Equilibrium price in intraday electricity markets

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Context

• Intra-day electricity Markets are organized in **continuous trading** with delivery for the next 36 hours

- Act as a forward market for delivery in a short period (few hours)

• Players (producers, retailers, speculators) can adjust their portfolio position between production and consumption to face expensive imbalance costs due to unexpected events.

• Significant development of Intra-day market to cope with the uncertainty of renewable generation: e.g. the exchanged volume in TWh has been multiplied by 10 in Germany in 10 years.

• Indra-day Market also contributes to a better equilibrium between supply and demand in electricity

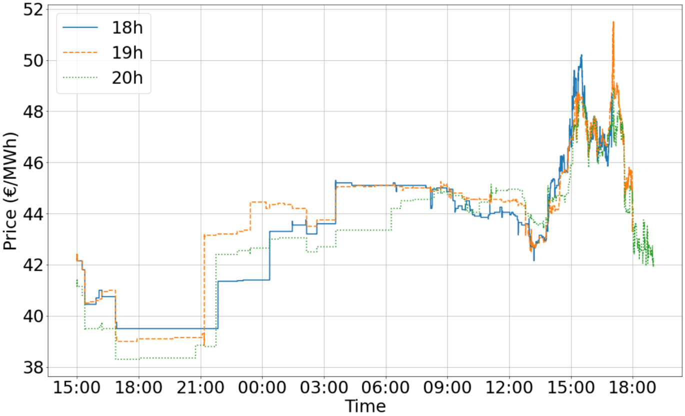


Figure: Intraday mid-prices on EEX market on August, 30$^{\text{th}}$, 2017 for deliveries at 18h, 19h and 20h up to 1 hour before maturity, picture extracted from Deschatre & Gruet (2021) with the kind permission of the authors.

Our motivation and goal

- Develop a tractable equilibrium model to understand
  - optimal trading strategies and coordination with generation
  - intraday price formation for a fixed hour of delivery
  - price volatility in relation with demand uncertainty and outages
  - Samuelson's effect: increase of volatility closer to maturity

Related literature:

- Optimal trading in intraday electricity markets with exogenous price: Aid, Gruet, P. (16), Tan and Tankov (18)
- Price formation in a mean-field Nash equilibrium: Feron, Tankov, Tinsi (21)

Preview of our main results

- Existence and uniqueness of an equilibrium intraday price in a model with
  - Heterogenous agents and market clearing condition for equilibrium
  - Demand uncertainty and production shocks/outages
- In case of no production shocks:
  - Closed-form expression of the equilibrium price and trading strategies, and intuitive economic interpretation
  - Necessary condition of heterogeneity of agents for Samuelson's effect to hold true
  - Sufficient condition for Samuelson's effect in terms of market access quality or production costs
- In case of production shocks:
  - Numerical approximation of equilibrium price showing some similarities with observed intraday price

Model

- N agents (power producers) who can trade electricity on Intraday market for a given hour of delivery T in order to satisfy demand of their customers

- **The demand forecast** over $[0, T]$ of each agent $i \in [\![1, N]\!]$ is given by

$$D_t^i \quad = \quad d_0^i + \int_0^t \sigma_i(s) \big(\underbrace{\rho_i dW_s^0 + \sqrt{1 - \rho_i^2} dW_s^i}_{d\widetilde{W}_t^i} \big),$$

with deterministic volatility σ_i, typically non-increasing in time.

- **Power production**: each agent i can generate a quantity of power ξ_i at T with a cost:

$$c_i(\xi_i) \quad = \quad \frac{1}{2} \beta_T^i |\xi_i|^2$$

  - *Uncertainty/shocks on production*: marginal cost $(\beta_t^i)_t$ modelled as a Markov chain with finite state E, and intensity matrix $\Lambda_i = \{\lambda_i(e, e') : e, e' \in E\}$.

Trading on intra-day markets

- X_t^i Inventory of agent i with a trading rate control $q_t^i = \dot{X}_t^i$:

$$X_t^i = x_0^i + \int_0^t q_s^i ds, \quad 0 \leq t \leq T.$$

- Expected total cost of agent i

$$\tilde{J}_i(q_i, \xi_i) = \mathbb{E}\Big[\underbrace{\int_0^T q_t^i (P_t + \gamma_i q_t^i) dt}_{\text{trading cost}} + \underbrace{c_i(\xi_i)}_{\text{production cost}} + \frac{\eta_i}{2} \big(\underbrace{D_T^i - X_T^i - \xi_i}_{\text{Imbalance from trade and production}} \big)^2 \Big]$$

  - $(P_t)_t$ intraday electricity price
  - $\gamma_i \geq 0$; temporary price impact $\rightarrow$ liquidity access that may differ from an agent to another one
  - $\eta_i \geq 0$: penalty for imbalance cost

- Agents are heterogeneous and characterized by the parameters $(\sigma_i, \rho_i, \Lambda_i, \gamma_i, \eta_i)$.

Single agent optimal trading problem

- Agent i: minimize over q^i and ξ_i, the expected total cost $\tilde{J}_i(q^i, \xi_i)$
- By relaxing the nonnegativity constraint on ξ, the optimal generation is given by:

$$\hat{\xi}_i = \frac{\eta_i}{\eta_i + \beta^i_T}(D^i_T - X^i_T).$$

(Negative generation can be interpreted as demand-side management)

- The objective of each agent i is then reduced to the minimization over q^i of:

$$J_i(q^i) = \tilde{J}_i(q^i, \hat{\xi}_i) = \mathbb{E}\Big[\int_0^T q^i_t(P_t + \gamma_i q^i_t)dt + \frac{1}{2}\frac{\eta_i \beta^i_T}{\eta_i + \beta^i_T}\big(D^i_T - X^i_T\big)^2\Big].$$

Market equilibrium

- Given a price $P = (P_t)$, denote by $\hat{q}^{i,P}$ the solution to agent i problem:

$$\sup_{q^i} J_i(q^i).$$

▸ A market equilibrium is a pair of price and trading rates $(\hat{P}, (\hat{q}^{i,\hat{P}})_i)$ satisfying the market clearing condition:

$$\sum_{i=1}^{N} \hat{q}_t^{i,\hat{P}} \quad = \quad 0, \quad 0 \leq t \leq T.$$

▸ $\hat{P}$ is called equilibrium (intraday) price, and $\hat{q}^i := \hat{q}^{i,\hat{P}}$ the equilibrium trading rates

Case without production shocks: $\Lambda_i \equiv 0$

Equilibrium price

The equilibrium price $\hat{P}$ is given by

$$\hat{P}_t = \sum_{i=1}^{N} F_i(t) c_i'(\hat{\xi}_t^i), \qquad \hat{\xi}_t^i := \frac{\eta_i}{\eta_i + \beta_0^i}(D_t^i - \hat{X}_t^i),$$

$$\text{with} \quad F_i(t) := \frac{1}{f_i(t)}\left(\sum_{j=1}^{N} 1/f_j(t)\right)^{-1}, \quad f_i(t) := \gamma_i + \frac{1}{2}\frac{\eta_i \beta_0^i}{\eta_i + \beta_0^i}(T - t).$$

Interpretation: the equilibrium price is a convex combination of the forecasted marginal cost of production at maturity.

Equilibrium trading rates

The equilibrium trading rates $\hat{q}^i$, $i \in [\![1, N]\!]$ are given by

$$\hat{q}^i_t = \frac{1}{2} \frac{c_i'(\hat{\xi}^i_t) - \hat{P}_t}{f_i(t)}.$$

Interpretation: the optimal trading consists in

- Buying if the forecasted marginal cost $c_i'(\hat{\xi}^i_t)$ is higher than the market price,
- Selling if the forecasted marginal cost $c_i'(\hat{\xi}^i_t)$ is lower than the market price.

The rate of trading is inversely proportional to the market access quality γ_i.

Properties of equilibrium price

The equilibrium price $\hat{P}$ can be also written as

$$\hat{P}_t = S_t - \sum_{i=1}^{N} \epsilon_i F_i(t)(\hat{X}_t^i - x_0^i),$$

$$\text{with} \quad S_t := \sum_{i=1}^{N} \epsilon_i F_i(t)(D_t^i - x_0^i), \quad \epsilon_i := \frac{\eta_i \beta_0^i}{\eta_i + \beta_0^i}.$$

- (S_t) is an uncontrolled process interpreted as the fundamental price as in Almgren and Chriss (01) model of intraday trading
- The factors $\epsilon_i F_i(t)$ reads as the permanent market impact of each agent
- In the case where all agents are identical, $\hat{P} = S$ due to the market clearing condition.

Volatility of the equilibrium intraday price

Volatility

The dynamics of the equilibrium price is given by

$$d\hat{P}_t = \sum_{i=1}^{N} \epsilon_i F_i(t)\sigma_i(t) d\widetilde{W}_t^i.$$

The volatility ζ_t defined as: $\zeta_t^2 = \frac{d<\hat{P}>_t}{dt}$, is deterministic and given by

$$\zeta_t^2 = \sum_{i=1}^{N} (1-\rho_i^2)\big(\epsilon_i F_i(t)\sigma_i(t)\big)^2 + \Big(\sum_{i=1}^{N} \rho_i \epsilon_i F_i(t)\sigma_i(t)\Big)^2.$$

Remark: Assume that all agents are identical

- When $\rho = 0$, then $\lim_{N\to\infty} \zeta_t = 0$, while the limit is strictly positive when $\rho \neq 0$. In other words, price is random because agents face a common economic factor

Samuelson's effect: increase of the volatility closer to maturity

Monotonicity of the volatility

- If all agents are identical, then

$$\zeta_t^2 = \frac{\epsilon^2}{N^2}\Big[\sum_{i=1}^{N}(1-\rho^2)(\sigma_i(t))^2 + \rho^2\Big(\sum_{i=1}^{N}\sigma_i(t)\Big)^2\Big].$$

▸ Recalling that σ_i is nonincreasing in time, it follows that ζ_t is also nonincreasing in time, hence the Samuelson's effect does not hold true!

▸ Heterogeneity is a necessary condition for the Samuelson's effect to hold.

- We provide two sufficient cases that induce Samuelson's effect:
  - Heterogeneity in market access quality γ_i
  - Heterogeneity in production costs $\epsilon_i = \frac{\eta_i\beta_0^i}{\eta_i+\beta_0^i}$.

General results

- Existence and uniqueness of equilibrium characterized by a coupled forward-backward system of SDEs with jumps

- No closed-form expression in the general case

A special case

We make the following assumptions:

a. For each $i \in [\![1, N]\!]$, the demand forecast is d_0^i, and is perfect with $\sigma_i = 0$;
b. The set $E = \{g, b\}$ consists of two states (*good* and *bad*), with $g < b$;
c. For each $i \in [\![1, N]\!]$, the Markov chain β^i has state space $E = \{g, b\}$, initial state g at time $t = 0$ and intensity matrix given by

$$\Lambda_i = \begin{pmatrix} -\lambda_i^g & \lambda_i^g \\ \lambda_i^b & -\lambda_i^b \end{pmatrix},$$

where λ_i^g and λ_i^b are fixed strictly positive real numbers.
d. Two groups of agents: $N = N_I + N_{II}$ with

$$\begin{aligned} \gamma_i = \gamma_I, \quad \eta_i &= \eta_I, \quad & \lambda_i &= \lambda_I, \quad & i &= 1, \dots, N_I, \\ \gamma_i = \gamma_{II}, \quad \eta_i &= \eta_{II}, \quad & \lambda_i &= \lambda_{II}, \quad & i &= N_I + 1, \dots, N. \end{aligned}$$

A Riccati system of equations

Consider for $i = 1, \dots, N$:

$$\begin{cases} y'_{i,g}(t) &= \frac{1}{\gamma_i} y_{i,g}(t)^2 + \lambda_i^g y_{i,g}(t) - \lambda_i^g y_{i,b}(t), \qquad y_{i,g}(T) = \frac{1}{2} \frac{\eta_i\, g}{\eta_i + g}, \\ y'_{i,b}(t) &= \frac{1}{\gamma_i} y_{i,b}(t)^2 - \lambda_i^b y_{i,g}(t) + \lambda_i^b y_{i,b}(t), \qquad y_{i,b}(T) = \frac{1}{2} \frac{\eta_i\, b}{\eta_i + b}. \end{cases}$$

and denote by $y_{I,g}, y_{I,b}$ (resp. $y_{II,g}, y_{II,b}$) the solutions to the system of equations above with coefficients $\gamma_I, \eta_I, \lambda_I$ (resp. $\gamma_{II}, \eta_{II}, \lambda_{II}$).

Approximate market equilibrium

For N large enough,

1. The equilibrium price is approximately given by $\hat{P}_t \simeq \tilde{P}_t$ with

$$\tilde{P}_t = \frac{\bar{\gamma}}{1-\bar{\gamma}\theta_t} \sum_{i=1}^{N} \underbrace{2y_{i,\beta_t^i}(t)(d_0^i - \hat{X}_t^i)}_{\Delta_t^i},$$

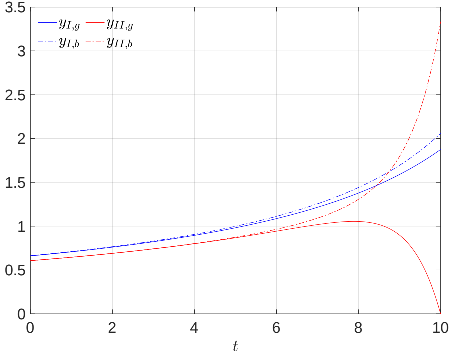
with $\bar{\gamma} = \frac{1}{\frac{N_I}{\gamma_I}+\frac{N_{II}}{\gamma_{II}}}$, $\theta_t = \frac{1}{\gamma_I}\sum_{i=1}^{N_I} a_t^i + \frac{1}{\gamma_{II}}\sum_{N_I+1}^{N} a_t^i$, with $a_t^i = \frac{1}{\gamma_i}(T-t)y_{i,\beta_t^i}(t)$.

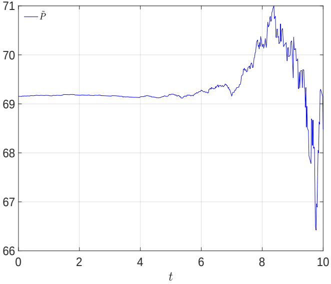
2. The equilibrium trading rates are approximately given by $\hat{q}_t^i \simeq \tilde{q}_t^i$ with

$$\tilde{q}_t^i = \frac{1}{2\gamma_I}\left(\Delta_t^i - (1-a_t^i)\hat{P}_t\right),$$

Numerical illustrations

First population: $N_I = 100$, $\lambda_I^g = \lambda_I^b = 0.1$, $\beta_I^g = 6$, $\beta_I^b = 7$;
Second population: $N_{II} = 100$, $\lambda_I^g = \lambda_I^b = 0.5$, $\beta_{II}^g = 0$, $\beta_{II}^b = 20$;





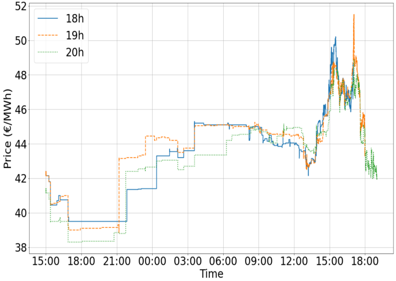


Figure: (Left) the approximated market equilibrium price (Right) Intraday mid-prices on EEX market on August, 30^{th}, 2017 for deliveries at 18h, 19h and 20h up to 1 hour before maturity

Thank you for your attention