

Irregularity and Predictability of ENSO

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Main Reference

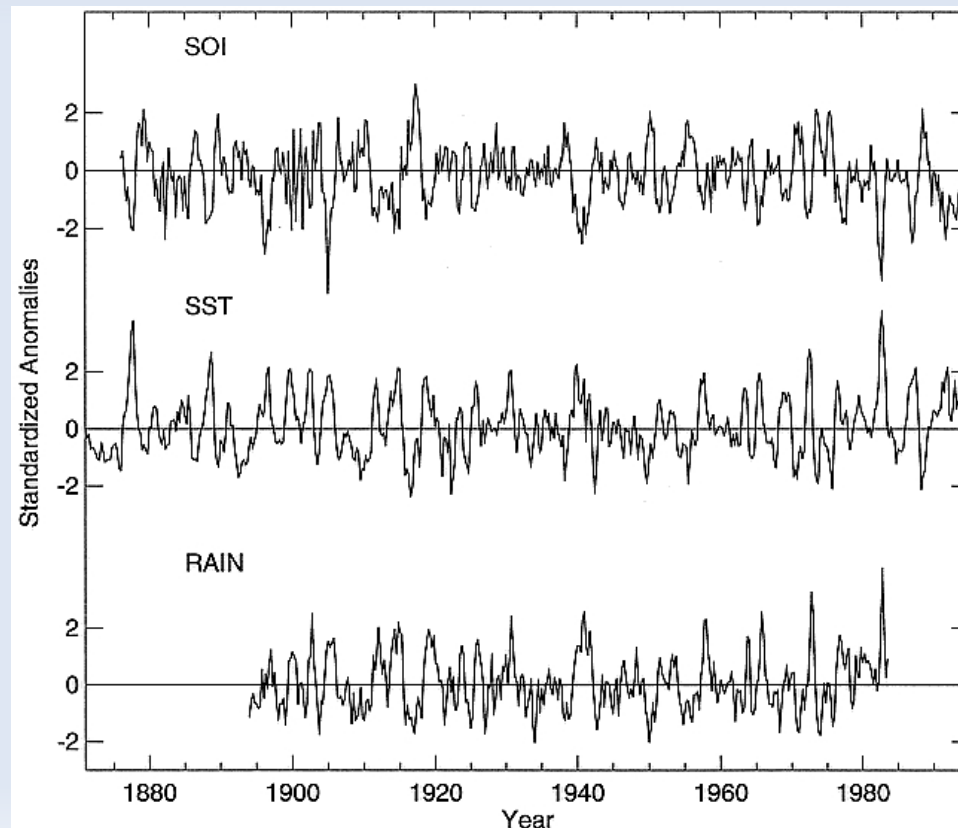
R. Kleeman. Stochastic theories for the irregularity of ENSO. *Phil. Trans. Roy. Soc. A.*, 2008. In press.

Conceptual Background

- Predictability is often intuitively related to the degree to which a system has any "regular" oscillations. This is an indicative but not comprehensive view.
- Some systems such as the solar system are highly regular and predictable for centuries in advance.
- Other systems with much less periodicity (such as earthquakes) are quite difficult to predict.
- Predictability is limited in "chaotic" or "stochastic" (randomly forced) dynamical systems. Such systems often are characterised by "broadband" spectral peaks.
- What about ENSO?

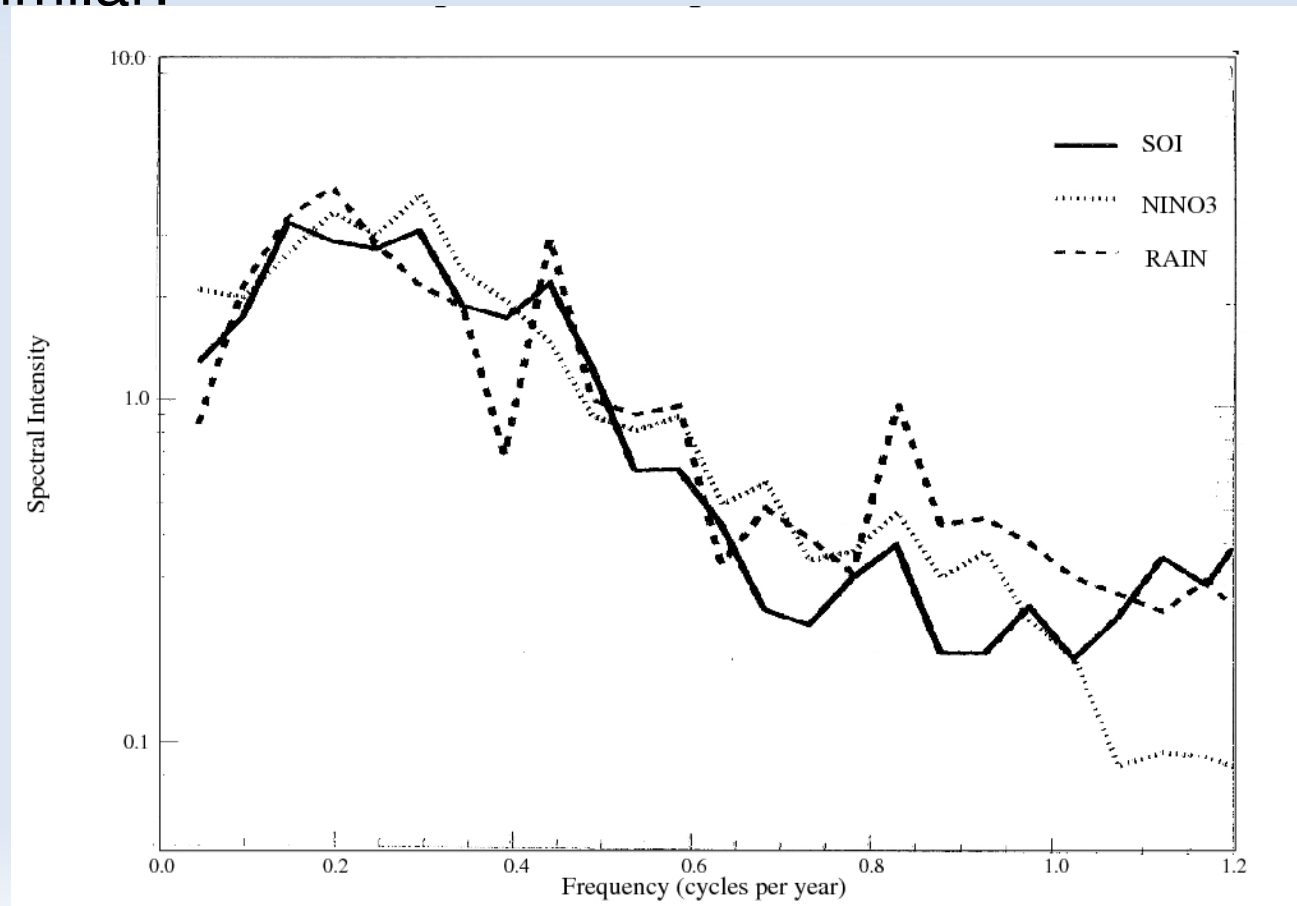
Observational Evidence

There are difficulties in defining ENSO irregularity due to the shortness of the length of time of reliable observations. Nevertheless the common indices of variability have "reasonable" definition for about a century or so.



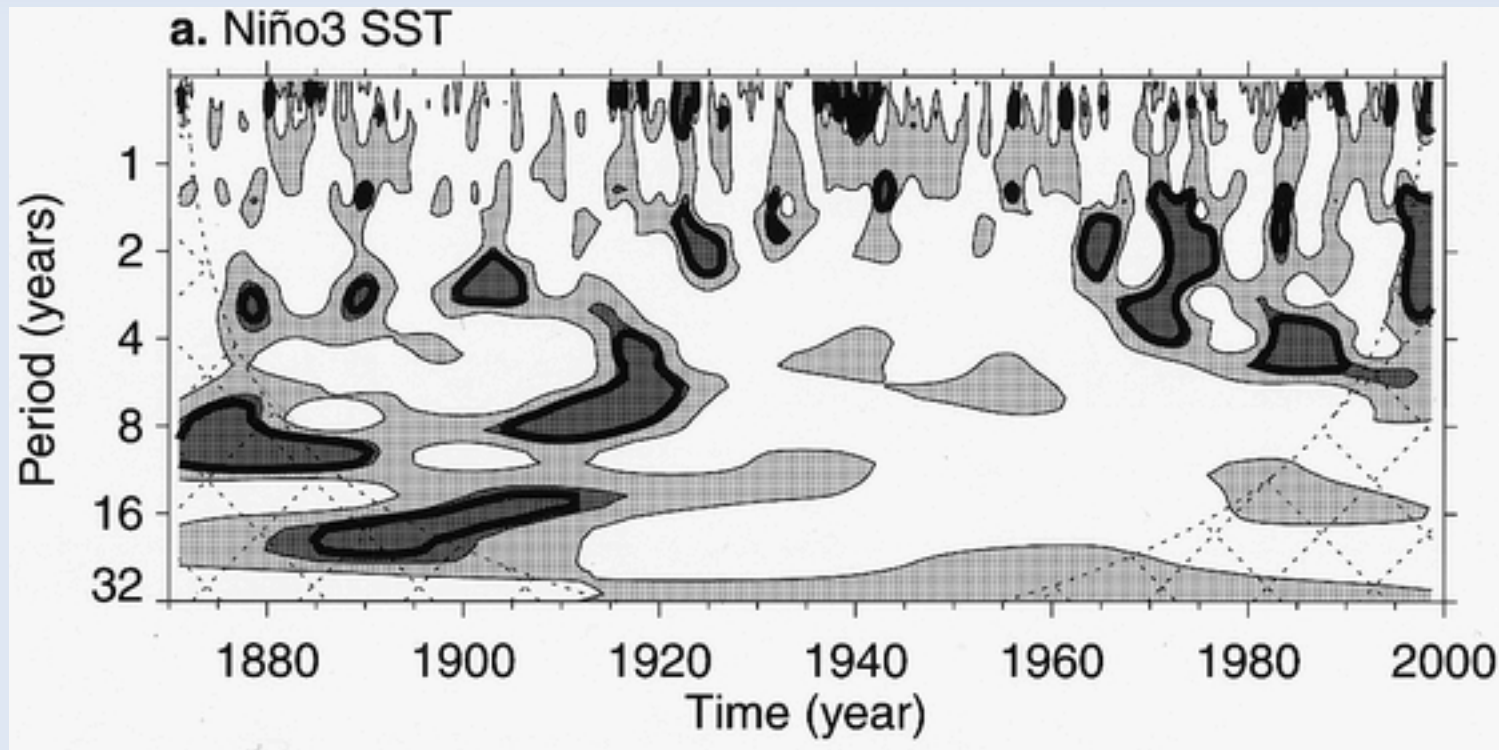
Observational Evidence

Clearly these indices are irregular but some periodicity can be discerned if the series are carefully scrutinized. The spectra of the different indices over this period of a century or so are remarkably similar:



Observational Evidence

Other interesting observational features of ENSO include (partial) phase locking to the annual cycle with warm events tending to peak in the Northern winter. Also there appears to be significant decadal variation in the spectrum as a wavelet analysis shows:



Theories of ENSO irregularity

- Two main major mechanisms have been proposed and consensus has yet to be achieved on which dominates. To describe these it is useful to divide the tropical system into slow and fast components. The first is set primarily by the ocean while the latter by the atmosphere. Slow phenomena include things like oceanic internal waves (Kelvin and Rossby) and the annual cycle while the latter includes things like tropical "weather" and the MJO. This is a conceptual division to aid in theoretical description.

Theory 1: ENSO as a chaotic oscillator

- In this theory the slow components of the coupled tropical system interact non-linearly producing a well known form of chaos.

Theory 2: ENSO as a stochastically forced oscillator

- In this theory the fast components of the system are able to randomly disrupt the slow components of the coupled tropical system.

ENSO as a chaotic oscillator

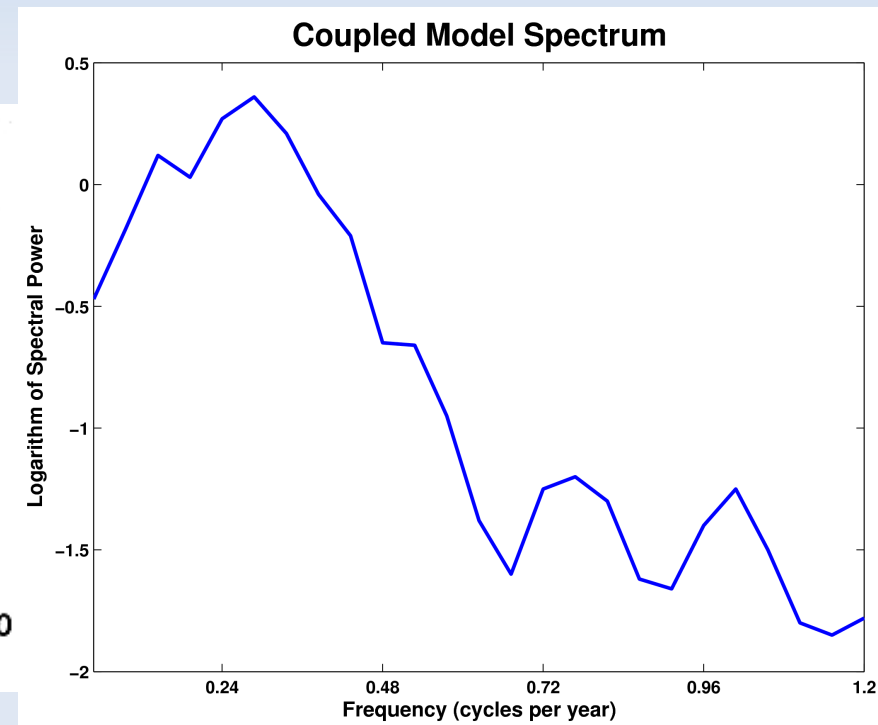
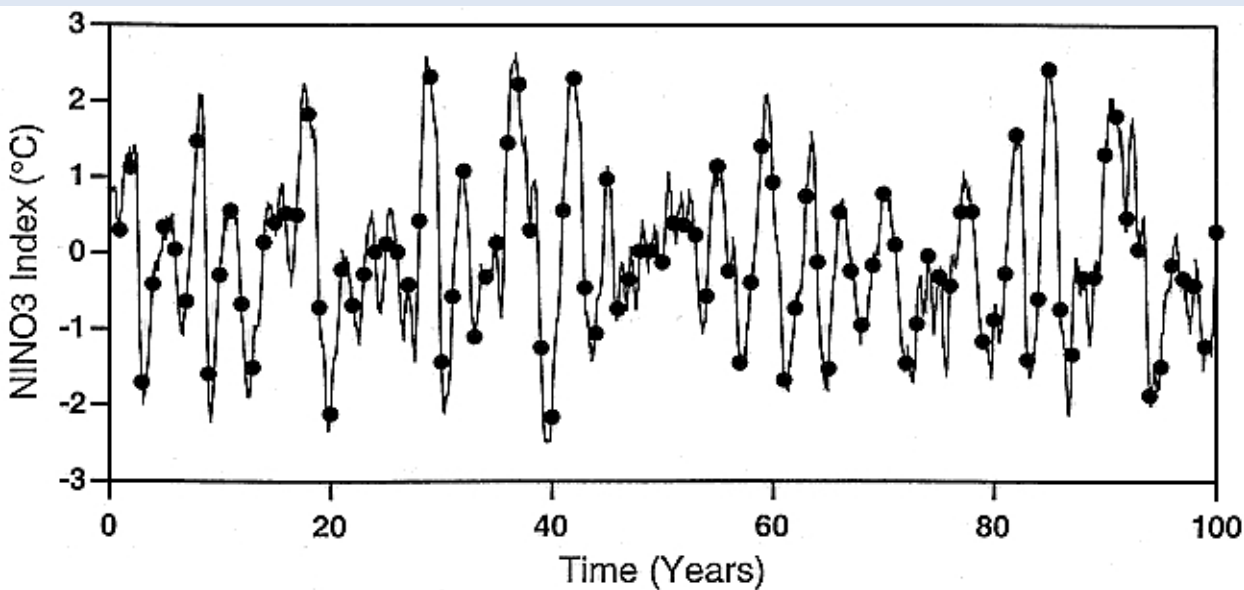
- The slow component of the coupled system has two major natural modes of variability. The first is the seasonal cycle which is an external forcing. The second group are internal modes which result fundamentally from the coupling of the atmosphere and ocean. Linear instability analysis of coupled models shows that there is often a dominant (i.e. most destabilized) mode. The structure of this mode resembles ENSO strongly.
- In general the external annual forcing causes the internal mode to "lock onto" a periodicity which is some multiple of the annual cycle e.g. 3 or 4 years. Which particular multiple is selected depends on the nature of the internal coupled mode i.e. what period it would have in the absence of the external forcing.
- If the natural period of the internal mode is adjusted to lie between two multiples of the annual period the system mode may transition irregularly between the two multiples due to non-linear interaction.

ENSO as a stochastic oscillator

- Linear instability analysis of coupled models (stochastic optimal analysis) reveals that they are particularly susceptible to forcing with specific large scale patterns.
- If variability from the fast component of the system "projects" significantly onto the above patterns of susceptibility then it can significantly perturb the coupled system.
- Coupled models with random forcing having the "right" large scale structure develop highly robust irregular oscillations with spectra matching the observations well. They can also reproduce warm event seasonal phase locking as well as produce strong decadal variation in their spectra again matching the observations.

ENSO as a stochastic oscillator

A sample stochastically forced coupled model. This model was used in student projects



Stochastic Optimals

A crucial aspect of the stochastic oscillator theory of irregularity is that the stochastic forcing should have significant projection onto certain large scale patterns. We review this.

$$u(t + \Delta t) = R(t + \Delta t, t)u(t) + \Delta t F$$

R is the linear propagator while F is the additive stochastic forcing. Note that this is a vector equation where the vector indices are for different spatial points and different variables.

Suppose we are interested in the variance growth of a particular index region of the model:

$$Var(t) = \sum_{i,j} X_{ij} cov(u^i(t), u^j(t))$$

With reasonable assumptions about the stochastic forcing we can solve the first stochastic equation and write this variance as

$$Var(t) = trace(ZC)$$

$$Z = \int_{t'}^t R^t(t, s) X R(t, s) ds$$

$$C_{ij} = cov(F_i, F_j)$$

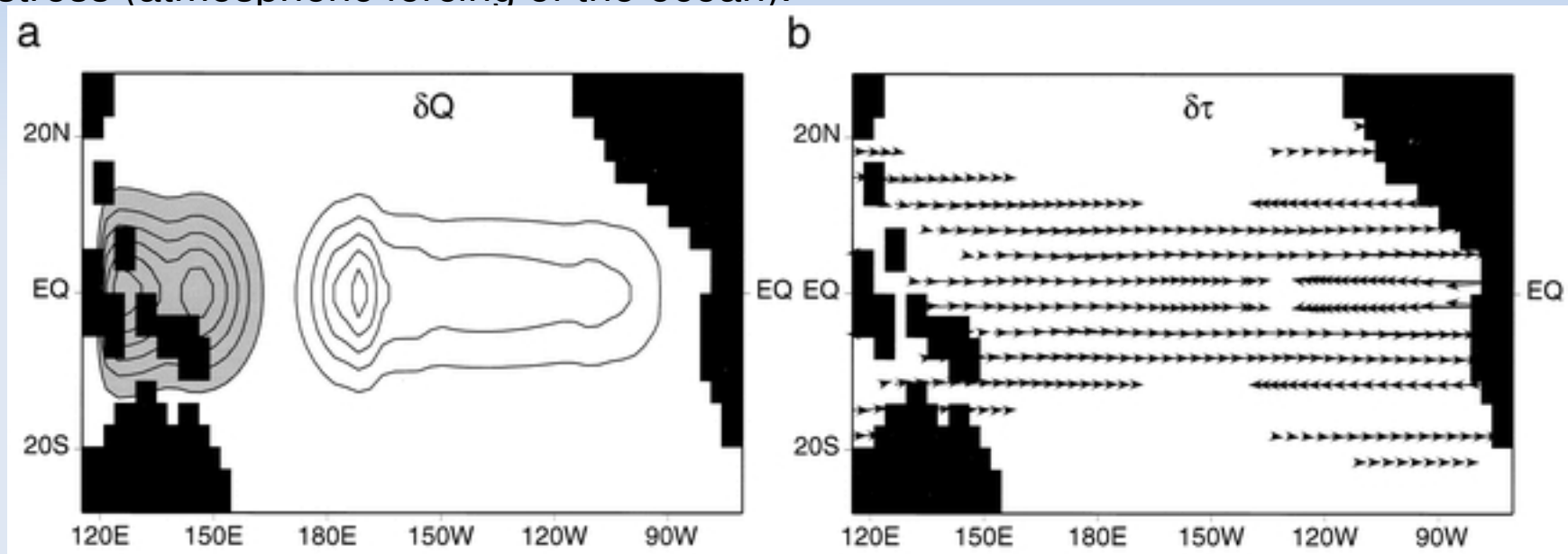
The matrices Z and C are usually both symmetric and positive so have orthogonal eigenvectors with non-negative eigenvalues. Moreover the first line above can be rewritten in terms of these eigenvectors and eigenvalues:

$$Var(t) = trace(ZC) = \sum_{m,n} p_m q_n (P_m, Q_n)^2$$

Thus for this to be large we need some of the eigenectors of Z and C to match and also have large eigenvalues. The latter measure system instability and noise variance respectively.

Stochastic Optimals

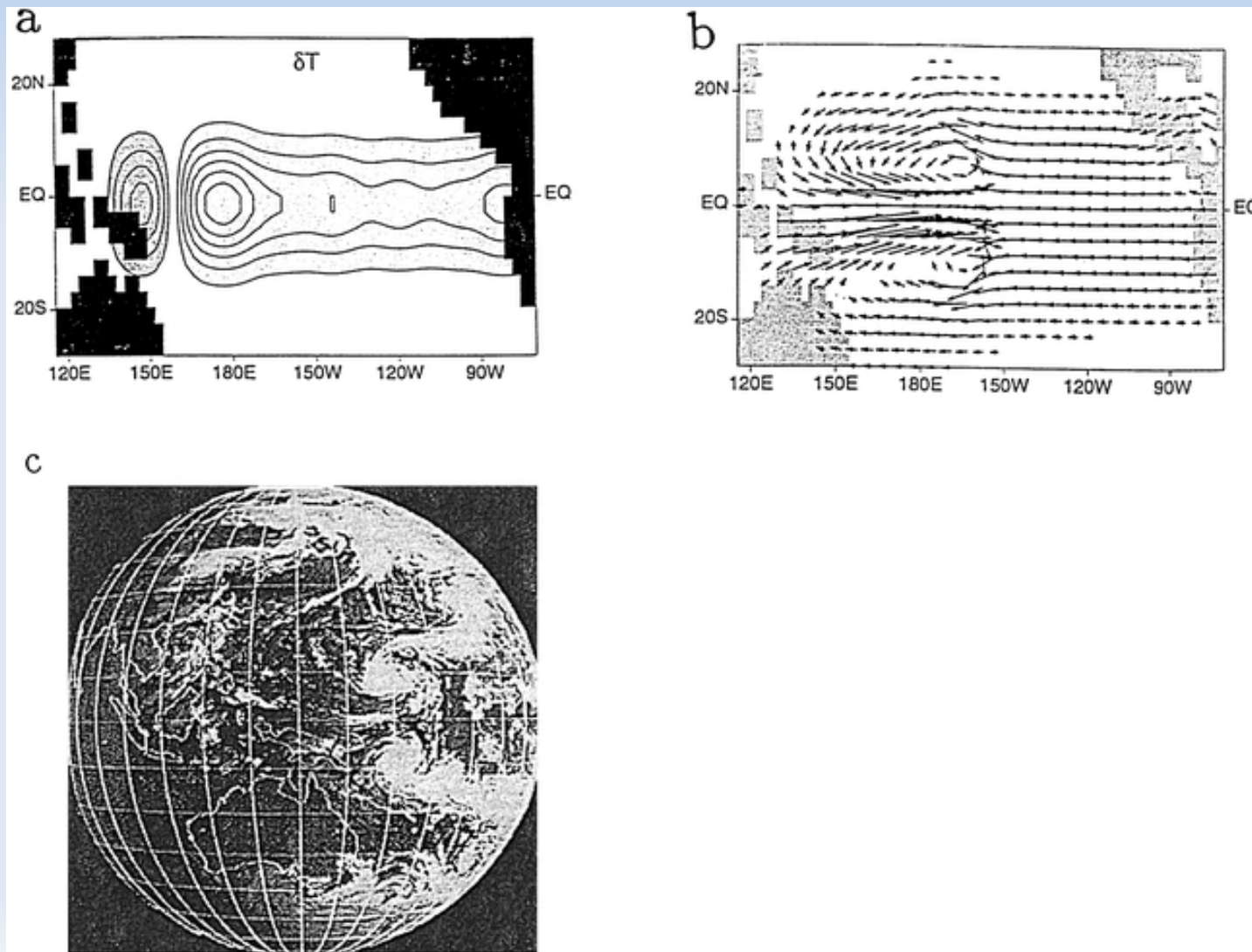
The eigenvectors of Z (i.e. the P_m) are called stochastic optimals. If we calculate them in a coupled model we find that their eigenvalues drop off rapidly so only a few are important for explaining how variance of important indices grows. Here are some examples for heat flux and wind stress (atmospheric forcing of the ocean).



These patterns can vary somewhat from coupled model to coupled model so the sensitivity to stochastic forcing can vary somewhat. Note that for the variance of our target variable to grow there must be a significant inner product between the stochastic optimals and the eigenvectors of the forcing covariance matrix C (i.e. the Q). These eigenvectors give patterns of stochastic forcing. The corresponding eigenvalue is the variance explained by the particular pattern. Such patterns are usually called EOFs since they are the eigenvectors of a covariance matrix.

Stochastic Optimals

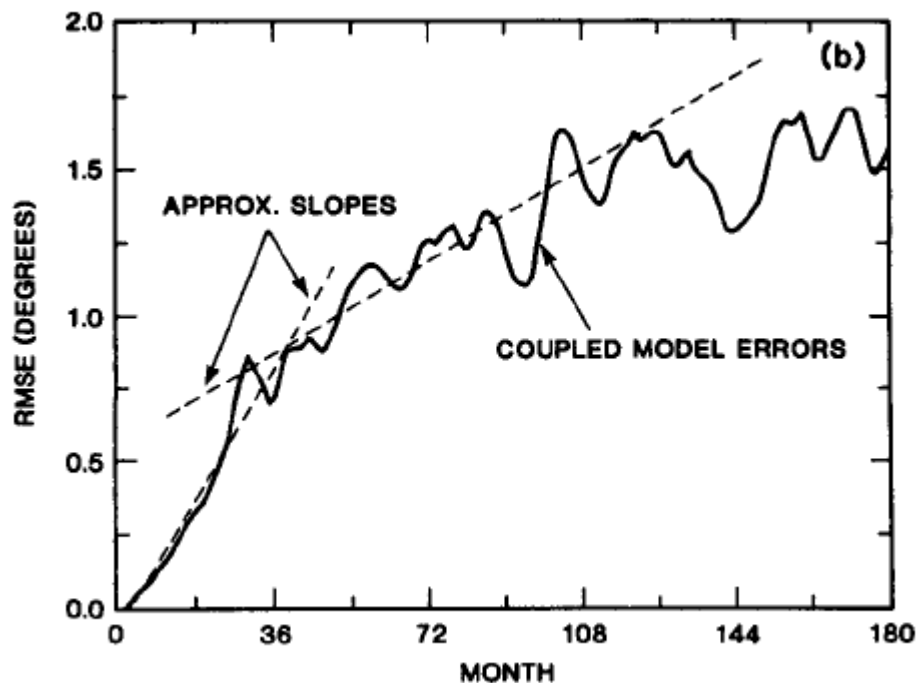
If the coupled model is perturbed by the previous pattern the system rapidly (1-2 weeks) develops a very characteristic response which in some coupled models strongly resembles a westerly wind burst.



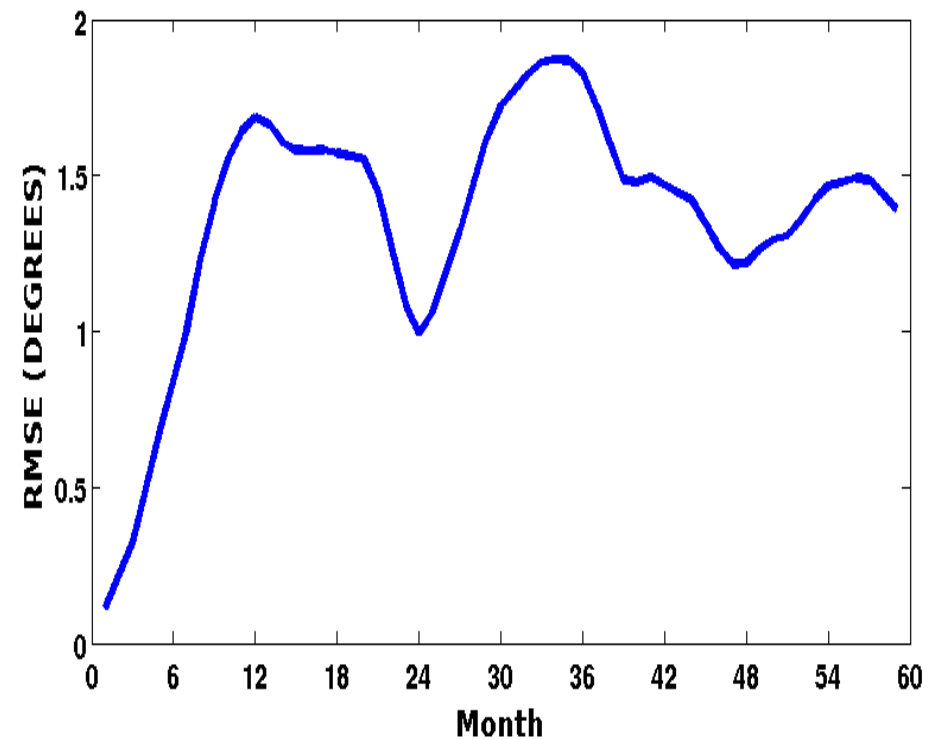
Predictability and Irregularity

The theoretical upper limit of predictability depends heavily on the mechanism for irregularity

Chaotic Oscillator Error Growth

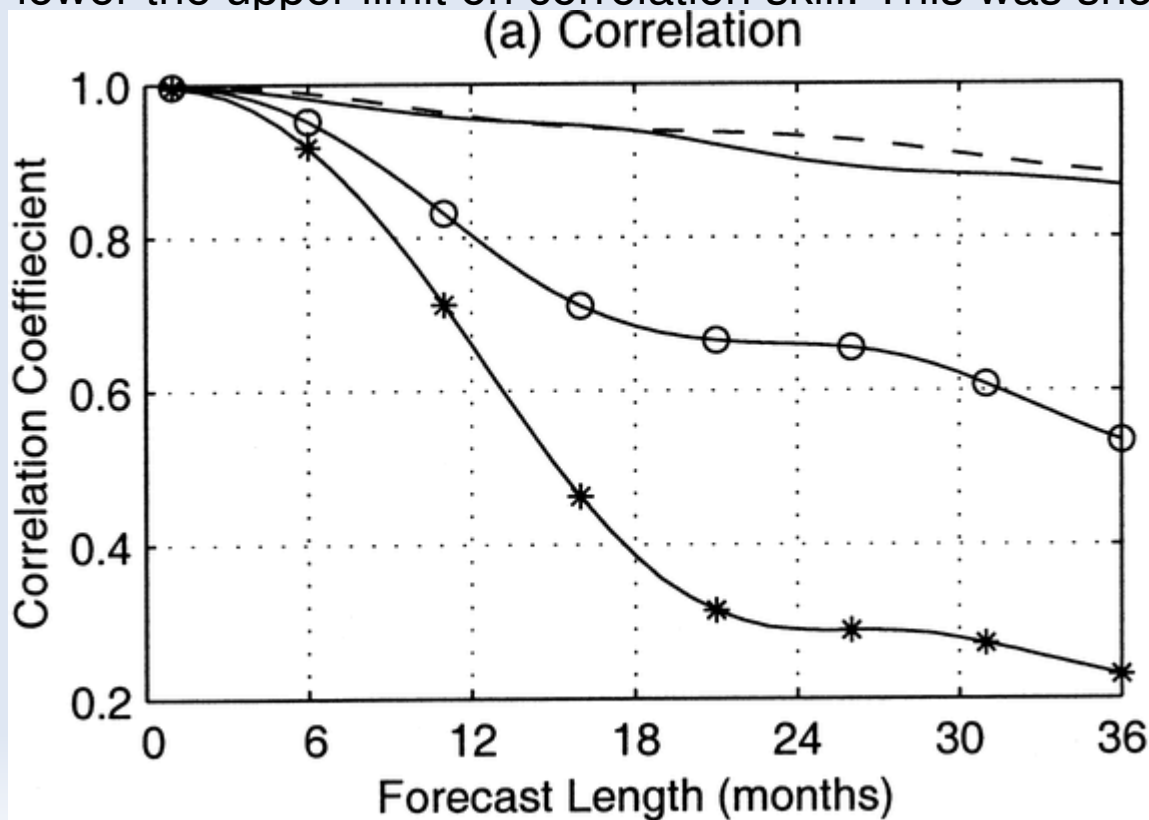


Stochastic Oscillator Error Growth



Predictability and Stability

Coupled model behavior usually varies strongly as the parameters are varied. The dominant variation observed is "stability". For some parameter ranges the model will oscillate in a self sustained fashion while for other ranges the oscillation will decay. Stability can strongly influence upper limits of predictability. In both models of irregularity RMSE increases as stability decreases as expected intuitively. Correlation skill however behaves differently. In the chaotic oscillator this measure of skill is often not highly sensitive to stability. In the stochastic oscillator there is a very strong dependency: The more stable the system the lower the upper limit on correlation skill. This was shown by Thompson and Battisti in 2000.



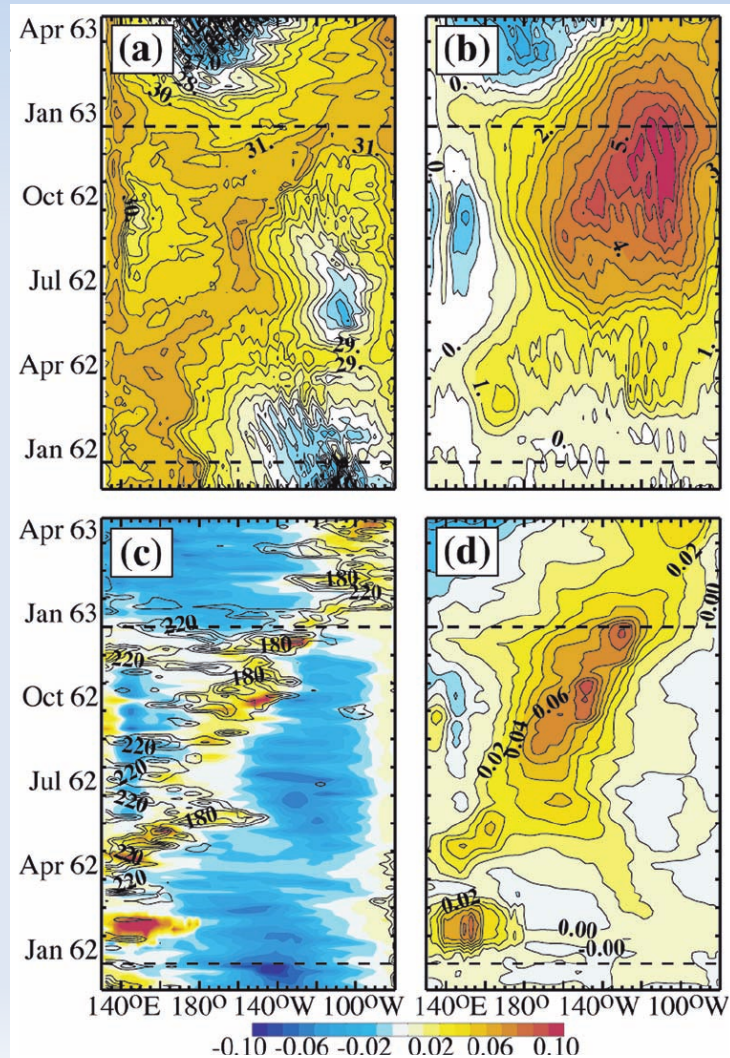
The asterisk curve is a model with a strongly decaying oscillation without stochastic forcing. The upper curves have oscillation which are almost self-sustaining without stochastic forcing. The open circles are intermediate cases.

The reason for this behaviour is that when the oscillation is "long lasting" it is able to resist disruption by the stochastic forcing for longer. More in next Lecture.

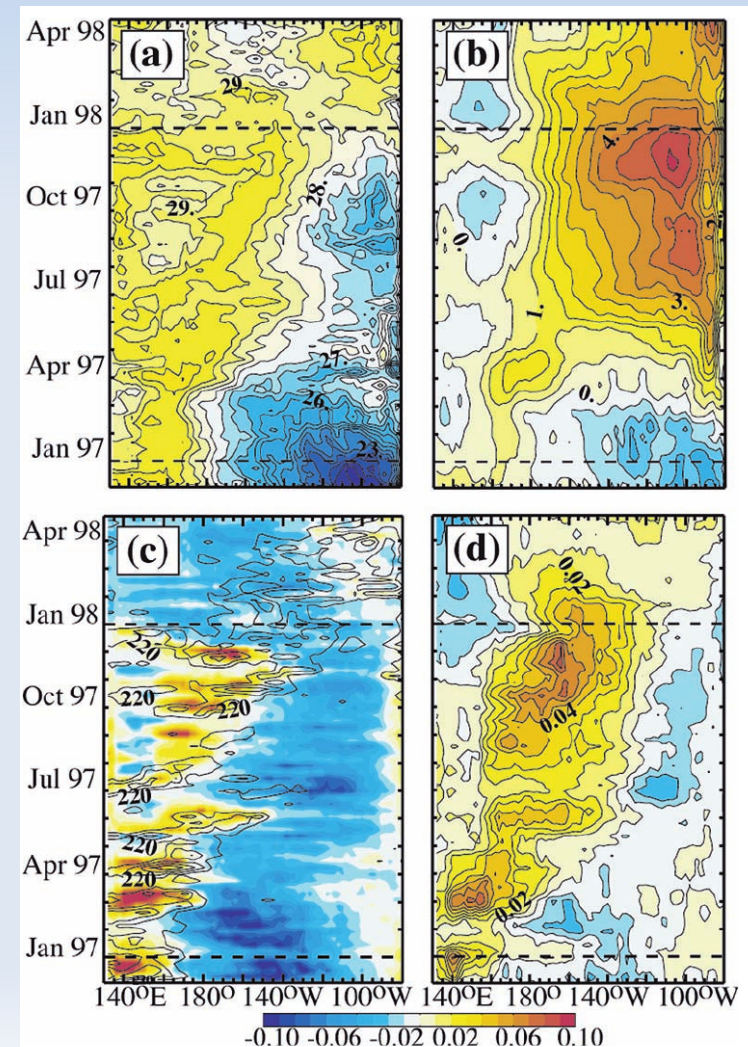
General Circulation Models

Most physically comprehensive models of ENSO. Can have problems with realism. One model due to Lengaigne et. al. (2004) is quite good and interesting from the viewpoint of irregularity and predictability.

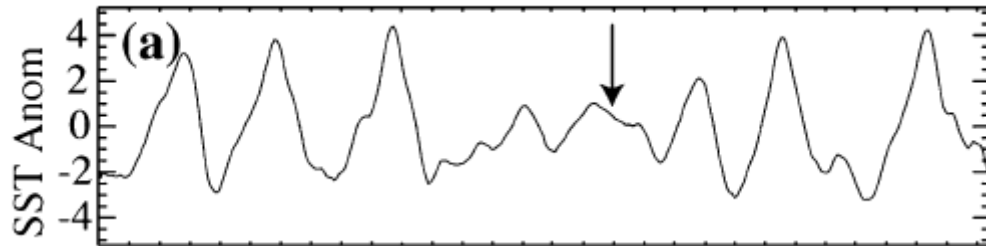
GCM



Observations

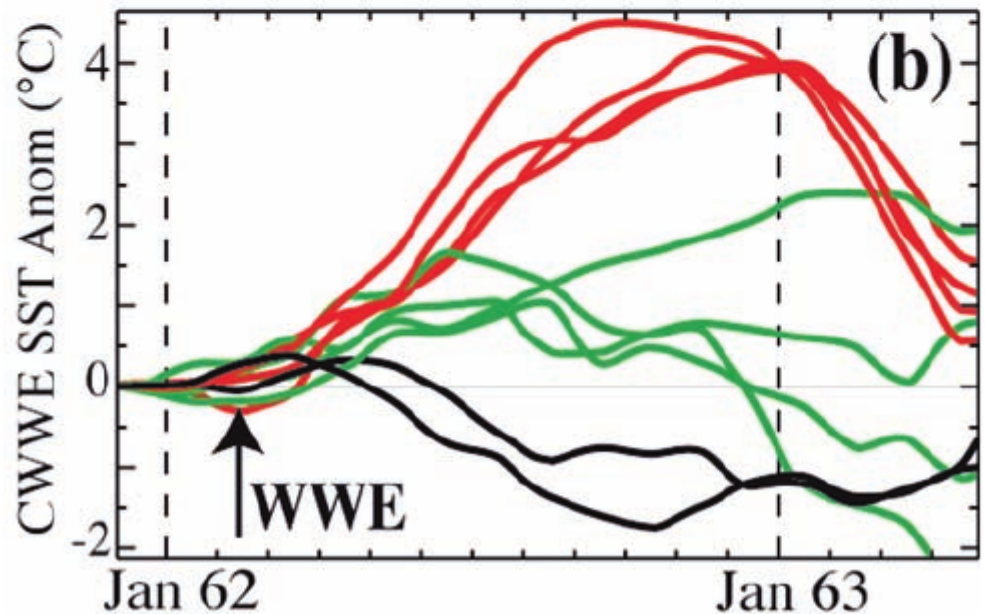
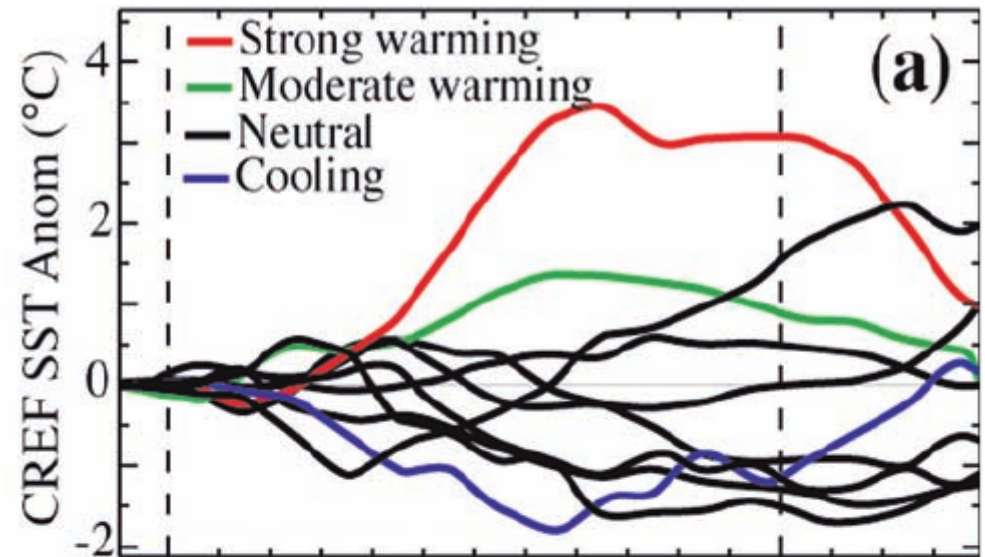


General Circulation Models



Irregularity is a little weak compared to the observations.

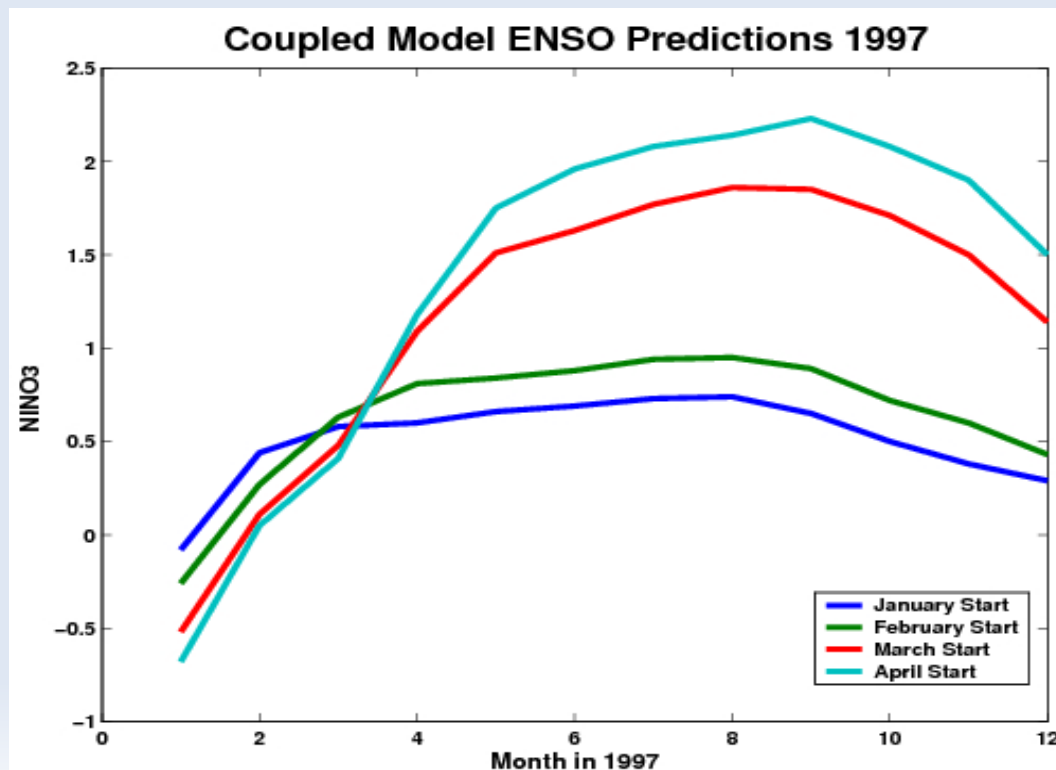
RMSE growth appears similar to that of a stochastic oscillator. Ensembles are very sensitive to anomalies resembling westerly wind bursts. These can shift ensembles strongly toward warm events.



Practical Implications

The theoretical upper limits on predictability are evidently potentially very important to future improved coupled models. If the chaotic scenario were the dominant mechanism of irregularity then useful ENSO predictions beyond 24 months may be possible. If the stochastic scenario dominates then we may be limited to 9-12 months.

Is it possible to discern these mechanisms in present practical forecasts of ENSO? The 1997 warm event was the largest for roughly a century and yet in real time (an acid test of prediction efforts) it was not very well forecast. A careful scrutiny of one (real time) forecast system shows the possible influence of stochastic forcing.



Notice the large increase in the predicted warm event between February and March 1997. What is different between the two months? In early March 1997 a large atmospheric westerly wind burst occurred around the dateline. The initialization for March included this strong forcing event while obviously that for February did not.....