scenarios** through random tree
- ➔ untested yet…

Simulation model

Objectives
Principle
Method

Model objectives

- get a grasp on the **long-term dynamic of oil price setting**, by accounting for
  - reserves exhaustion
  - different production cost ranges
  - technical constraints (exploration and production)
- combining **classical approaches**
- test the existence of a Hubbert peak on oil production

Model principles

- enrich original Hotelling model to assess for
  - the (random) **discovery of new reserves** through exploration
  - the **technical constraints** on oil production
  - the cost difference between the production of various types of oil (Ricardo), e.g.
    - Arabian light
    - West Texas intermediate
    - North Sea brent
- account for the **exploration strategy** of the agent through **heuristics**

Model hypothesis

- constant and inelastic demand
- 5 cost-differentiated types of oil available, spread into 330 unknown reserves of 3 different sizes
  - **randomness on both size and future production cost** of discoverable reserves
- infinitely and immediately available backstop technology
- knowledge of the agent
  - a priori statistical knowledge of the reserves to be discovered
  - **updated knowledge** along successive discoveries
- constant discovery cost per reserve

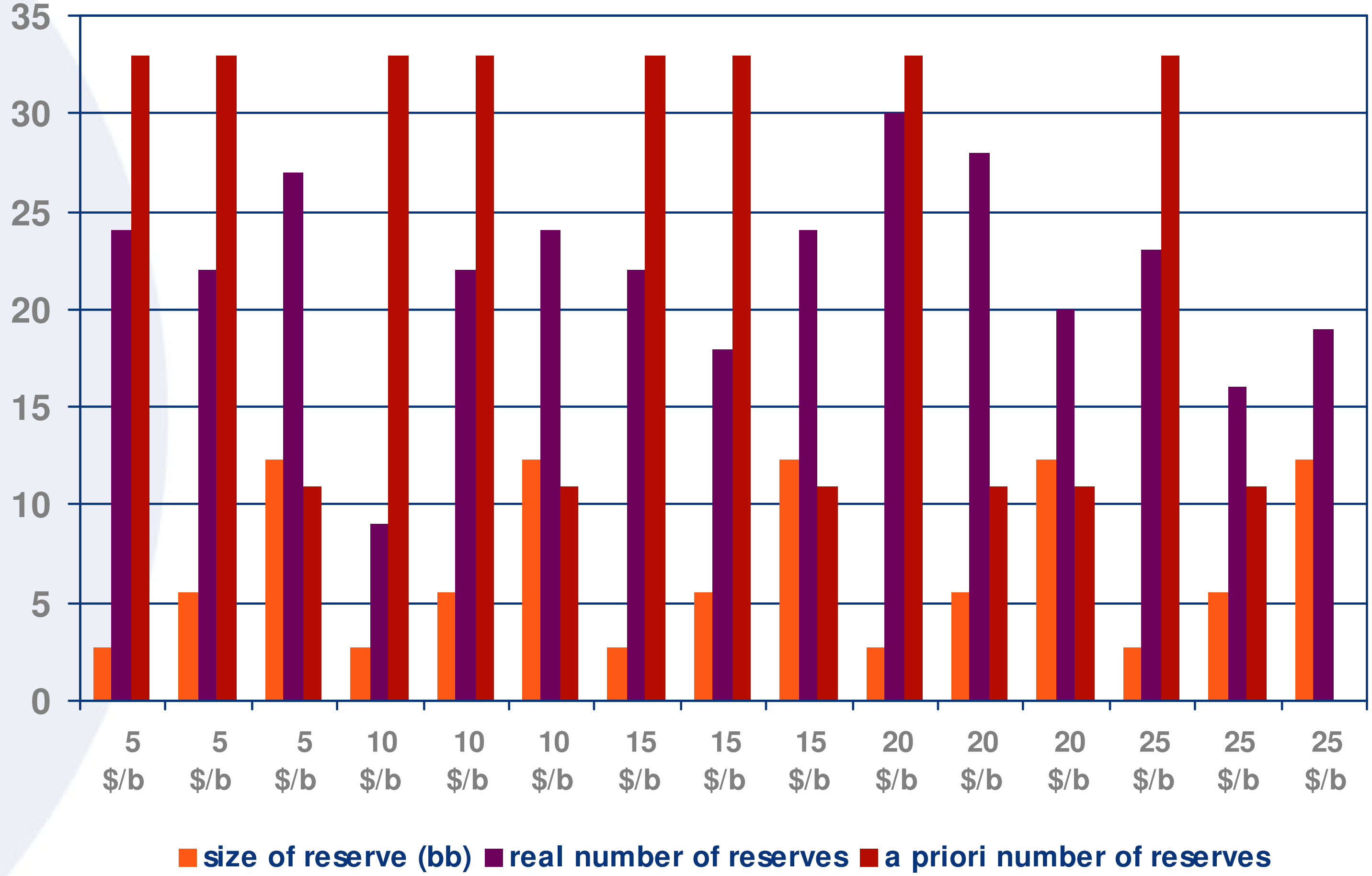
Model description

- 5 cost-differentiated types of oil available, spread into 330 unknown reserves of 3 different sizes

- 2 types of randomness
  - initial modelled distribution of oil reserves on earth
    - follows a uniform law (expected number of reserves from each of the 15 categories is 22)
    - this draw is made once and for all (sense of considering scenarios here?)
  - order in which these reserves are discovered
    - what defines scenarios
    - number of scenarios is tremendous: $\frac{330!}{(22!)^{15}} \equiv 10^{377}$

Model description (2)

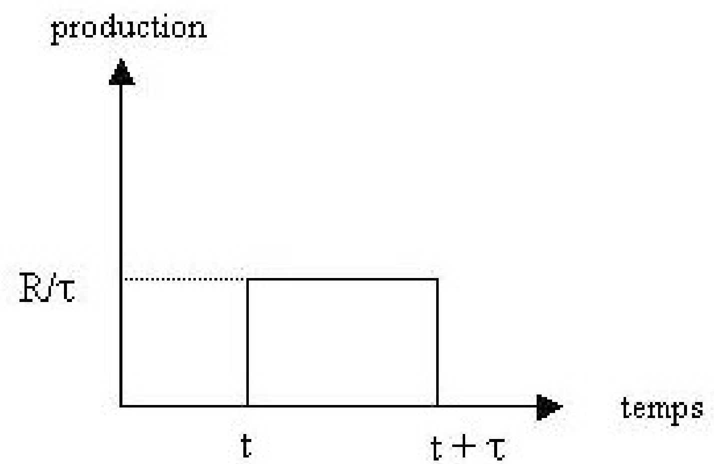
- 5 cost-differentiated types of oil available, spread into 330 unknown reserves of 3 different sizes
  - overestimation of cheap and small reserves ; underestimation of big and expensive reserves



Model description (3)

- demand is satisfied by putting new reserves into production, in the **ascending cost order**

  - under a **technical constraint**: a reserve yields a constant rate of production during τ years



profile of a producing reserve

Model exploration heuristics

- idea: short-sighted vision ➔ explore when it seems worth it, but to meet demand only for **the following time step** *(no dynamic feature)*

1. for each time period

   - **the agent owns a reserves portfolio** inherited from his exploration/production decisions in the past

   - it then computes for each period an **exploration level** such that

     E[Cost$_{exploration}$] + E[marginal Cost$_{production}$(new port.)]

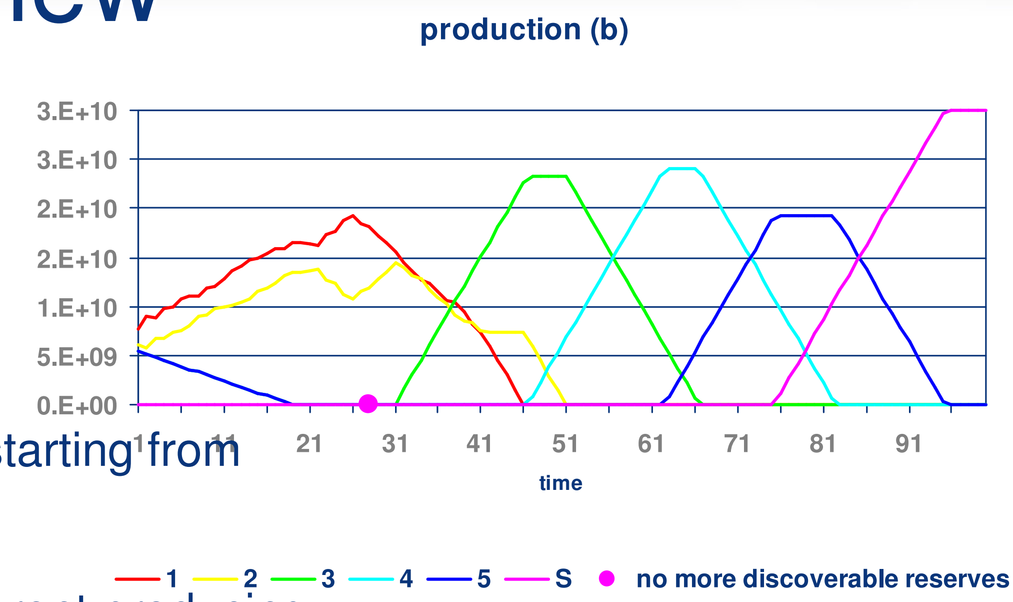
     is less or equal than

     E[marginal Cost$_{production}$(old port.)]

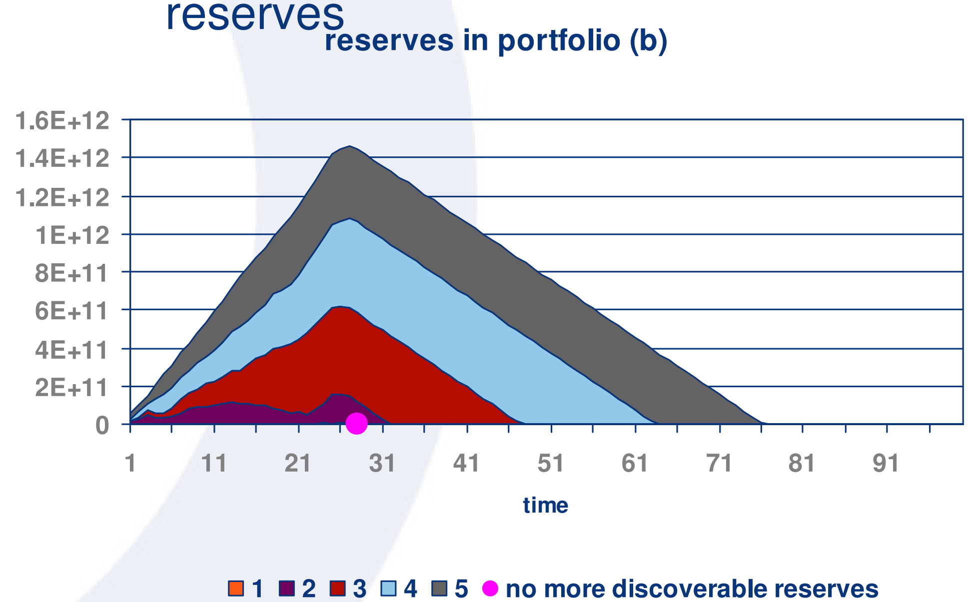
2. it proceeds with exploration, which **randomly returns size and future production cost** of the discovered reserve

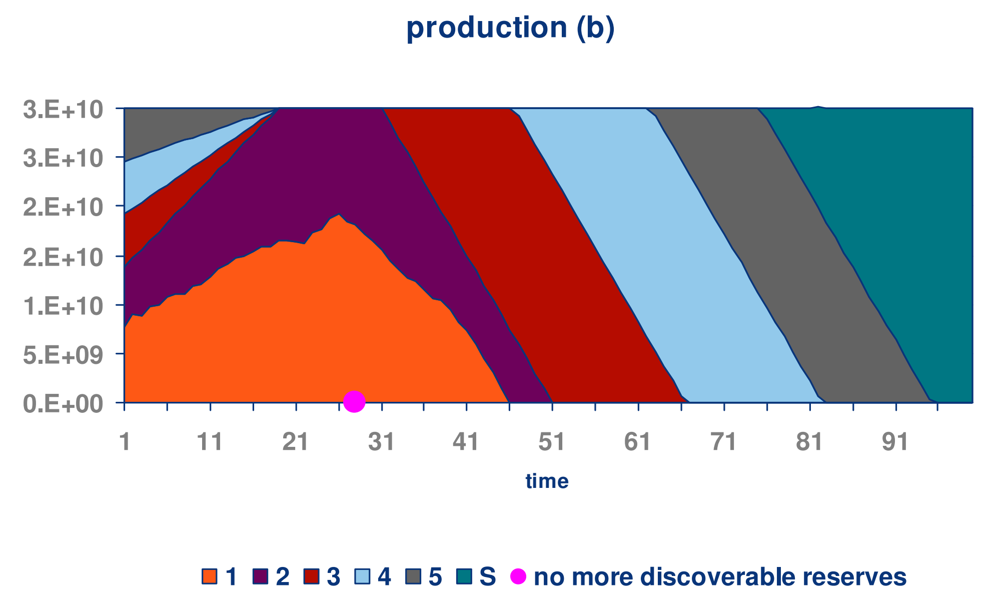
Simulation overview

- simulation over 100 time steps
- total reserves: 2,000 billions barrels
- demand: 30 billions barrels



1. peaks are occurring one after the other, starting from cheapest oil types
2. there is always an overlapping of the different producing reserves



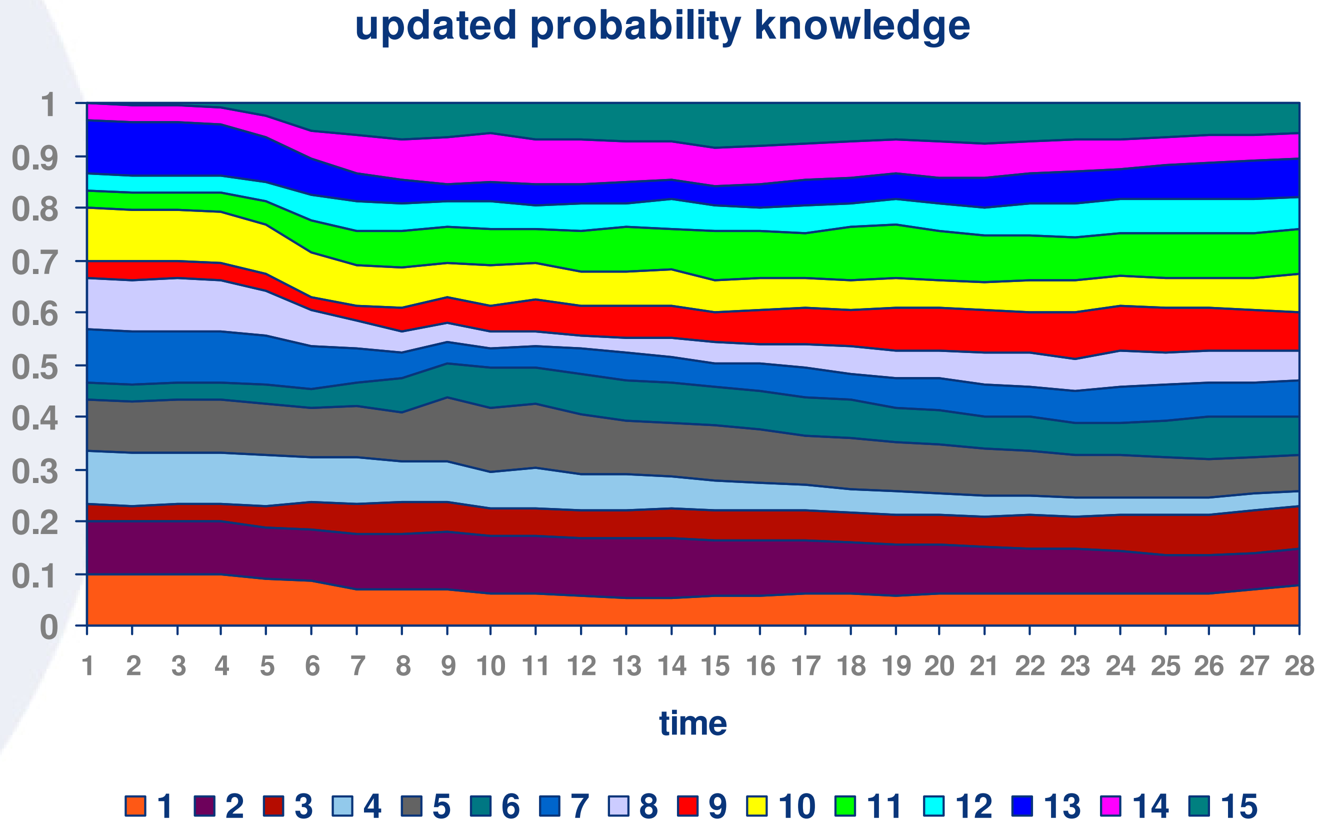


Agent knowledge

- the agent first has to explore to be able to produce oil
  - it starts with an **a priori statistical knowledge** of the reserves to be discovered, which might be **far from the actual reality**
  - it will then **use the outcome of its further discoveries** to update this knowledge

Agent knowledge (2)

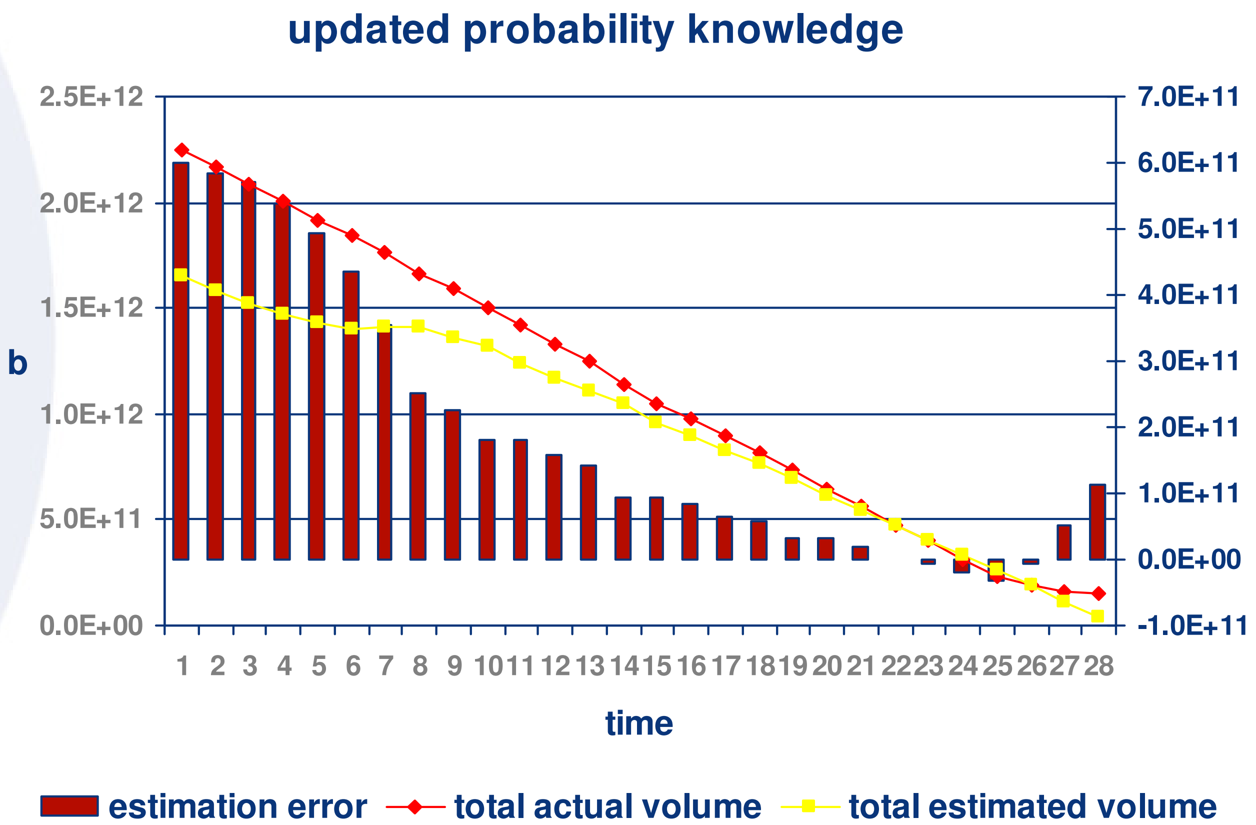
- updated knowledge of reserves allows agent **to better infer outcome of exploration**
  - for each of the 15 cost-size oil categories



Inferring depletion date

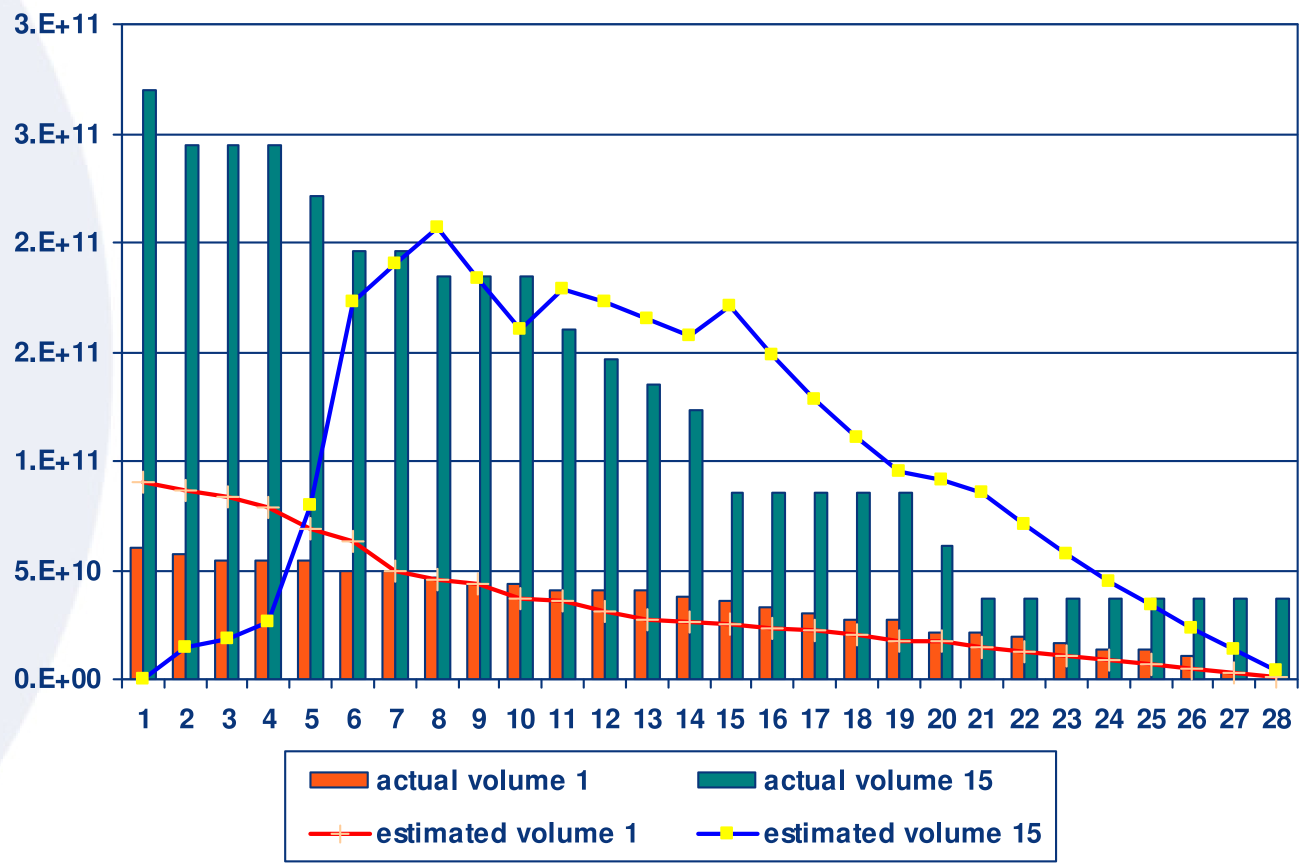
- updated knowledge of the reserves allows agent **to better infer outcome of exploration**

  - total volume estimation error



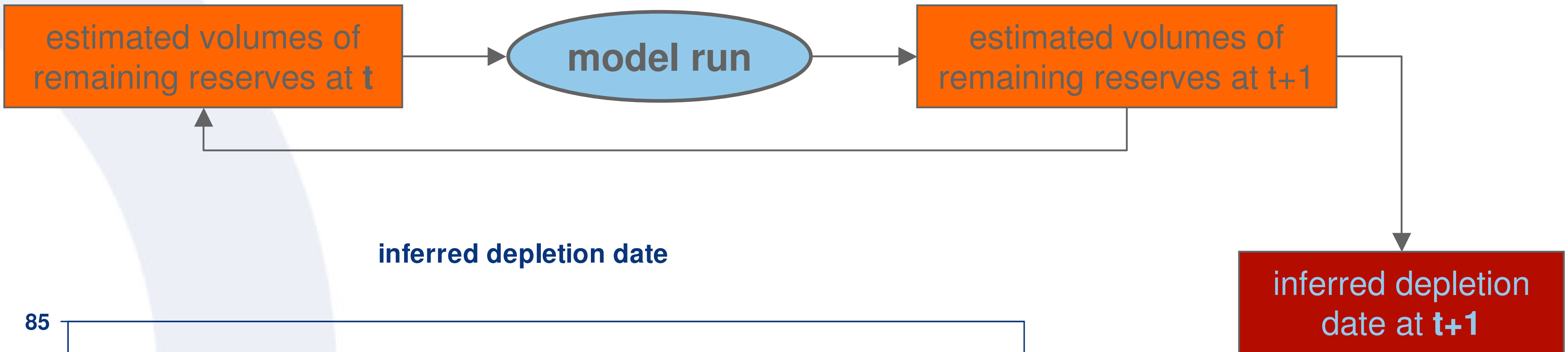
Inferring depletion date (2)

- updated knowledge of the reserves allows agent **to better infer outcome of exploration**
  - example for 2 oil categories

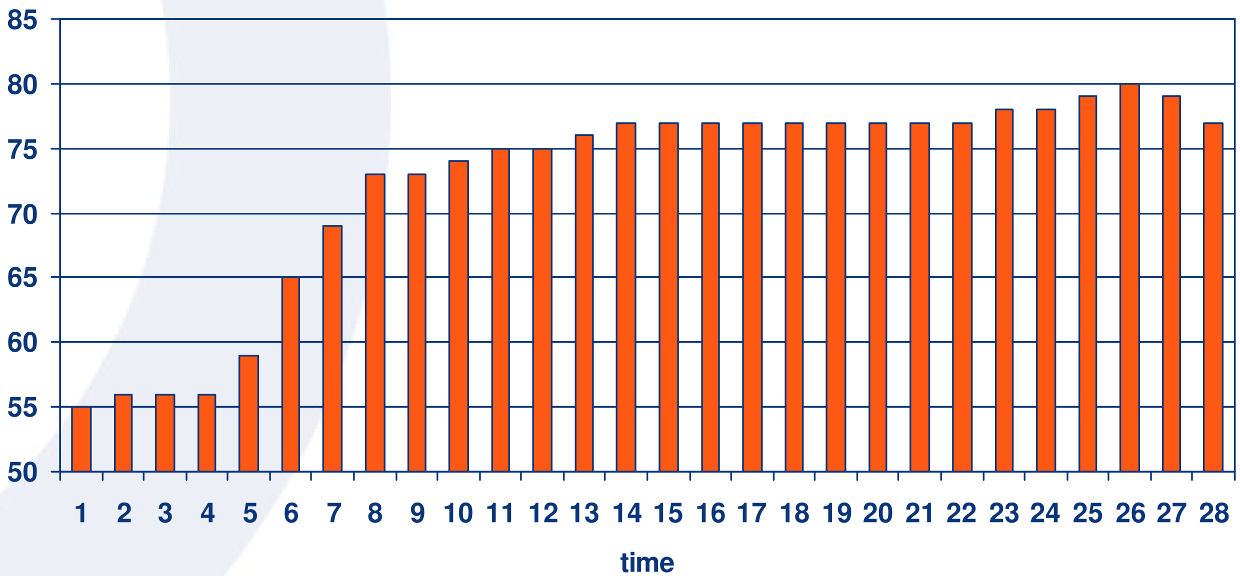


Inferring depletion date (3)

- estimated volumes $V_{total\ remaining}$ of remaining reserves allow agent **to infer a depletion date** T



inferred depletion date



Inferring Hotelling rent

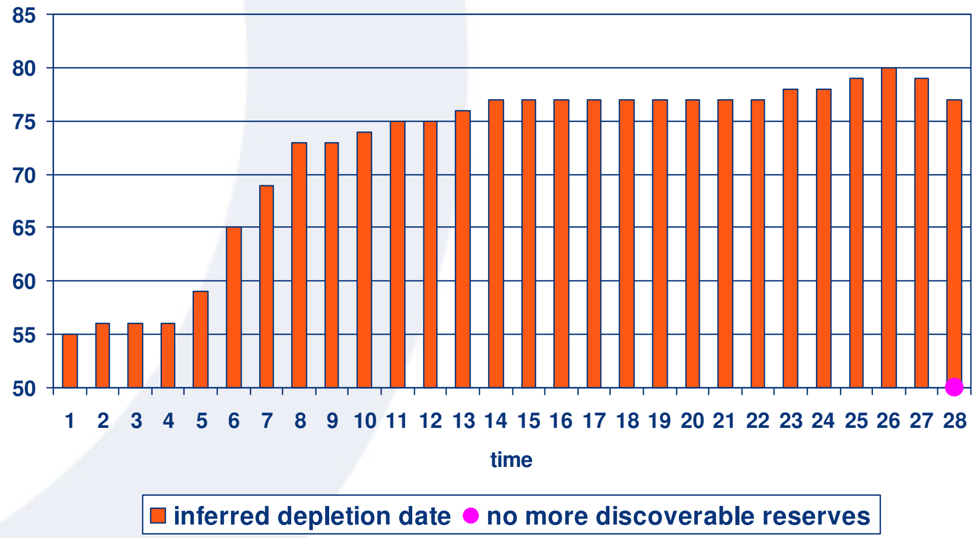
- inferred depletion date *T* allow agent **to update the Hotelling rent** $\boldsymbol{rent_H}$ **at each time step**

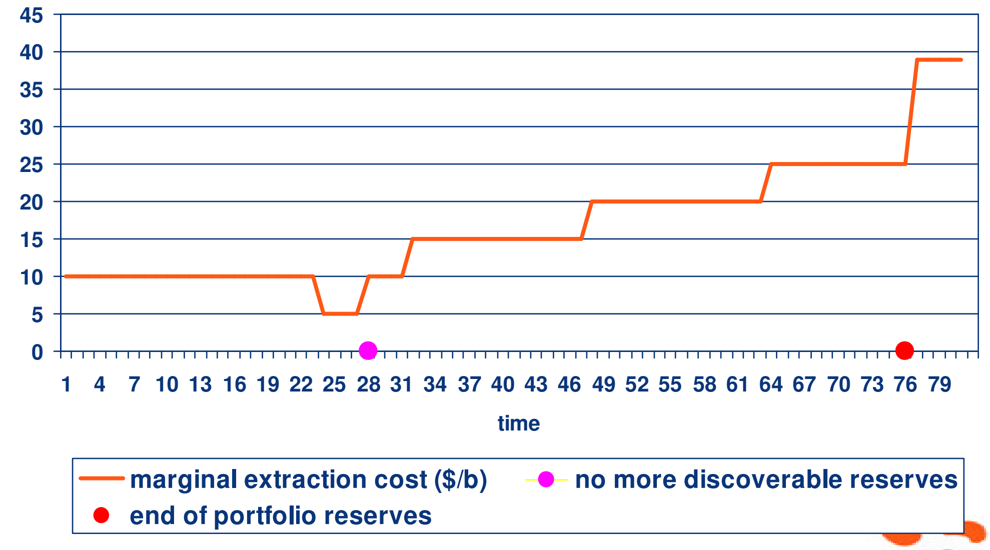
- **discount rate *r* also is affected** by the depletion date being non deterministic

Inferring Hotelling rent (2)

- inferred depletion date *T* at each time step
- discount rate *r* at each time step
- marginal extraction cost *marginal Cost$_{production}$*

$$rent_H = (p_s - p_e)\exp\left(r(T - T_0)\right)$$

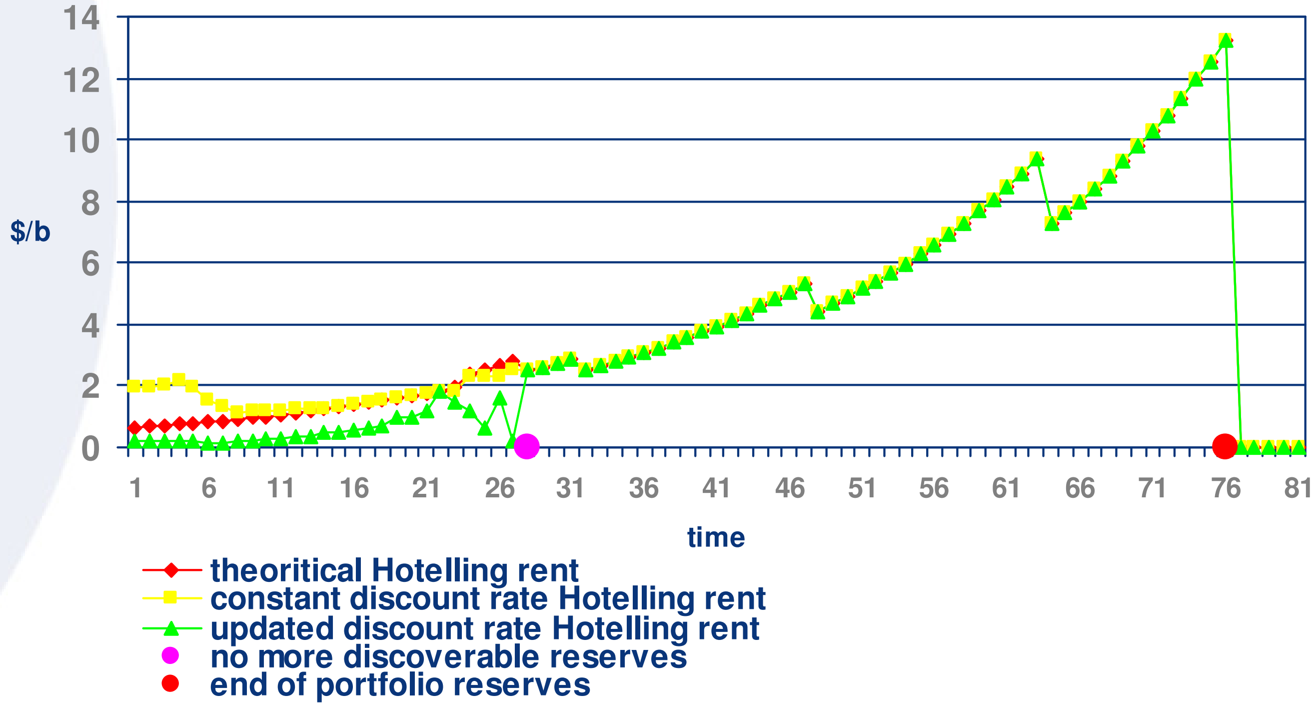




Inferring Hotelling rent (3)

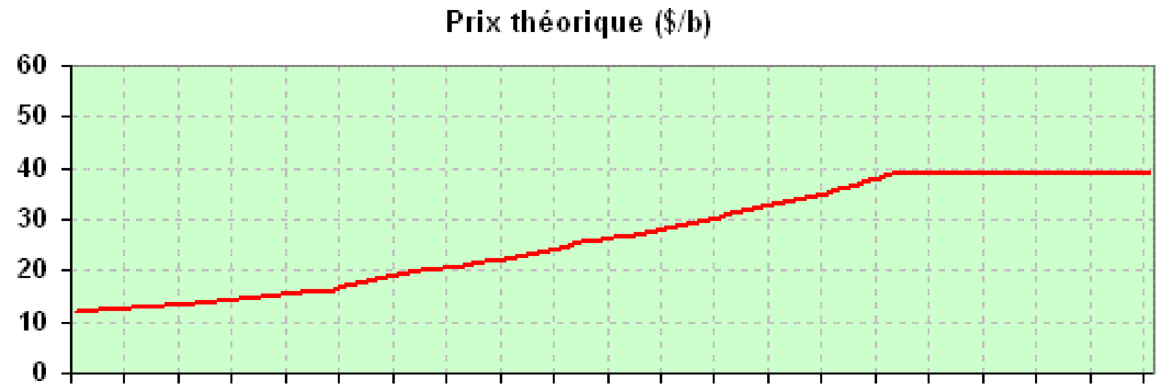
- inferred depletion date *T* at each time step
- discount rate *r* at each time step
- marginal extraction cost *marginal Cost_{production}*

$$rent_H = (p_s - p_e)\exp\left(r(T - T_0)\right)$$



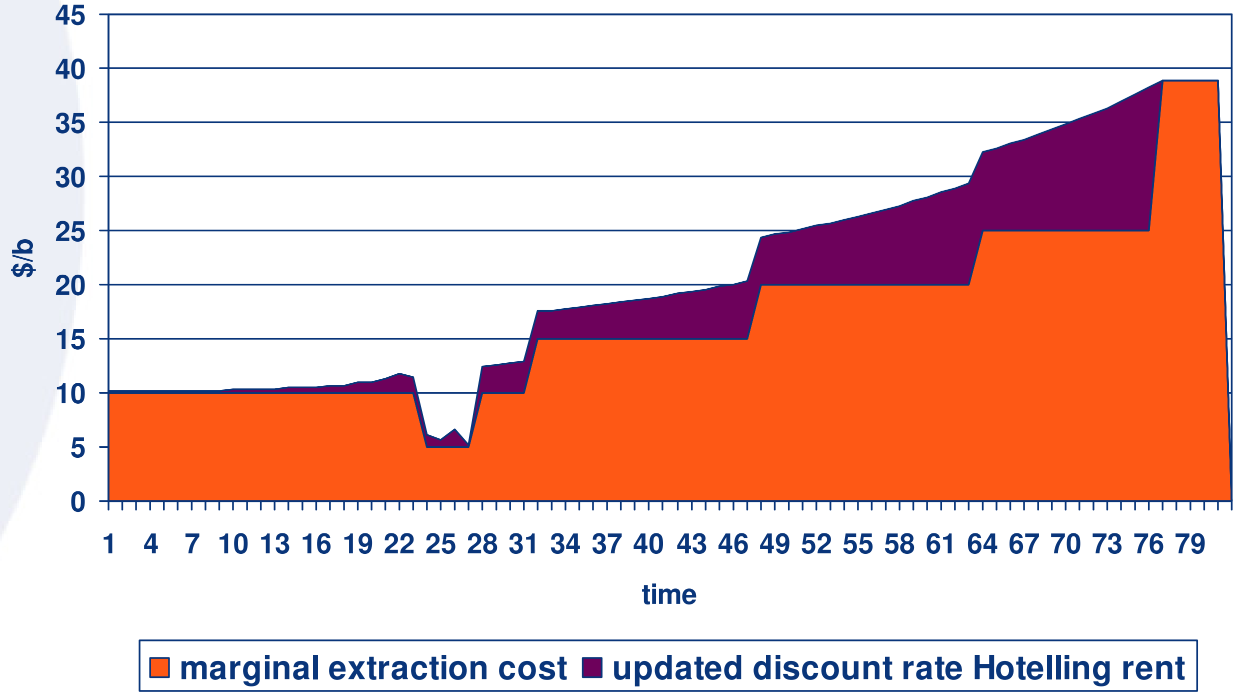
Price output

- theoretical definition
  - price for each time step is the **expectation of the marginal production cost of one barrel**
  - more precisely, price is defined as the expectation of the marginal production cost of **1/τ barrel during τ years**
  - due to the variety of possible scenarios, such price might depend (among others) on realization



Price output (2)

- rational definition
  - for a given agent, price corresponds to the **sum of all costs necessary to meet demand**
    - **extraction cost** of marginal reserve
    - **exploration cost** ➔ supposed to be covered by production **differential rents**
    - **Hotelling rent**



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