Energy transition: a mean-field game approach

Roxana Dumitrescu

[1]King's College

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Outline

Climate change and energy transition

- To avoid the worst impacts of climate change, emissions need to be reduced by almost half by 2030 and reach net-zero by 2050.
- To achieve this, we need to end our reliance on fossil fuels and invest in alternative sources of energy that are clean, accessible, affordable, sustainable, and reliable.
- Renewable energy sources (sun, wind, water...) - emit little to no greenhouse gases or pollutants into the air.
- Fossil fuels still account for more than 80 % of global energy production, but cleaner sources of energy are gaining ground.

Energy transition

Renewable capacities increase worldwide

- Addition of 260 GW of renewables in 2020 (which represents 80% of all added capacities) [1].
- Almost 2800 GW of renewables worldwide (36% of total capacities), 730 GW is wind, 714 GW is solar [1].

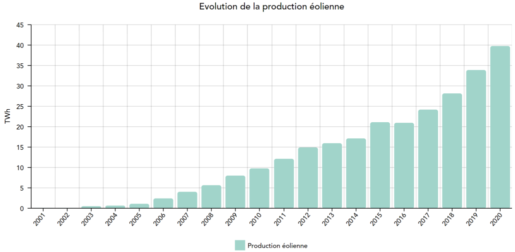

Figure : RTE

[1] IRENA, RENEWABLE CAPACITY STATISTICS 2021

Energy transition

Renewables are variable

- To maintain reliability the SO must continously match the demand for electricity with supply on a second-by-second basis
- With the growing penetration of renewables on the grid, there are higher levels of non-controllable, variable generation resources $\mapsto$ SO must direct less controllable resources to match both variable demand and variable supply

Energy transition

◇ **Model**: Entry-exit game in electricity markets

Focus on the **long-term evolution of the dynamics** of the **energy supply**

- R. Aid, R. D., P.Tankov, "Entry-exit in energy markets: a mean field game approach", Journal of Dynamics and Games, 2021
- Dumitrescu, Leutscher, and Tankov, "Energy transition under scenario uncertainty: a mean-field game approach." arXiv:2210.03554 (2022).

Energy transition

◇ **Model**: Entry-exit game in electricity markets

- We build a stylized equilibrium model of electricity market with conventional and renewable agents, interacting through the market price, allowing for entry and exit decisions (2 classes of agents).
- Conventional (e.g., gas) producers with fixed capacity and variable cost, aim to exit the market at the optimal time
- Renewable (e.g., wind) projects with variable capacity and zero marginal cost aim to enter the market at the optimal time
- The producers interact through the price resulting from a demand-supply equilibrium, which determines gains from production.
- **Our goal:** understand the effects of this interaction and of the market mechanisms on the long-term price levels and the renewable penetration.

Mathematical tool: Optimal stopping mean-field games

Mean-field games

Foundations and applications

- Introduced by Lasry and Lions (2006, 2007) and Huang, Caines, Malhamé (2006) using PDE tools to describe large-population games with symmetric interactions in a tractable way
- Numerous applications in economics, finance, engineering, epidemiology etc. For a very recent review of applications: Carmona (2020).
- The literature on MFGs with regular control well developped, in contrast few papers on optimal stopping MFG

Mean-field games

N-players game formulation

- Each player controls its state $X_t^i \in \mathbb{R}^d$ by taking an action $\alpha_t^i \in A \subset \mathbb{R}^k$:

$$dX_t^i = b(t, X_t^i, \bar{\mu}^{N-1}_{X_t^{-i}}, \alpha_t^i)dt + \sigma(t, X_t^i, \bar{\mu}^{N-1}_{X_t^{-i}}, \alpha_t^i)dW_t^i,$$

W^i are independent and $\bar{\mu}^{N-1}_{X_t^{-i}}$ is the empirical distribution of other players.

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  W^i are independent and $\bar{\mu}^{N-1}_{X_t^{-i}}$ is the empirical distribution of other players.

- Each player minimises the cost

$$J^i(\boldsymbol{\alpha}) = \mathbb{E}\left[\int_0^T f(t, X_t^i, \bar{\mu}^{N-1}_{X_t^{-i}}, \alpha_t^i)dt + g(X_T^i, \bar{\mu}^{N-1}_{X_T^{-i}})\right],$$

- We look for a Nash equilibrium $\hat{\boldsymbol{\alpha}}: \ \forall i, \ \forall \alpha^i, \ J^i(\hat{\boldsymbol{\alpha}}) \leq J^i(\alpha^i, \hat{\boldsymbol{\alpha}}^{-i})$.

Mean-field games

Towards a mean-field game

When the number of agents is large, it is natural to consider the following limiting version of the game:

- The *representative player* controls its state X^α depending on the deterministic flow $(\mu_t)_{0\leq t\leq T}$, which corresponds to the distribution of states of all players:

$$dX_t^\alpha = b(t, X_t^\alpha, \mu_t, \alpha_t)dt + \sigma(t, X_t^\alpha, \mu_t, \alpha_t)dW_t.$$

- The aim of the player is to minimize the cost

$$\inf_{\alpha\in A} J^\mu(\alpha), \quad J^\mu(\alpha) = \mathbb{E}\left[\int_0^T f(t, X_t^\alpha, \mu_t, \alpha_t)dt + g(X_T^\alpha, \mu_T)\right] \quad (*)$$

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- A mean-field equilibrium is a flow $(\mu_t)_{0\leq t\leq T}$ such that $\mathcal{L}(\hat{X}_t^\mu) = \mu_t$, $t \in [0, T]$, where $\hat{X}^\mu$ is the solution to $(*)$.

Mean-field games

Approaches

- **PDE approach**: developed by Lasry and Lions (2006, 2007) and Huang, Malhamé and Caines (2006) → coupled system of partial differential equations: *Hamilton-Jacobi-Bellman (backward)* and *Fokker-Planck-Kolmogorov (forward)*.
- **FBSDE approach**: introduced by Carmona and Delarue (2012) → coupled *forward-backward stochastic differential equations* with coefficients which depend on the law of the solution.
- **Compactification methods**: Allow to solve the problem under mild assumptions by relaxing the concept of equilibrium.
  - → Controlled martingale problem (Lacker (2015)).
  - → Linear-programming approach (Bouveret, Dumitrescu, Tankov (2020), Dumitrescu-Leutscher-Tankov (2021)).

Outline

Optimal stopping MFG

N-players game problem

- Consider N agents X^i, $i = 1, \dots, N$ with dynamics

$$dX_t^i = b(t, X_t^i)dt + \sigma(t, X_t^i)dW_t^i, \quad X_0^i \in \mathcal{O},$$

where W^i, $i = 1, \dots, N$ are independent.

In the talk, to simplify notation, we assume either $\mathcal{O} = \mathbb{R}$ or $\mathcal{O} \subset \mathbb{R}$ with unattainable boundary; $\mathbb{R}^n$ and absorbing boundary can also be considered.

Optimal stopping MFG

N-players game problem

- Each agent aims to solve the following *optimal stopping problem*:

$$\sup_{\tau} \mathbb{E}\left[\int_0^{\tau} f\left(t, X_t^i, m_t^{N-1}\right) dt + g\left(\tau, X_\tau^i, \mu^{N-1}\right)\right],$$

where

$$m_t^{N-1}(dx) = \frac{1}{N-1}\sum_{k=1;k\neq i}^{N-1} \delta_{X_t^k}(dx)\mathbf{1}_{t\leq\tau^k},$$

and

$$\mu^{N-1}(dt, dx) = \frac{1}{N-1}\sum_{k=1;k\neq i}^{N-1} \delta_{(\tau^k, X_\tau^k)}(dt, dx),$$

with τ^k is the stopping time chosen by the player k.

- Look for Nash equilibria.

Optimal stopping MFG

MFG formulation

As $N \to \infty$, we expect m^N converge to deterministic limit m.

- State process of the *representative agent*

$$dX_t = b(t, X_t)dt + \sigma\left(t, X_t\right)dW_t,$$

- The optimal stopping problem for the agent takes the form

$$\sup_{\tau} \quad \mathbb{E}\left[\int_0^{\tau} f\left(t, X_t, m_t\right)dt + g(\tau, X_{\tau}, \mu)\right]. \tag{1}$$

Optimal stopping MFG

- Given the solution $\tau^{m,\mu}$ of the problem (1) for the agent facing a mean-field $((m_t)_{t\in[0,T]}, \mu)$, find $((m_t)_{t\in[0,T]}, \mu)$ such that

$$m_t(B) = \mathbb{P}\left[X_t \in B, t < \tau^{\mu,m}\right], B \in \mathcal{B}(\mathcal{O}), t \in [0, T]. \qquad (2)$$

and

$$\mu = \mathcal{L}\left(\tau^{\mu,m}, X_{\tau^{\mu,m}}\right). \qquad (3)$$

Solution of the optimal stopping MFG: fixed point of (2) – (3).

Linear programming approach for optimal stopping

Single agent problem

- We start with the **single agent problem** (no mean-field terms here):

$$\sup_{\tau \in \mathcal{T}} \quad \mathbb{E}\left[\int_0^\tau f\left(t, X_t\right) dt + g(\tau, X_\tau)\right],$$

$$\text{s.t.} \quad dX_t = b\left(t, X_t\right) dt + \sigma\left(t, X_t\right) dW_t,$$

$$X_0 \sim m_0^\star.$$

Linear programming approach for optimal stopping

- For any $\tau \in \mathcal{T}$, define the **flow of subprobability measures** m^τ and **the probability measure** μ^τ by

$$m_t^\tau(B) = \mathbb{P}\left(X_t \in B, t < \tau\right), \quad B \in \mathcal{B}(\mathcal{O}), \ \ t \in [0, T],$$
$$\mu^\tau(C) = \mathbb{P}((\tau, X_\tau) \in C), \quad C \in \mathcal{B}([0, T] \times \bar{\mathcal{O}}).$$

- *Objective functional.* We can *rewrite the expected reward*

$$\mathbb{E}\left[\int_0^\tau f\left(t, X_t\right) dt + g\left(\tau, X_\tau\right)\right]$$
$$= \int_0^T \int_{\mathcal{O}} f(t, x) m_t^\tau(dx) dt + \int_{[0,T]\times\bar{\mathcal{O}}} g(t, x) \mu^\tau(dt, dx).$$

Linear programming approach for optimal stopping

- *Constraint.* By applying Itô's formula to $u \in C_b^{1,2}([0,T] \times \mathbb{R})$ up to time τ, we deduce that the set of tuples (μ^τ, m^τ), $\tau \in \mathcal{T}$ is *included* in the set:

Definition

Let $\mathcal{R}$ be the set of (μ, m) such that for all $u \in C_b^{1,2}([0,T] \times \mathcal{O})$

$$\int_{[0,T]\times\bar{\mathcal{O}}} u(t,x)\mu(dt,dx) = \int_{\mathcal{O}} u(0,x)m_0^*(dx) + \int_0^T \int_{\mathcal{O}} (\partial_t u + \mathcal{L}u)(t,x)m_t(dx)dt.$$

Linear programming approach for optimal stopping

The **linear programming formulation** consists in solving the problem

$$V^{LP} := \sup_{(\mu,m)\in\mathcal{R}} \int_0^T \int_{\mathcal{O}} f(t,x) m_t(dx) dt + \int_{[0,T]\times\bar{\mathcal{O}}} g(t,x)\mu(dt,dx).$$

The initial problem is embedded in this one.

MFG linear programming formulation

Fix a pair $(\bar{\mu}, \bar{m})$.

- Let $\Gamma[\bar{\mu}, \bar{m}]$ be defined by

$$\Gamma[\bar{\mu}, \bar{m}](\mu, m) = \int_0^T \int_{\bar{\mathcal{O}}} f(t, x, \bar{m}_t) m_t(dx) dt + \int_{[0,T]\times\bar{\mathcal{O}}} g(t, x, \bar{\mu}) \mu(dt, dx).$$

MFG linear programming formulation

- We say that $(\mu^\star, m^\star)$ is an *LP MFG Nash equilibrium* if $(\mu^\star, m^\star) \in \mathcal{R}$ and for all $(\mu, m) \in \mathcal{R}$,

$$\Gamma[\mu^\star, m^\star](\mu, m) \leq \Gamma[\mu^\star, m^\star](\mu^\star, m^\star).$$

The real number $\Gamma[\mu^\star, m^\star](\mu^\star, m^\star)$ is called a *Nash value*.

MFG linear programming formulation

- **Existence of an equilibria** The set of LP MFG Nash equilibria is compact and nonempty.
- **Uniqueness of the Nash value** Under a special dependence of f and g on m, μ: for (μ^1, m^1) and (μ^2, m^2) two LP Nash equilibria,

$$\Gamma[\mu^1, m^1](\mu^1, m^1) = \Gamma[\mu^2, m^2](\mu^2, m^2).$$

Entry-exit model for electricity markets

Conventional producers

- Each **conventional producer** has marginal cost function $C_t^i : [0,1] \to \mathbb{R}$. $C_t^i(\xi)$ is the unit cost of increasing capacity if operating at ξ.
  We assume

  $$C_t^i(\xi) = C_t^i + c(\xi),$$

  where C_t^i is the baseline cost:

  $$dC_t^i = k(\theta(t) - C_t^i)dt + \delta\sqrt{C_t^i}dW_t^i, \;\; C_0^i = c_i,$$

  and $c : [0,1] \to \mathbb{R}$ is increasing smooth with $c(0) = 0$.

Entry-exit model for electricity markets

Conventional producers

- By maximizing its profit per unit, for a given price p, the producer offers fraction $F(p - C_t^i)$ of its capacity, where $F = c^{-1}$.
- Gain of the producer at price level p is $G(p - C_t^i)$, where

$$G(x) = \int_0^x F(z)dz, \;\; x \geq 0, \;\; G(x) = 0, \;\; x < 0.$$

Entry-exit model for electricity markets

Conventional producers

- Producer i aims to exit the market at the optimal time τ_i, where the optimization problem is

$$\max_{\tau} \mathbb{E}\left[\int_0^{T\wedge\tau} e^{-\rho t}(G(P_t - C_t^i) - \kappa_C)dt + K_C e^{-(\gamma_C+\rho)\tau\wedge T}\right],$$

where P_t is the electricity price, K_C is the cost of assets recovered upon exit, κ_C is the fixed running cost and γ_C is the depreciation rate.

Entry-exit model for electricity markets

Conventional producers

- The total conventional supply at price level p, including baseline conventional supply, is given by

$$\int_\Omega F(p-x)\omega_t^n(dx) + F_0(p) = \sum_{i=1}^n \frac{1}{n} F(p - C_t^i)\mathbf{1}_{\tau^i > t} + F_0(p),$$

with $\omega_t^n(dx)$ the distribution of costs of conventional producers who have not yet exited the market, i.e.

$$\omega_t^n(dx) = \frac{1}{n}\sum_{i=1}^n \delta_{C_t^i}(dx)\mathbf{1}_{\tau^i > t}.$$

Entry-exit model for electricity markets

Renewable producers

- Renewable producers aim to enter the market at the optimal time σ_i.
- To enter they pay the cost K_R after which the plant generates $S_t^i \in [0,1]$ units of electricity per unit of time at zero cost, where

$$dS_t^i = \bar{\kappa}(\bar{\theta} - S_t^i)dt + \bar{\delta}\sqrt{S_t^i(1 - S_t^i)}d\overline{W}_t^i, \; S_0^i = s_i \in [0,1].$$

- The renewable producers always bid their full intermittent capacity and solve:

$$\max_{\sigma} \mathbb{E}\left[\int_{\sigma\wedge T}^{T} e^{-\rho t}(P_t S_t^i - \kappa_R)dt - K_R e^{-\rho\sigma_i\wedge T} + K_R e^{-\rho T - \gamma_R(T - \sigma\wedge T)}\right],$$

where K_R is the fixed cost, κ_R is the running cost and γ_R is the depreciation rate.

Entry-exit model for electricity markets

Renewable producers

- We denote by $\eta^n_t(dx)$ the distribution of output of renewable producers who have entered the market :

$$\eta^n_t(dx) = \frac{1}{n}\sum_{i=1}^{n} \delta_{S^i_t}(dx)\mathbf{1}_{\sigma_i \leq t}.$$

- The total renewable supply at time t is given by $R^n_t = \int_0^1 x\eta^n_t(dx)$.

Entry-exit model for electricity markets

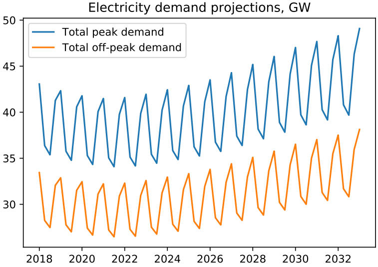
Price formation

- Agents are coupled through the market price, by matching exogenous demand process $\overline{D}_t$, to the aggregate supply function.

$$P_t := \inf\{P : (\overline{D}_t - R_t^n)^+ \leq \int_\Omega F(P-x)\omega_t^n(dx) + F_0(p)\} \wedge \overline{P},$$

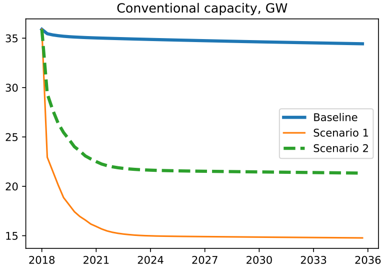
where $\overline{P}$ is the cap in the market.
When cap $\overline{P}$ is reached, demand is not entirely satisfied by producers.

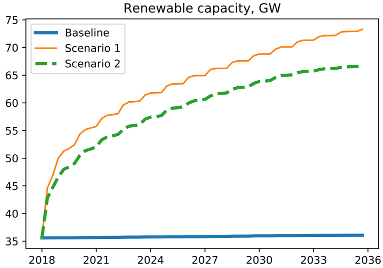
Numerical illustration: demand projections



We distinguish peak / off-peak price/demand for more realistic projections.

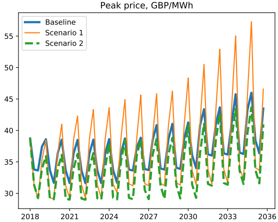
Numerical illustration: capacity

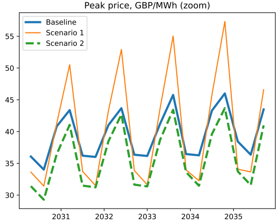


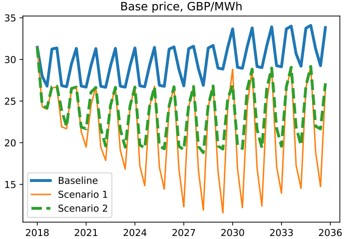


Conventional / renewable capacity evolution in three scenarios.
Baseline: costs estimated for UK market, no subsidy
Scenario 1: 30% renewable subsidy.
Scenario 2: renewable subsidy + a mechanism to keep conventional producers in the market.

Numerical illustration: price evolution







Entry-exit game for electricity markets

Extended model with common noise

- We consider a discrete-time version of the previous model and add a random carbon price. Study the impact on the pace of decarbonization of the electricity industry.
- Mathematical point of view: MFG of optimal stopping with common noise

Entry-exit game for electricity markets

Extended model with common noise

- Conventional producers $\mapsto$ Stochastic baseline cost $\mapsto$ decide when to exit the market
- Renewable producers $\mapsto$ Stochastic capacity factor $\mapsto$ decide when to enter the market.
- Carbon price impacts the cost of the conventional producers and the demand.
- Supplies from conventional and renewable producers = Demand $\mapsto$ Electricity price.
- The optimization problems are coupled thorugh the electricity price $\mapsto$ Non cooperative game.
- We look for Nash equilibria.

Entry-exit game for electricity markets

Extended model with common noise

- The demand process is given by

$$D_t = d(t) + \beta(Z_t - Z_0),$$

where

  - $d(t)$ is a determistic function
  - $\beta \geq 0$: carbon price increases imply that carbon-intensive sectors of the industry are forced to electrify and contribute to electricity demand.

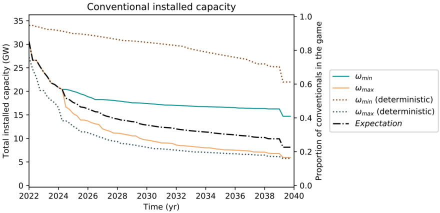
- The marginal unit cost of conventional producer i is given by

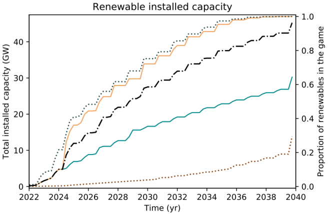
$$C_t^i(\xi) = C_t^i + \tilde{\beta} Z_t + c(\xi),$$

where

  - $\tilde{\beta} \geq 0$ represents the emission intensity

Entry-exit game for electricity markets





MFG for energy transition without common noise vs. with common noise

Thank you for your attention!