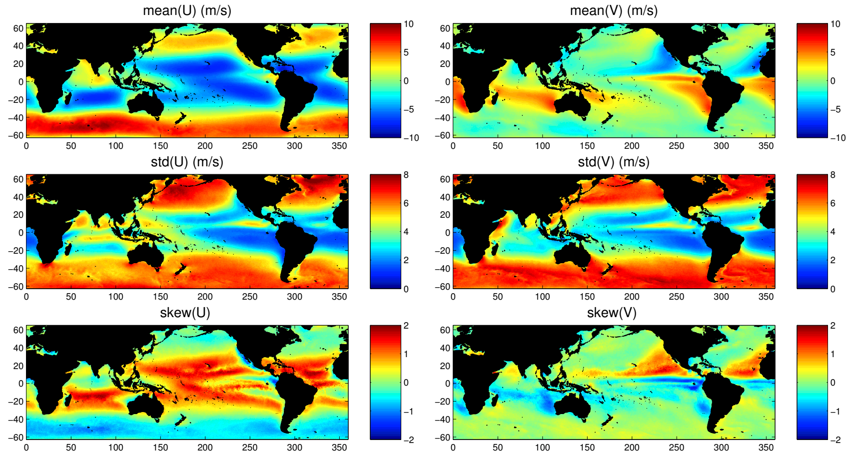
Case Study: Sea Surface Winds

- *Air/Sea Exchange*
  - ocean and atmosphere interact through respective boundary layers, exchanging momentum, energy, freshwater, and gases
  - fluxes depend on surface winds, in general nonlinearly
  - ocean currents largely driven by surface winds
- *Sea State*
  - sea state important for shipping, recreation
  - determined by both local and remote winds
- *Power Generation*
  - wind power potentially significant source of energy
  - generation rate scales as cube of wind speed; extreme events important

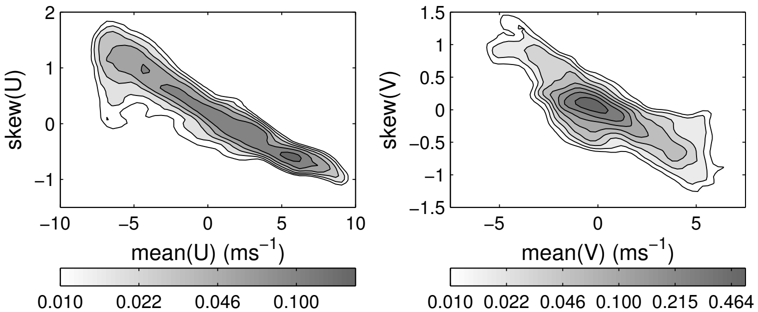
Vector Wind Moments



Zonal Wind　　Meridional Wind

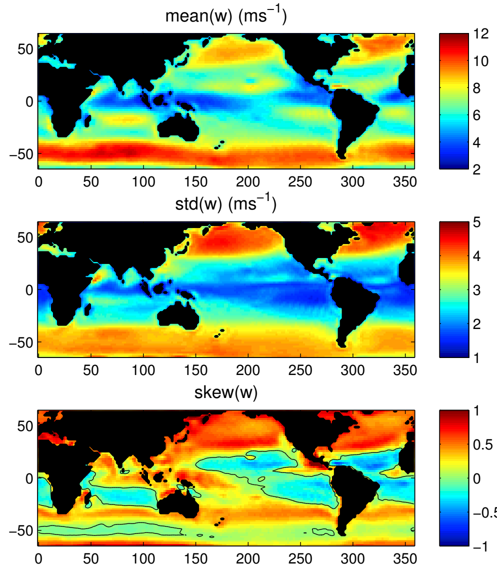
Mean and Skewness of Vector Wind

- Joint pdfs of mean and skew for zonal and meridional winds



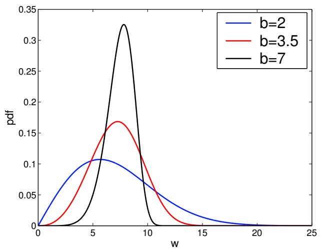
(note logarithmic contour scale)

Wind Speed Moments



Wind Speed pdf: Weibull distribution

- The pdf of wind speed w has traditionally (and empirically) been represented by 2-parameter Weibull distribution:

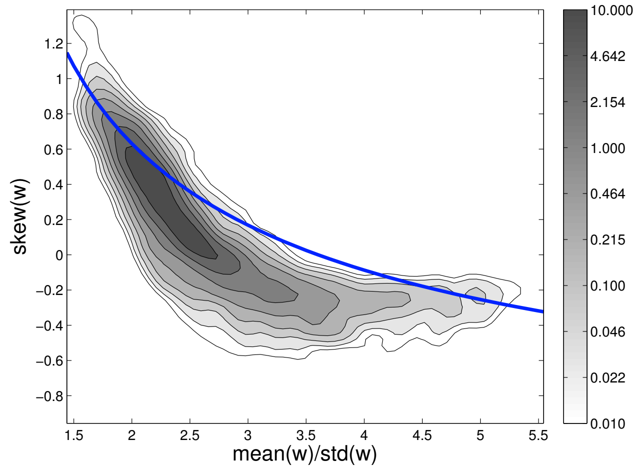


$$p(w) = \frac{b}{a}\left(\frac{w}{a}\right)^{b-1} \exp\left[-\left(\frac{w}{a}\right)^{b}\right]$$

- a is the scale parameter (pdf centre)
- b is the shape parameter (pdf tilt)
- $p_w(w)$ is unimodal

Wind Speed pdfs: Observed

- Observed speed moments fall around Weibull curve



Boundary Layer Dynamics

- Horizontal momentum equations:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} = -\frac{1}{\rho}\nabla p - f\hat{\mathbf{k}}\times\mathbf{u} - \frac{1}{\rho}\frac{\partial(\rho\overline{\mathbf{u}'u_3'})}{\partial z}$$

- Momentum tendency due to:
  - advection (transport by flow; secondary importance on daily timescales)
  - pressure gradient force
  - Coriolis force
  - turbulent momentum flux (in vertical)
- Integrated momentum budget over boundary layer h:

$$\frac{d\mathbf{u}}{dt} = -\frac{1}{\rho}\nabla p - f\hat{\mathbf{k}}\times\mathbf{u} + \frac{1}{h}\left(\overline{\mathbf{u}'u_3'}(0) - \overline{\mathbf{u}'u_3'}(h)\right)$$

Surface Wind Stress

- Surface wind stress is turbulent momentum flux across air/sea interface:

$$\tau_s = \rho_a\overline{\mathbf{u}'u_3'}(0)$$

where

$$\begin{aligned} u &= \text{along-mean wind component} \\ v &= \text{cross-mean wind component} \\ \mathbf{u} &= (u, v) \\ u_3 &= \text{vertical wind component} \end{aligned}$$

- Flux parameterised in terms of $\mathbf{u}$ by bulk drag formula:

$$\tau_s = \rho_a c_d w \mathbf{u}$$

where $w = \| \mathbf{u} \|$ is the wind speed.

Surface Momentum Budget

- To close momentum budget, need parameterisation of turbulent momentum flux at $z = h$
- Use specified "entrainment velocity" W_e

$$\overline{\mathbf{u}' u_3'}(h) = W_e(\mathbf{U} - \mathbf{u})$$

$\Rightarrow$ Surface layer momentum budget

$$\begin{aligned} \frac{d\mathbf{u}}{dt} &= -\frac{1}{\rho}\nabla p - f\hat{\mathbf{k}} \times \mathbf{u} - \frac{c_d}{h} w \mathbf{u} + \frac{W_e}{h}(\mathbf{U} - \mathbf{u}) \\ &= \mathbf{\Pi} - \frac{c_d}{h} w \mathbf{u} - \frac{W_e}{h}\mathbf{u} \end{aligned}$$

where

$$\mathbf{\Pi} = -\frac{1}{\rho}\nabla p - f\hat{\mathbf{k}} \times \mathbf{u} + \frac{W_e}{h}\mathbf{U}$$

Mechanistic Model: SDE

- Decomposing $\mathbf{\Pi}$ into mean and fluctuations:

$$\begin{aligned} \Pi_u(t) &= \langle \Pi_u \rangle + \sigma \dot{W}_1(t) \\ \Pi_v(t) &= \sigma \dot{W}_2(t) \end{aligned}$$

where $\dot{W}_i$ is Gaussian white noise

$$\left\langle \dot{W}_i(t_1)\dot{W}_j(t_2) \right\rangle = \delta_{ij}\delta(t_1 - t_2)$$

we obtain stochastic differential equation

$$\begin{aligned} \frac{du}{dt} &= \langle \Pi_u \rangle - \frac{c_d}{h} w u - \frac{W_e}{h} u + \sigma \dot{W}_1 \\ \frac{dv}{dt} &= -\frac{c_d}{h} w v - \frac{W_e}{h} v + \sigma \dot{W}_2 \end{aligned}$$

Mechanistic Model: pdf

- Solution of associated Fokker-Planck equation for stationary pdf:

$$p_{uv}(u,v) = \mathcal{N}_1 \exp\left(\frac{2}{\sigma^2} \left\{ \langle \Pi_u \rangle u - \frac{W_e}{2h}(u^2 + v^2) - \frac{1}{h}\int_0^{\sqrt{u^2+v^2}} c_d(w')w'^2 \, dw' \right\} \right)$$

- Changing to polar coordinates and integrating over angle gives wind speed pdf:

$$p_w(w) = \mathcal{N} w I_0 \left(\frac{2\langle \Pi_u \rangle w}{\sigma^2} \right) \exp\left(-\frac{2}{\sigma^2} \left\{ \frac{W_e}{2h} w^2 + \frac{1}{h}\int_0^w c_d(w')w'^2 \, dw' \right\} \right)$$

Mechanistic Model: Predictions