Pacific Institute *for the* Mathematical Sciences

PIMS 25ᵗʰ Anniversary Network-Wide Colloquium

February 11, 2021 1:30 PM PST

Ben Green, University of Oxford

On the divisors of a typical integer

Abstract: Pick a random positive integer n between X and $2X$. What do its divisors look like, typically? This can be a hard question. For instance, Erdős conjectured in the 1940s that almost surely n has two divisors d, d' with $d < d' < 2d$, but this was only proven more than 30 years later, by Maier and Tenenbaum. I will give an introduction to this subject and then turn to some recent joint work with Ford and Koukoulopoulos in which we delve deeper into this kind of question, showing that a typical n in fact has many divisors in some dyadic range $y < d < 2y$ and giving a bound for this problem which we conjecture to be optimal. This work involves a mix of ideas from probability, number theory and analysis and the results feature some of the strangest exponents I have ever seen in a problem. The talk should be accessible to a general audience.

Biography: Ben Green works in additive combinatorics and related areas, such as harmonic analysis and approximate algebraic structure. The underlying relationship is that ideas rooted in harmonic analysis are used to give a quantitative description of randomness versus structure. Among many important results, the results with Terence Tao stating that the prime numbers contain arbitrarily long arithmetic progressions, is a landmark. His work has revolutionized the field and Dr. Green has been awarded many prizes, including the Salem Prize and the Sylvester Medal of the Royal Society. Dr. Green is a Fellow of the Royal Society.

Ben Green is the Waynflete Professor of pure mathematics at the University of Oxford. Previously he was a Professor at the University of Cambridge. His paper with Terence Tao on arithmetic progressions in the primes was written when he was a PIMS postdoctoral fellow at the University of British Columbia.

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