Short-Term Wind Forecasting Using Spatio-Temporal Covariance Models

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Background

Why is wind speed/power forecasting important?

- It helps to integrate wind power effectively into the energy system.
- It enables system operators to anticipate fluctuations in wind power generation and maintain grid stability and reliability.
- It enables market participants to make informed decisions and manage risks associated with the variability of wind power generation.
- In general, it can ultimately reduce the levelized cost of energy and increase the overall system value.

Background

What are the tools and techniques are used for wind forecasting?

- Time series models
  - Browell and Gilbert (2017) used regime-switching (RS) AR to predict daily wind power.
  - Browell et al. (2018) adopted RS AR and VAR models with exogenous variables to predict wind speed at multiple locations.
  - Jia, Sezer, & Wood (2022) applied RS mixture AR and VAR models to predict wind speed at 23 weather stations in Alberta.
- Covariance models
  - Gneiting (2002); Gneiting et al. (2006) proposed a parametric family of covariance models. The prevailing wind was considered as fixed in a Lagrangian framework.
  - Ezzat et al. (2018, 2019) proposed a calibrated RS covariance method where the prevailing wind was considered a random variable.

Background

Covariance models:

- Tobler's law: everything is related to everything else, but near things are more related than distant things.
- They are used when wind is modeled by spatio-temporal (ST) Gaussian processes which are stationary and completely determined by mean and covariance functions.
- The spatio-temporal covariance functions must be positive semi-definite (p.s.d.) and there are limited choices: e.g. Matérn class, Gneiting class, etc.
- We study the Gneiting class and 1) introduce a new covariance function for accounting asymmetry, 2) allow the model to be regime-switching, 3) relax the constraint of the interaction parameter.

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ST Gaussian processes

- A ST process $\{Y(\mathbf{x}, t) : \mathbf{x} \in \mathbf{S}, t \in [0, T]\}$ is Gaussian if for any $n \geq 1$ and $(\mathbf{x}_1, t_1), \ldots, (\mathbf{x}_n, t_n)$, $Y(\mathbf{x}_1, t_1), \ldots, Y(\mathbf{x}_n, t_n)$ has multivariate Gaussian distribution with

$$\mathrm{E}[Y(\mathbf{x}_i, t_i)] = \mu(\mathbf{x}_i, t_i) \tag{1}$$

$$\mathrm{Cov}(Y(\mathbf{x}_i, t_i), Y(\mathbf{x}_j, t_j)) = C(\mathbf{x}_i, \mathbf{x}_j; t_i, t_j) \tag{2}$$

for some functions

$$\mu : \mathbf{S} \times [0, T] \to \mathbb{R},$$
$$C : (\mathbf{S} \times [0, T]) \times (\mathbf{S} \times [0, T]) \to \mathbb{R}.$$

ST Gaussian processes

- Often we need second-order stationarity and isotropy to make inference on the function C:

$$\text{Cov}(Y(\mathbf{x}_1, t_1), Y(\mathbf{x}_2, t_2)) = C(\|\mathbf{x}_1 - \mathbf{x}_2\|, |t_1 - t_2|) \tag{3}$$
$$= C(\|\mathbf{h}\|, |u|). \tag{4}$$

- It is difficult to come up with a stationary ST covariance function from scratch. Hence we use known parametric covariance functions.

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Gneiting Class

- **Gneiting class** (Gneiting, 2002) is built on top of purely spatial and purely temporal correlation functions.
- Purely spatial model:

$$C_S(\mathbf{h}) = (1-\nu)\exp(-c\|\mathbf{h}\|^{2\gamma}) + \nu\delta_{\mathbf{h}=0}, \tag{5}$$

  where $\delta_{\mathbf{h}=0} = 1$ if $\mathbf{h} = 0$ and 0 o.w., $\nu \in [0,1], c > 0, \gamma \in (0, 1/2]$.
- Purely temporal model:

$$C_T(u) = (1 + a|u|^{2\alpha})^{-1}, \tag{6}$$

  where $a > 0$ and $\alpha \in (0, 1]$.

Gneiting Class

- Separable model:

$$C_{\mathsf{SEP}}(\mathbf{h}, u) = C_{\mathsf{S}}(\mathbf{h}) C_{\mathsf{T}}(u) \tag{7}$$

$$= \left[(1-\nu)\exp(-c\|\mathbf{h}\|^{2\gamma}) + \nu\delta_{\mathbf{h}=0}\right](1 + a|u|^{2\alpha})^{-1}. \tag{8}$$

- Fully symmetric model:

$$C_{\mathsf{FS}}(\mathbf{h}, u) = \frac{1-\nu}{1 + a|u|^{2\alpha}}\left(\exp\left(\frac{c\|\mathbf{h}\|^{2\gamma}}{(1 + a|u|^{2\alpha})^{\beta\gamma}}\right) + \frac{\nu}{1-\nu}\delta_{\mathbf{h}=0}\right), \tag{9}$$

where $\beta \in [0, 1]$.

Asymmetric Covariance Functions - Lagrangian Framework

- Lagrangian reference frame is used to achieve asymmetry in correlation due to transport effects of prevailing wind flows.
- For a spatial random field on $\mathbb{R}^d$ with stationary covariance function C_S, suppose the entire field moves time-forward with velocity $\mathbf{V} \in \mathbb{R}^d$, then the resulting ST random process has the covariance

$$C(\mathbf{h}, u) = \mathsf{E}_{\mathbf{V}}[C_S(\mathbf{h} - \mathbf{V}u)]. \tag{10}$$

- The above function is generally asymmetric.

Asymmetric Covariance Functions - Lagrangian Framework

- General stationary model (Gneiting et al., 2006):

$$C_{\mathrm{STAT}}(\mathbf{h}, u) = (1-\lambda) C_{\mathrm{FS}}(\mathbf{h}, u) + \lambda C_{\mathrm{LGR}}(\mathbf{h}, u), \tag{11}$$

$$C_{\mathrm{LGR}}(\mathbf{h}, u) = C_{\mathrm{S}}(\mathbf{h} - \mathbf{V} u). \tag{12}$$

- Lagrangian correlation functions:
  - Triangular:

$$\left(1 - \frac{1}{2v} |\mathrm{h}_1 - vu| \right)_+, \tag{13}$$

  - New:

$$\left(1 - \frac{1}{2\|\boldsymbol{v}\|} \left| \frac{\mathbf{h}^\top \boldsymbol{v}}{\|\boldsymbol{v}\|} - \|\boldsymbol{v}\| u \right| \right)_+, \tag{14}$$

  - Askey (Askey, 1973):

$$\left(1 - \frac{1}{2\|\boldsymbol{v}\|} \|\mathbf{h} - \boldsymbol{v} u\| \right)_+^{3/2}. \tag{15}$$

Covariance Models - Regime-Switching

- The regime-switching models show better performance in wind forecasting than traditional time-series and covariance models.
- Regime-switching general stationary model:

$$C_{\mathrm{STAT},k}(\mathbf{h}, u) = (1 - \lambda_k) C_{\mathrm{FS}}(\mathbf{h}, u) + \lambda_k C_{\mathrm{LGR},k}(\mathbf{h}, u_k), \tag{16}$$

$$C_{\mathrm{LGR},k}(\mathbf{h}, u) = \left(1 - \frac{1}{2\|\boldsymbol{v}_k\|} \left| \frac{\mathbf{h}^\top \boldsymbol{v}_k}{\|\boldsymbol{v}_k\|} - \|\boldsymbol{v}_k\| u_k \right| \right)_+, \tag{17}$$

where k is the regime index.

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Markov Chain Gaussian Field

- A more practical framework is a p-th order Markov Chain Gaussian Field (MCGF) where time is discrete and we only model covariances up to time lag p. The system state is updated one time-step at a time using a transition probability conditional on the history up to p time lags.
- Markov chain Gaussian field: $\{\boldsymbol{Y}(\mathbf{S}, t) : t \in [0, \infty)\}$.
  - For a Gaussian process defined on field $\mathbf{S}$ with time lag p,

$$P(\boldsymbol{Y}(\mathbf{S}, t) | \boldsymbol{Y}(\mathbf{S}, t-1), \ldots, \boldsymbol{Y}(\mathbf{S}, 0)) = P(\boldsymbol{Y}(\mathbf{S}, t) | \boldsymbol{Y}(\mathbf{S}, t-1), \ldots, \boldsymbol{Y}(\mathbf{S}, t-p)). \quad (18)$$

  - It is different from Gaussian Markov random fields where the Markov property is applied spatially on $\mathbf{S}$.

Markov Chain Gaussian Field

Reasons of considering MCGF:

- For short-term wind speed modeling, future relies on short time lags.
- For RS models, defining the transition probabilities is more elegant in a Markovian framework.
- The covariance functions only need to be valid for temporal domain $[-p, p]$, instead of $\mathbb{R}$.

Covariance Models

- Fully symmetric model:

$$C_{\mathsf{FS}}(\mathbf{h}, u) = \frac{1-\nu}{1+a|u|^{2\alpha}} \left(\exp\left(\frac{c\|\mathbf{h}\|^{2\gamma}}{(1+a|u|^{2\alpha})^{\beta\gamma}} \right) + \frac{\nu}{1-\nu} \delta_{\mathbf{h}=0} \right), \tag{19}$$

where $\beta \in [0, 1]$.

- During parameter estimation, we found that the constraint $\beta \leq 1$ is binding and results will be improved if we allow $\beta > 1$.
- While letting $\beta > 1$ may not give a valid correlation function, we can prove that the constraints can be relaxed under certain conditions.

Covariance Models

- Goal: allow $\beta > 1$ in C_{FS}; that is, prove that C_{FS} gives a valid covariance matrix for **S** when $\beta > 1$.
- Steps:
  1. Let $\mathbf{G} = \{\mathbf{g}_1, \ldots, \mathbf{g}_m\}$ be a finite discrete rectangular grid that covers **S**.
  2. Let C be a function on $\mathbf{G} \times \mathbf{G} \times [-p, p]$, and we build the block-Toeplitz matrix $\boldsymbol{\Sigma}$ consisting of blocks

$$\Sigma_{i,j,u} = C(\mathbf{g}_i, \mathbf{g}_j, u).$$

  We assume:

  1. C is non-negative.
  2. $\boldsymbol{\Sigma}$ is p.s.d. This can be verified numerically.
  3. $C(\mathbf{g}_i, \mathbf{g}_i, |u|) = C_{\mathsf{T}}(u)$ is a p.s.d. function on $\{-p, -p+1, \ldots, p\}$ such that $C_{\mathsf{T}}(u) = C_{\mathsf{T}}(-u)$ and $C_{\mathsf{T}}(0) = 1$.

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Covariance Models

Theorem

Under Assumptions 1-3, the following function is a valid covariance function for MCGF $\boldsymbol{W}(\cdot, t)$.

$$C_M(\mathbf{x}_1, \mathbf{x}_2, t_1, t_2) = \begin{cases} C_T(|t_1 - t_2|), & \text{if } \mathbf{x}_1 = \mathbf{x}_2 \\ (1-\nu)C(G(\mathbf{x}_1), G(\mathbf{x}_2), |t_1 - t_2|), & \text{otherwise,} \end{cases}$$

where $\mathbf{x}_1, \mathbf{x}_2 \in \mathbf{S}$ *and* $t_1, t_2 \in \{t-p, \ldots, t\}$. *That is, for any* $\mathbf{X} = \{\mathbf{x}_1, \ldots, \mathbf{x}_n\} \subseteq \mathbf{S}$, *and any time lag* $u \in \{0, 1, \ldots, p\}$, C_M *gives a valid covariance matrix for the joint distribution of* $\boldsymbol{w} = (\boldsymbol{W}(\mathbf{X}, t), \ldots, \boldsymbol{W}(\mathbf{X}, t-u))$.

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The proof is based on the following construction of a ST process. Let $\mathbf{Y}(\cdot, t)$ be a Gaussian process with covariance matrix $\mathbf{\Sigma}$ on $G(\mathbf{X})$. We let

$$\widetilde{\boldsymbol{W}}(\mathbf{x}, t) = \boldsymbol{Y}(G(\mathbf{x}), t) + \boldsymbol{Z}(\mathbf{x}, t), \tag{20}$$

where $\boldsymbol{Z}(\mathbf{x}, t)$ is a ST Gaussian process with no spatial correlation and temporal correlation given by $\nu C_{\mathrm{T}}(u)$. Then $\{\widetilde{\boldsymbol{W}}(\mathbf{x}, t)\}$ is a valid ST process and its covariance matrix is given by C_M. Therefore, C_M must be positive semi-definite.

Remarks:

- Function C can be any functions that satisfy Assumptions 1-3.
- Example:
  - Let

$$C(\mathbf{h}, u) = \frac{1}{1 + a|u|^{2\alpha}} \exp\left(\frac{c\|\mathbf{h}\|^{2\gamma}}{(1 + a|u|^{2\alpha})^{\beta\gamma}} \right), \tag{21}$$

then

$$C_M = \begin{cases} (1 + a|u|^{2\alpha})^{-1}, & \text{if } \mathbf{h} = \mathbf{0} \\ \dfrac{1-\nu}{1 + a|u|^{2\alpha}} \exp\left(\dfrac{c\|\widetilde{\mathbf{h}}\|^{2\gamma}}{(1 + a|u|^{2\alpha})^{\beta\gamma}} \right), & \text{otherwise,} \end{cases} \tag{22}$$

is the fully symmetric model.

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Covariance Models - Summary

- Propose a new correlation function to account both prevailing wind speed and direction.

$$C_{\mathrm{LGR}}(\mathbf{h}, u) = \left(1 - \frac{1}{2\|\boldsymbol{v}\|}\left|\frac{\mathbf{h}^\top \boldsymbol{v}}{\|\boldsymbol{v}\|} - \|\boldsymbol{v}\| u\right|\right)_+ . \qquad (23)$$

- Expend the covariance models to be regime-switching.

$$C_{\mathrm{STAT},k}(\mathbf{h}, u) = (1 - \lambda_k) C_{\mathrm{FS}}(\mathbf{h}, u) + \lambda_k C_{\mathrm{LGR},k}(\mathbf{h}, u_k). \qquad (24)$$

- Allow β to be higher than 1.

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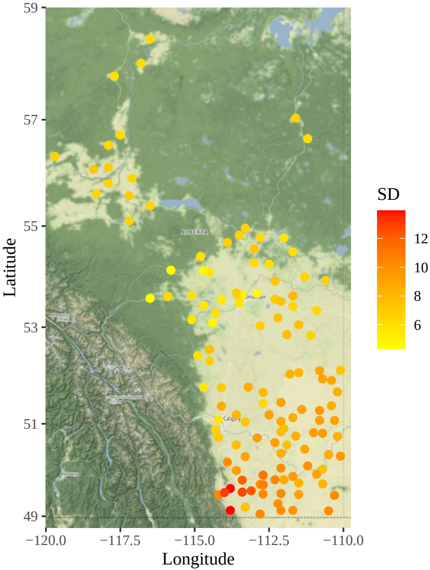
- We fit above models to predict hourly wind speed at 131 Alberta weather stations for up to 6-h ahead.
- Time span: 6-year
  - Training: 2016-2019
  - Test: 2020-2021
- To fit regime-switching models, we identify atmospheric regimes with ERA5 database on four atmospheric variables (Browell et al., 2018) using k-means and hidden Markov models.



Wind Speed Forecasting

- Candidate models:
  - Empirical model
  - Fully symmetric model (FS) and that with relaxed constraints on β (FS2)
  - Regime-switching general stationary model (STAT) and that with relaxed constraints on β (STAT2)
  - For regime-switching models, we also obtain soft forecasts: the weighted averages of forecasts under each regime.

Modeling Results - Summary

- All parametric models are better than the empirical model, on average.
- The STAT2 model with soft forecast under HMM performs the best, followed by the FS2 model.
- Compared with the FS model, all general stationary models that take regime-dependent prevailing wind into consideration perform better.
- Considering atmospheric information and prevailing wind is not beneficial for short forecast horizons (1-2 h), and the Empirical model works the best.

Modeling Results

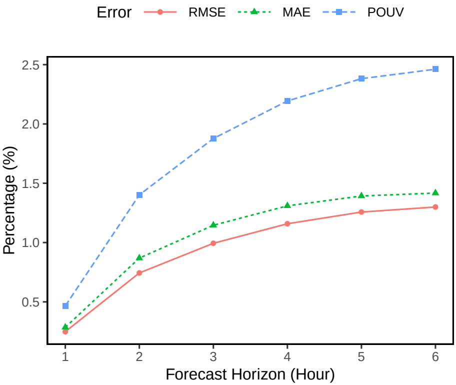


Figure: Percentage Improvement of STAT-soft Under HMM Over FS

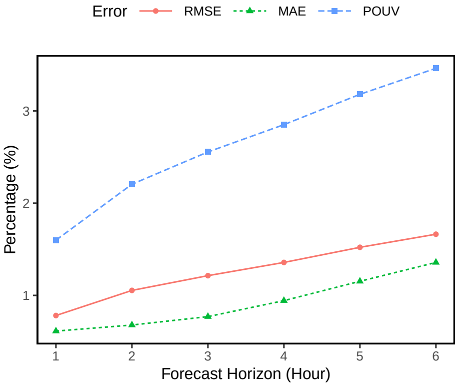


Figure: Percentage Improvement of FS2 Over FS

Modeling Results - Station

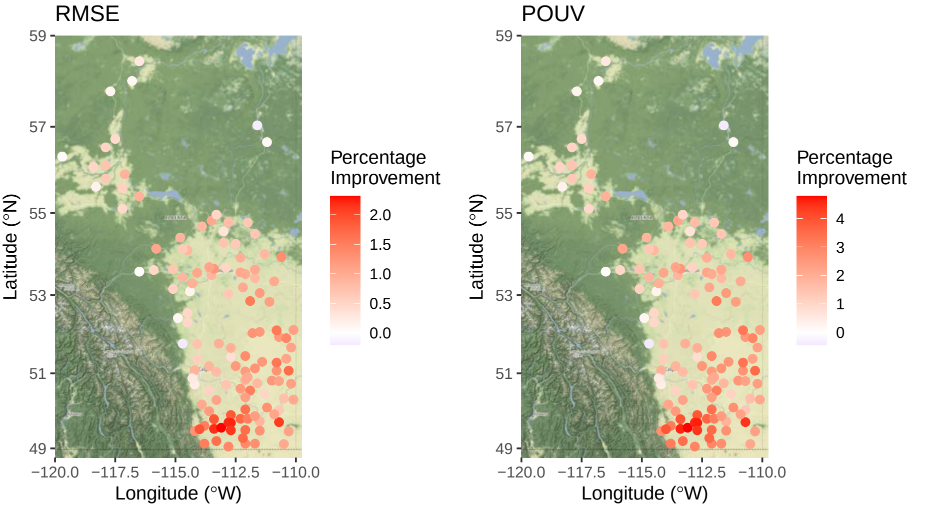


Figure: Percentage Improvement of STAT-soft Over FS by Weather Stations.

Modeling Results - Station

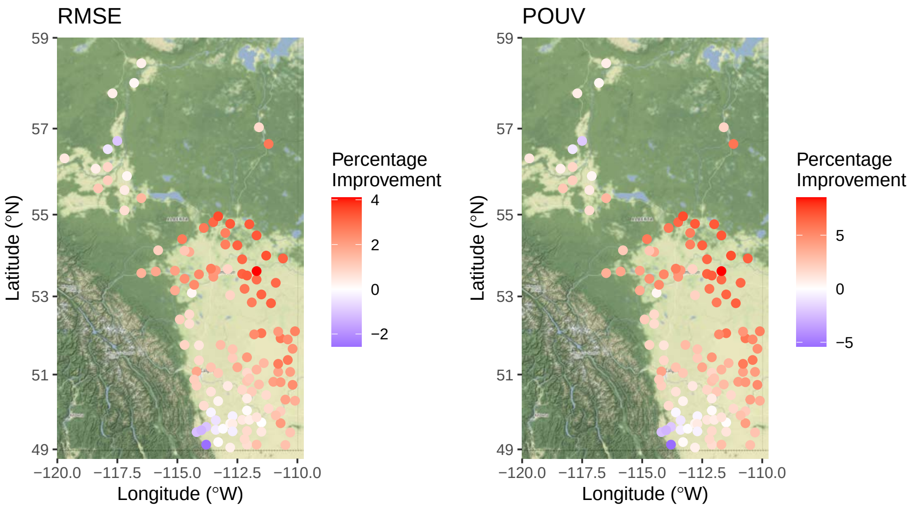


Figure: Percentage Improvement of FS2 Over The FS by Weather Stations.

Future Work

- Allow prevailing wind speed and direction to vary by time and location.
- Allow different models for different areas.
- Propose new models for areas with more complex wind patterns.

Future Work

- We are writing an R package called `mcgf` to
  - simulate MCGF with aforementioned (regime-switching) covariance structures,
  - estimate parameters of the covariance functions by weighted least squares or maximum likelihood estimation.
  - predict future values by simple kriging.
- We have finished simulation and parameter estimation parts of the package, and now we are finishing the rest.
- The package is available on: `https://github.com/Tianxia-Jia/mcgf`.

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Thank You!

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