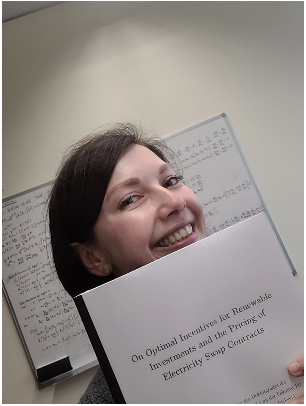
A Principal-Agent Model for Optimal Incentives in Renewable Investments

Workshop on Forecasting and Mathematical Modeling for Renewable Energy
Pacific Institute for Mathematical Sciences
Bristish Columbia, Canada, June 2023

René Aïd   Annika Kemper   Nizar Touzi

Université Paris-Dauphine   Bielefeld University   Ecole Polytechnique


On Optimal Incentives for Renewable
Investments and the Pricing of
Electricity Swap Contracts

Agenda

Motivation

Motivation

- Climate change mitigation requires carbon emission reductions: substitution of carbon emissive electricity production technologies by non-emissive technologies.
- Alternative technologies like wind-farms and solar are intermittent.
- Externatility cost of intermittency: The more wind farms are installed, the more it is costly for the grid (Joskow (2011), Borenstein (2012), Hirth (2013), Gowrisankaran (2016)).
- Optimal pace of substitution considering both externalities: carbon and intermittency.

The story

- We consider a regulator who wishes to decrease the installed capacities of emissive electricity production X^1 and to increase those of renewable production X^2 over a period of time T.
- The emissive technology has a per unit negative externality cost of k_1 and the renewable has a positive externality k_2 (avoided emissions).
- Because renewable energy production is intermittent, we assume there is a cost for the system to handle it. We assume it is proportional to the volatility of the energy production with factor h.
- The regulator is facing two types of potential firms
  - A single firm deciding the rate of investment a^i in each technology $i = 1, 2$ and the rate of investment b^i in volatility reduction devices.
  - Two firms deciding each for one single technology.
- For sake of simplicity (and mainly tractability), we assume that the price of electricity is constant equal to p.

Model main features

- Principal-Agent framework with the Regulator acting as the Principal and the Firm(s) acting as the Agent(s)
- Firms invest in technologies according to their cost and to the price of electricity to maximise the expected CARA utility of profit.
- Firms do not see or feel carbon externality.
- The Regulator bears social costs of carbon and volatility.
- Incentive mechanism is a function of realised trajectories of capacities, not investment rates.
- Incentives can be positive (subsidies) or negative (taxes).

Main results

- Optimal incentive mechanisms consists in lumpsum payment plus time-dependent prices for differences between initial capacities and realised capacities.
- In particular, it is optimal to pay for avoided emissions.
- The incentive price for volatility reduction is roughly its social cost.
- Incentive price for clean energy is positive (subsidy) but lower than its social value (risk-sharing effect)
- Incentive price for emissive technology is negative (tax) but lower in absolute value than its social cost (risk-sharing effect)

Model

Single Firm

The Firm (Agent)

- Available capacity of production technologies dynamics

$$dX_t^j = \left(a_t^j - \delta_j X_t^j\right)dt + b_t^j dW_t^j \;, \quad j = 1, 2.$$

with a^j and b^j the investment efforts in installed capacity and in volatility control. W^1, W^2 independent.

- Firm's criterion

$$V^{\mathrm{A}}(\xi) := \sup_{\nu := (a,b)} J_A(\xi, \nu) := \mathbb{E}^{\nu}\Big[U_{\mathrm{A}}\Big(\xi + \int_0^T p(X_t^1 + X_t^2) - c(\nu_t)dt\Big)\Big],$$

with $U_{\mathrm{A}}(x) = -e^{-\eta_A x}$ and

$$c(a, b) := \sum_{j=1,2} g_j(a) + \phi_j\left(b_j\right),$$

$$g_j(a) := \ell_j a_j + \frac{1}{2} q_j {a_j}^2 + \underbrace{\frac{1}{2}\varepsilon a_1 a_2,}_{\text{congestion}} \quad \phi_j\left(b_j\right) := ({b_j}^{-2} - {\sigma_j}^{-2})\Phi_j, \quad b_j \leq \sigma_j.$$

The Regulator (Principal)

- The Regulator's criterion is

$$J_P(\xi, \nu) := \mathbb{E}^{\nu}\Big[U_P\Big(-\xi + \int_0^T k_1 X_t^1 + k_2 X_t^2 dt - p(X_t^1 + X_t^2)dt - \frac{1}{2} h d\langle X^1 + X^2\rangle_t\Big)\Big]$$

  where $U_{\mathrm{P}}(x) = -e^{-\eta_P x}$.

- Denote $\nu^\star(\xi)$ the best-response of the firm to the incentive mechanism ξ.
- The regulator's objective is

$$V^{\mathrm{sb,m}} := \sup_{\xi \in \mathcal{C}} J_P(\xi, \nu^\star(\xi))$$

  where $\mathcal{C}$ is the set of random variables mesurable w.r.t. the state variables $X^i_\cdot$, $i = 1, 2$, and together with a participation constraint of the firm

$$V^{\mathrm{A}} \geq R_0 =: U_{\mathrm{A}}(L_0).$$

Two Interacting Firms

The case of two interacting firms

The dynamics of the available capacity of production technologies are the same

$$dX_t^i = \left(a_t^i - \delta_i X_t^i\right)dt + b_t^i dW_t^i \ , \quad i = 1, 2.$$

but now a^i and b^i the investment efforts in installed capacity and in volatility control are only controled by firm i with criterion

$$J_i(\xi_i, \nu_1, \nu_2) := \mathbb{E}^{\nu_i}\Big[U_i\Big(\xi_i + \int_0^T pX_t^i - c_i(a_t, b_t)dt\Big)\Big],$$

with $U_i(x) = -e^{-\eta_i x}$ and

$$c_i(a, b) := g_i(a) + \phi_i(b_i), \quad g_i(a) := \ell_i a_i + \frac{1}{2}q_i {a_i}^2 + \underbrace{\frac{1}{2}\varepsilon a_1 a_2}_{\text{congestion}},$$

Firms' best-responses

For a given couple of incentive mechanisms (ξ_1, ξ_2), the best-responses of the two firms $(\nu_1^\sharp, \nu_2^\sharp)$ are given by the Nash equilibrium:

$$J_1(\xi_1, \nu_1, \nu_2^\sharp) \leq J_1(\xi_1, \nu_1^\sharp, \nu_2^\sharp), \quad \text{for all} \quad \nu_1$$
$$J_2(\xi_2, \nu_1^\sharp, \nu_2) \leq J_2(\xi_2, \nu_1^\sharp, \nu_2^\sharp), \quad \text{for all} \quad \nu_2.$$

Firms' optimal value

For a given couple of incentive mechanisms (ξ_1, ξ_2), the values of the firms optimisation problem are

$$V^1(\xi_1, \xi_2) = J_1(\xi_1, \nu_1^\sharp, \nu_2^\sharp), \quad V^2(\xi_1, \xi_2) = J_2(\xi_2, \nu_1^\sharp, \nu_2^\sharp).$$

The Regulator (Principal)

The regulator's criterion is

$$J_P(\xi_1, \xi_2, \nu_1, \nu_2) := \mathbb{E}^{\nu_1,\nu_2}\Big[U_P\Big(-\left(\xi_1+\xi_2\right) + \int_0^T \big[(k_1 - p)X_t^1 + (k_2 - p)X_t^2\big]dt$$
$$- \int_0^T \frac{1}{2} h d\langle X^1 + X^2\rangle_t\Big)\Big]$$

where $U_P(x) = -e^{-\eta_P x}$. The regulator's objective is

$$V^{\mathrm{sb,c}} := \sup_{\xi_1, \xi_2 \in \mathcal{C}} J_P(\xi_1, \xi_2, \nu_1^\sharp(\xi_1, \xi_2), \nu_2^\sharp(\xi_1, \xi_2))$$

where $\mathcal{C}$ is the same set of random variables mesurable w.r.t. the state variables $X_\cdot^j$, and together with two participation constraints of the firms

$$V^i \geq R_0^i =: U_i(L_0^i), \quad i = 1, 2.$$

Single Firm

The firm's Hamiltonian

$$H(x, z, \gamma) := \sup_{a,b} \Big\{ \sum_{i=1,2} p x_i + (a_i - \delta_i x_i) z_i + \frac{1}{2} \gamma {b_i}^2 - c(a, b) \Big\}$$

- The firm's Hamiltonian gives the instantaneous rate of value the firm can make when optimaly responding to payment rates z_i and γ_i.
- The firm best-responses are given by

$$\hat{a}_i(z) = q_j \frac{z_i - \ell_i}{q_i q_j - \varepsilon^2} - \varepsilon \frac{z_j - \ell_j}{q_i q_j - \varepsilon^2}, \quad \hat{b}_i(\gamma) = \Big(- \frac{\Phi_i}{2\gamma_i} \Big)^{\frac{1}{4}}.$$

Admissible incentive mechanisms [Cvitanic, Possamaï & Touzi (2018)]

ξ can be written w.l.o. optimality $\xi = Y_T^{y_0,Z,\Gamma}$ with y_0 a real number and $Z = (Z^1, Z^2)$ and $\Gamma = (\Gamma^1, \Gamma^2)$ payment rates such that

$$Y_T^{y_0,Z,\Gamma} = y_0 + \int_0^T \sum_{i=1,2} Z_t^i dX_t^i + \frac{1}{2}\Gamma_t^i d\langle X^i\rangle_t + \frac{1}{2}\eta_A \big(Z_t^i\big)^2 d\langle X^i\rangle_t - H_i(X_t, Z_t, \Gamma_t)dt.$$

- y_0: Payment to satisfy participation constraint
- Z^i (resp. Γ_t^i): Payment/Charge for variations in X^i (resp. $\langle X^i\rangle$)
- $\frac{1}{2}\eta_A \big(Z_t^i\big)^2$: Payment of risk-aversion premia
- $-H_i(X_t, Z_t, \Gamma_t)$: Taking back all the value induced by optimal efforts

For all contracts of this form, it holds that

$$V^{\mathrm{A}}(Y_T^{y_0,Z,\Gamma}) = U_{\mathrm{A}}(y_0).$$

Optimal incentives

1. The payment rates Z and Γ are deterministic functions of time $Z_t^i = z_i(t)$, $\Gamma_t^i = \gamma_i(t)$, solutions of a nonlinear system.
2. The optimal incentive mechanisme is given by $\xi = \xi^F + \xi_1^V + \xi_2^V$, where

$$\xi^F = L_0 - \int_0^T H(X_0, z(t), \gamma(t))dt,$$

$$\xi_i^V = \int_0^T \pi_i^d(t)(X_t^i - X_0^i)dt + \frac{1}{2}\int_0^T \pi_i^v(t)d\langle X^i\rangle_t.$$

and where the optimal energy prices are

$$\pi_i^d(t) = -\dot{z}_i(t) - p - \delta_i z_i(t),$$

$$\pi_i^v(t) = -h - \eta_P\left(\frac{k_1}{\delta_i}\left(1 - e^{-\delta_i(T-t)}\right) - z_i(t)\right)^2.$$

No depreciation rates $\delta_i = 0$ and Risk-neutral agent $\eta_A = 0$

1. The payments rates are given by

$$z_i(t) = k_i(T - t), \quad \gamma_i(t) = -h, \quad i = 1, 2.$$

2. The optimal energy prices are constant given by

$$\pi_i^d = k_1 - p, \qquad \pi_i^v = -h.$$

3. The variable parts of the contract reduces to

$$\xi_i^V = (k_i - p) \int_0^T (X_t^i - X_0^i) dt - \frac{1}{2} h \langle X^i \rangle_T.$$

Comments

1. Variable part realised payments or charges depends on the initial state.
2. For tech. $i = 1$, we want it to decrease compared to X_0^1: the firm is paid when tech. $i = 1$ truly decreases and is charged if it increases.
3. Example of payment for environmental service. (story of bottlenose dolphins in Solomon Islands from Harstad (JET, 2016))
4. The volatility is exactly charged its externality value h.
5. Payments per technology are decoupled.

No depreciation $\delta_i = 0$, no congestion $\varepsilon = 0$, Risk-averse firm $\eta_A > 0$,

1. The payments rates are given by

$$z_i(t) = \frac{1 + \hat{b}_i(\gamma_i(t))^2 \eta_P q_i}{1 + \hat{b}_i(\gamma_i(t))^2(\eta_P + \eta_A) q_i} k_i(T - t),$$

$$\gamma_i(t) = -h - \eta_A z_i(t)^2 - \eta_P \big(k_i(T - t) - z_i(t)\big)^2$$

2. The optimal energy prices simplify to

$$\pi_i^d(t) = -\dot{z}_i(t) - p, \quad \pi_i^v(t) = -h - \eta_P \big(k_i(T - t) - z_i(t)\big)^2.$$

1. Coupling between payment rate for the installed capacity and volatility of each technology, but no coupling between investment in technologies.
2. The firm is paid less for the installed capacities when paid to reduce volatility.
3. The price for capacity investment π_i^d depends now on the volatility cost h and the price for volatility reduction π_i^v depends on the costs of technology.

No depreciation $\delta_i = 0$, congestion $\varepsilon > 0$, Risk-averse firm $\eta_A > 0$,

1. The payments rates are given by:

$$z_i(t) = \frac{k_i(T-t)}{1+\eta_A G(\gamma_1(t),\gamma_2(t))} - \varepsilon\eta_A \frac{k_j(T-t)\hat{b}_j(\gamma_j(t))^2}{Q_m \hat{b}_j(\gamma_j(t))^2(\eta_P+\eta_A)+q_i},$$

$$\gamma_i(t) = -h - \eta_A z_i(t)^2 - \eta_P\big(k_i(T-t) - z_i(t)\big)^2$$

with $Q_m := q_1 q_2 - \frac{1}{2}\varepsilon^2$.

Comments

1. G is a (little complicated) positive function.
2. Now, the payment rate of technology i includes the congestion effect: the monopoly is paid less to control the speed of investment in each technology.

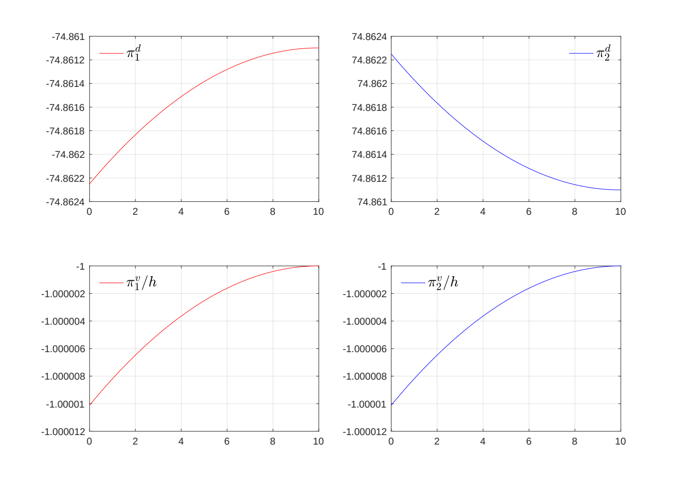


Figure: Second-best incentive prices for a monopoly. Parameters value: $p = 0$ €/Mwh, $k = (-100\,,\,100)$, $\ell = (100\,,\,400)$, $q = (1\,,\,1)$, $\Phi = (2000^4\,,\,2000^4)$, $\varepsilon = 0.25$, $\eta_A = 10^{-3}$, $\eta_p = 10^{-3}$, $h = 50^4$.

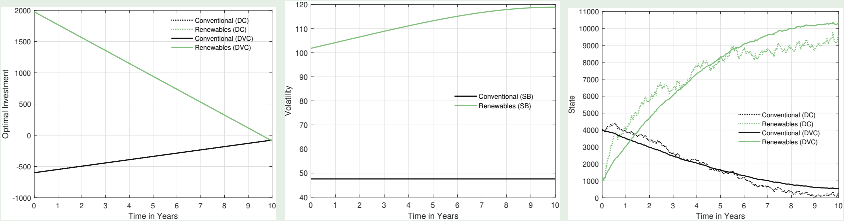


Figure: (Left) Investment rates $\hat{a}_i(z_i(t))$, (Middle) Investment rates $\hat{b}_i(\gamma_i(t))$ (Right) Available capacities X^i.

Two Interacting Firms

Firms' Hamiltonian and best-response functions

$$H_i(x, z, \gamma) := h_i(\hat{a}(x, z, \gamma), \hat{b}(x, z, \gamma), x, z, \gamma).$$

where

$$h_i(a, b, x, z, \gamma) := px_i + (a_i - \delta_i x_i)z_{ii} + (a_j - \delta_j x_j)z_{ij} + \frac{1}{2}\left(\gamma_{ii} {b_i}^2 + \gamma_{ij} {b_j}^2\right) - c_i(a, b),$$

and $(\hat{a}(x, z, \gamma), \hat{b}(x, z, \gamma))$ forms a Nash equilibrium of the payoffs (h_i, h_j).

We have

$$\hat{a}_i(z) = q_j \frac{z_{ii} - \ell_i}{q_i q_j - \frac{1}{4}\varepsilon^2} - \frac{1}{2}\varepsilon \frac{z_{jj} - \ell_j}{q_i q_j - \frac{1}{4}\varepsilon^2}, \quad \hat{b}_i(\gamma) = \left(\frac{\Phi_i}{-2\gamma_{ii}}\right)^{\frac{1}{4}}.$$

Admissible incentive mechanisms

ξ^i can be written w.l.o. optimality $\xi^i = Y_T^{i,y_0^i,Z^i,\Gamma^i}$ with $Z^i = (Z^{ii}, Z^{ij})$ and $\Gamma^i = (\Gamma^{ii}, \Gamma^{ij})$ such that

$$Y_T^{i,y_0^i,Z^i,\Gamma^i} = y_0^i + \int_0^T Z_t^{ii} dX_t^i + Z_t^{ij} dX_t^j + \frac{1}{2}\int_0^T \Gamma_t^{ii} d\langle X^i\rangle_t + \Gamma_t^{ij} d\langle X^j\rangle_t$$
$$+ \frac{1}{2}\int_0^T \eta_i \Big[\big(Z_t^{ii}\big)^2 d\langle X^i\rangle_t + 2 Z_t^{ii} Z_t^{ij} d\langle X^j\rangle_t\Big] - \int_0^T H_i(X_t, Z_t, \Gamma_t) dt$$

- Same interpretation as in the single firm case
- New terms: crossed-payment rates Z^{ij} and Γ^{ij} (no payment rates for covariation $\langle X^i, X^j\rangle$ because of Brownians independence)

Optimal incentive for a firm

1. The payment rates Z and Γ are deterministic functions of time $Z_t^{ij} = z_{ij}(t)$, $\Gamma_t^{ij} = \gamma_{ij}(t)$ solutions of a nonlinear system.
2. The optimal incentive mechanisms ξ_i are given by $\xi_i = \xi_i^F + \xi_{ii}^V + \xi_{ij}^V$, where

$$\xi_i^F = L_0^i - \int_0^T H_i(X_0, z(t), \gamma(t))dt,$$

$$\xi_{ij}^V = \int_0^T \pi_{ij}^d(t)(X_t^j - X_0^j)dt + \frac{1}{2}\int_0^T \pi_{ij}^v(t)d\langle X^j\rangle_t + (X_T^j - X_0^j)z_{ij}(T),$$

where by the optimal energy prices are

$$\pi_{ii}^d(t) = -\dot{z}_{ij}(t) - p, \quad \pi_{ij}^d(t) = -\dot{z}_{ij}(t),$$
$$\pi_{ii}^v(t) = -h - \eta_j z_{ji}(t)^2 - \eta_P\big(k_i(T-t) - z_{ii}(t) - z_{ij}(t)\big)^2, \quad \pi_{ij}^v(t) = \eta_i z_{ij}(t)^2.$$

No risk-aversion $\eta_i = 0$, no congestion $\varepsilon = 0$

1. The payments rates reduce to the single firm case, in the sense that

$$z_{ii} = z_i, \quad \gamma_{ii} = \gamma_i, \quad z_{ij} = 0, \quad \gamma_{ij} = 0, \quad i \neq j.$$

   There are no cross-payments between firms.
2. The optimal incentive mechanism per technology for the single firm case is the same as the optimal incentive per interacting firm.
3. The regulator of the single firm achieves the same value of certainty equivalent as the regulator of the interacting firms.

No depreciation $\delta_i = 0$, no congestion $\varepsilon = 0$, risk-aversion $\eta_i > 0$

1. The payments rates are given by the nonlinear system of equations:

$$z_{ii}(t) = \frac{1 + \hat{b}_i(\gamma_{ii}(t))^2 \eta_{jP} q_i}{1 + \hat{b}_i(\gamma_{ii}(t))^2(\eta_i + \eta_{jP}) q_i} k_i(T - t), \quad {\eta_{jP}}^{-1} := {\eta_P}^{-1} + {\eta_j}^{-1}$$

$$z_{ij}(t) = \frac{\hat{b}_j(\gamma_{jj})^2 \eta_{jP} q_j}{1 + \hat{b}_j(\gamma_{jj})^2(\eta_j + \eta_{iP}) q_j} k_j(T - t),$$

$$\gamma_{ii}(t) = -h - \eta_i z_{ii}(t)^2 - \eta_j z_{ji}(t)^2 - \eta_P \big(k_i(T - t) - z_{ii}(t) - z_{ji}(t)\big)^2.$$

It holds that $\gamma_{ij} \equiv 0$ for $i \neq j$.

- Payments rates for the two technologies are fully coupled.
- Cross-payments between firms. The firm i receives a payment for the installed capacity j controlled by firm j.
- The firm i is compensated for the volatility of production j induced by the incentive payment z_{ij}. Risk-sharing problem between three players.

$\delta_i = 0,\ \eta_i > 0,\ \varepsilon > 0$

1. The payment rates are given by:

$$z_{ii}(t) = k_i(T-t)F_{ii}(\gamma_{ii}(t), \gamma_{jj}(t)) - \varepsilon k_j(T-t)F_{ij}(\gamma_{jj}(t)) + \varepsilon F_{0i}(\gamma_{ii}(t)),$$
$$z_{ij}(t) = \frac{\eta_P}{\eta_P + \eta_i}\Big(k_j(T-t) - z_{jj}(t)\Big)$$
$$\gamma_{ii}(t) = -h - \eta_i z_{ii}(t)^2 - \eta_j z_{ji}(t)^2 - \eta_P\Big(k_i(T-t) - z_{ii}(t) - z_{ji}(t)\Big)^2,$$

where F_{ii}, F_{ij} and F_{i0} are explicit functions.
It holds that $\gamma_{ij} \equiv 0$ for $i \neq j$.

- Payments rates for the two technologies are fully coupled (not much worse...)

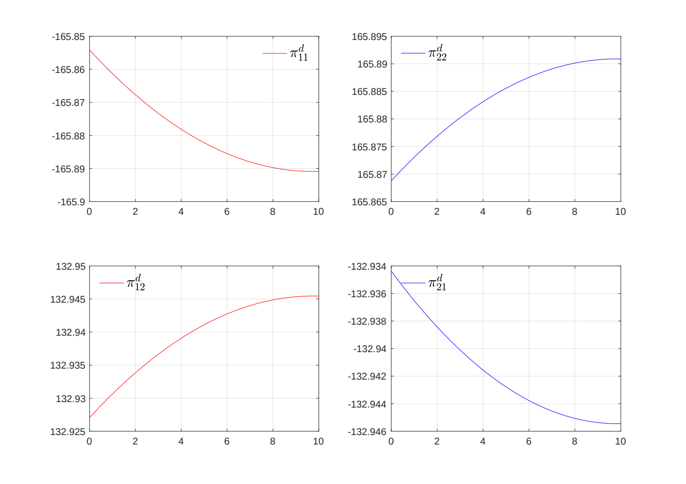


Figure: Second-best incentive prices for competing firms. Parameters value: $p = 0$ €/Mwh, $k = (-100\,,\,100)$, $\ell = (100\,,\,400)$, $q = (1\,,\,1)$, $\Phi = (2000^4\,,\,2000^4)$, $\varepsilon = 0.25$, $\eta = (10^{-3}\,,\,10^{-3})$, $\eta_p = 10^{-3}$, $h = 50^4$.

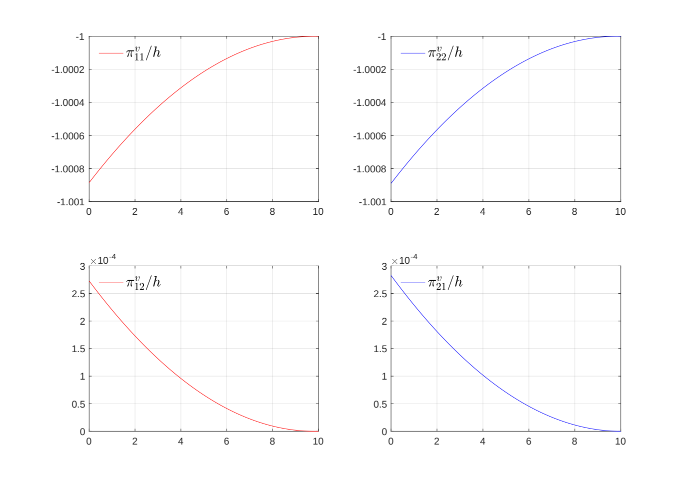


Figure: Second-best incentive prices for competing firms. Parameters value: $p = 0$ €/Mwh, $k = (-100\,,\,100)$, $\ell = (100\,,\,400)$, $q = (1\,,\,1)$, $\Phi = (2000^4\,,\,2000^4)$, $\varepsilon = 0.25$, $\eta = (10^{-3}\,,\,10^{-3})$, $\eta_p = 10^{-3}$, $h = 50^4$.

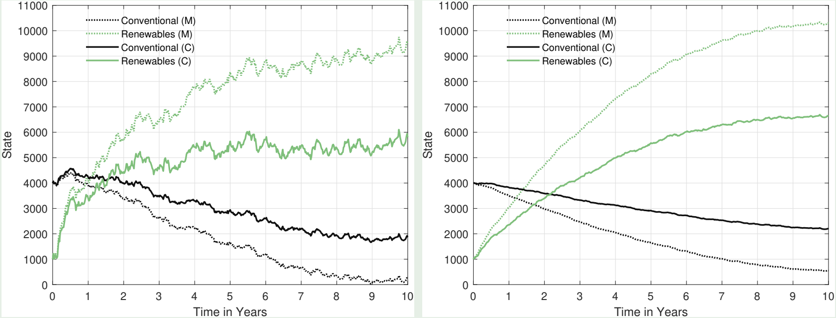


Figure: Capacities dynamics X^i under the singe firm and the two-interacting firms framework (Left) without volatility reduction incentive (Right) with volatility reduction incentive.

Conclusions

- Optimal incentive mechanisms consists in lumpsum payment plus time-dependent prices for differences between initial capacities and realised capacities.
- In particular, it is optimal to pay for avoided emissions.
- The incentive price for volatility reduction is roughly its social cost.
- The incentive prices for capacity are lower than their social value because of risk-sharing effect.

Limits and perspectives

- Complexity of the optimal mechanisms (cross-payments) calls for suboptimal but simpler incentives.
- Budget constraint.