Final Report
CRM-PIMS Summer School in Probability
June 15-July 10 2015

Summary

The CRM-PIMS Summer School in Probability took place in Montreal for the first time from June 15 to July 10, 2015. The lectures were held at McGill University, McConnell Engineering building. The details of the organization can be found on the school website:
http://problab.ca/ssprob2015/
A grand total of 115 participants attended the school including the speakers. As can be seen from the participant list (attached to this report), the demographics were very diverse with a strong Canadian, American and European contingent. Most of the participants (95) were Ph.D. students. The funding of the School came from PIMS, CRM, MSRI, Clay Institute, NSF, and he Montreal Probability Group (CRM probability laboratory).

The program of the school included four days of courses Monday, Tuesday, Thursday and Friday with one free day on Wednesday. There were two major courses of six hours per week that were given by Alice Guionnet (MIT) and Remco van der Hoftad (TU Eindhoven). There were also three mini-courses of three hours each given by Shankar Bhamidi (North Carolina, Chapel Hill), Jonathan Mattingly (Duke) and Louigi Addario-Berry (McGill). A one-hour guest lecture was also given by Peter Winkler (Dartmouth). The scientific content of these lectures are detailed below. Each day of the school included around four to five hours of lectures. The students seemed to appreciate the free day on Wednesday to assimilate the materials. The participation by the students was, in our opinion and in the opinion of the lecturers, quite extraordinary. The students asked many questions at every lecture and followed the material until the end. This is to the merit of the lecturers, especially Guionnet and van der Hofstadt, who had prepared detailed lecture notes (with exercises) beforehand allowing the students to follow along. In addition of the senior lectures, the schedule included twenty-seven short talks (30 minutes) by the participants. These participants were mostly advanced PhD students that were presenting their thesis work. It was widely acknowledged by the organizers and the senior participants that the quality of the presentation of the short talks as well as their scientific content were of very high quality. The subjects of the short talks are discussed in more details below.

Scientific Highlight

The course of Alice Guionnet was entitled *Random matrices, free probability, and the enumeration of maps*. Random matrices are random variables taking values in spaces of matrices. The study of random matrices has undergone major breakthroughs in the recent years, especially in the study of universal properties of their spectrum. Guionnet started the course by showing different methods to prove the convergence of the spectral measure based on the compuation of moments. The other was analytical and based on the convergence of the Stieltjes measure. She then discussed how the fluctuations of the spectral measure can be studied by refining these methods. She also mentioned recent results on the universal local fluctuations of the eigenvalues for the beta-ensemble which is a general class of random matrices. She briefly discussed the

applications of free probability to the problem of convergence for joint moments of products of matrices. Finally, she concluded the course by connecting the problem of finding the fine asymptotics of linear statistics of random matrices (Wigner matrices and beta-ensembles) to the topological expansion that arises when developing generating functions of these statistics. The problem of enumerating maps (or geometries) then corresponds to the control of these statistics using the so-called loop equations.

The course of Remco van der Hofstadt was *High-dimensional Percolation and Random Graphs*. Percolation is one of the most important model of modern probability. Percolation is a simple models exhibiting phase transition. On a graph, every edge chooses to be open with probability p. One can then study the probability of connections of two vertices on open edges. The goal of the lectures was to develop general techniques to prove common behaviors for percolation in high-dimension. The lectures started by studying percolation on a tree and branching random walks. The relevant connective properties were derived for this mean-field model. Van der Hoftadt then showed a recent proof of Dominil-Copin and Tassion on the uniqueness of the phase transition. The lecture then proceeded to the proof of mean-field behavior of percolation in high-dimension using the *lace expansion* to prove the so-called triangle condition. Recent results by Fitzner and van der Hoftadt were presented that brings the upper critical dimension (above which mean-field behavior is expected) from 19 to 11. The conjecture for the upper critical dimension is d=6.
(Last week ?)

The mini-courses of three hours each were by Addario-Berry (*Random minimum spanning trees*), *Dynamic random network models*, and Mattingly (*Stabilization by noise*).
Addario-Berry gave an introduction to the probabilistic study of minimum spanning trees, from the perspective of coalescent theory. There are three basic and natural random discrete coalescent procedures, corresponding to constant, additive, and multiplicative gelation kernels. The first of these corresponds to Kingman's coalescent; the second to the additive coalescent. The multiplicative case turns out to precisely build mean-field random minimum spanning trees. After situating the minimum spanning tree problem in this manner, Addario-Berry presented a novel proof of Frieze's zeta(3) limit theorem for the weight of the minimum spanning tree, and introduced the link between minimum spanning trees and critical percolation.

Bhamidi's lectures were focussed on his recent work on universality for scaling limits of random network models. As part of a programme to study the metric structure of minimum spanning trees, Addario-Berry and coauthors recently derived the scaling limit of the critical Erdös-Rényi random graph. Bhamidi and his coauthors have shown that this scaling limit is also the limit for many other random graph models; he provided an introduction to this impressive line of research. In his final lecture, he also described his recent work on the superstar model – a random graph process designed to model networks in which there is "condensation", or a small number of nodes which have macroscopic effects on the connectivity of the network.

Mattingly's lectures provided a beautiful introduction to techniques he and coauthors have recently developed to study the existence and uniqueness of invariant measures for Markov processes in infinite-dimensional state spaces. Mattingly presented techniques involving the use of Lyapunov and super-Lyapunov functions, and the use of Poisson equations to propagate Lyapunov functions between different parts of phase space. He closed by presenting his new

proof, with Martin Hairer, of Harris's ergodic theory, and theorem addressing uniqueness of ergodic invariant measures in settings where different initial conditions lead to mutually singular diffusions.

The guest lecture by Peter Winkler was entitled "Permutons" and focused on the study of patterns emerging in random permutations.

The short talks covered a wide range of topics in modern probability. We attached the abstract for more details. As a glimpse, there were presentations on KPZ universality, Random Trees, Schramm-Loewner Evolution on the Ising model, Spin Glasses, SDE with singular drift, Random Walks in Random Environment, Random Graphs, Cover Times of Random Walks, etc.

Conclusion

We believe that the 2015 CRM-PIMS Summer School in Probability was a great success considering the strong attendance and the wide range and deep scientific content of the lectures. This was the first time that the PIMS School in Probabiltiy was organized in Montreal. We hope that the school will return in the near future, building on the success of this year and improving the organization based on the feedback of the participants. We are grateful for the opportunity to organize such an important event for the international probability community and for the financial support provided by the PIMS, CRM, MSRI, Clay Institute, NSF, and the Montreal Probability Group (CRM probability laboratory).

Local organizing committee

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