A Graph of Matrices

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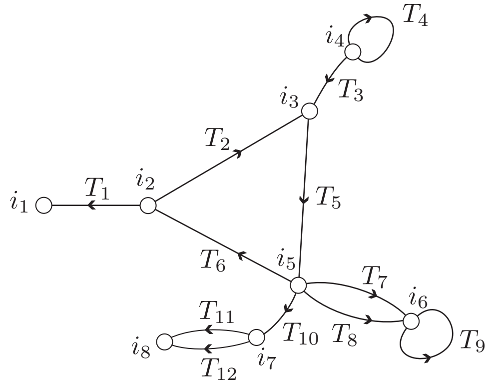
Free probability is a variation of probability theory for matrix valued random variables. It has many aspects: combinatorial, analytic, theoretical, and applied. I will discuss a problem on a graph of matrices arising from a random matrix problem in free probability.

Let $G = (E, V)$ be a graph and T a map from E to the $N \times N$ matrices. We write the matrix elements of $T(e)$ as $\{t_{ij}^{(e)}\}$ and let

$$S_G(T) = \sum_{i:V \to [N]} \prod_{e \in E} t^{(e)}_{i_{s(e)} i_{t(e)}}$$

where i runs over all functions from V to $[N] = \{1, 2, 3, \ldots, N\}$. For example if the the graph G is



the corresponding sum is

$$S_G(T) = \sum_{i_1, i_2, \ldots, i_7 = 1}^{N} t^{(1)}_{i_1 i_2} t^{(2)}_{i_3 i_2} t^{(3)}_{i_3 i_4} t^{(4)}_{i_4 i_4} t^{(5)}_{i_5 i_3} t^{(6)}_{i_2 i_5} t^{(7)}_{i_6 i_5} t^{(8)}_{i_6 i_5} t^{(9)}_{i_6 i_6} t^{(10)}_{i_7 i_5} t^{(11)}_{i_8 i_7} t^{(12)}_{i_8 i_7}$$

The question we wish to address is the dependence of $S_G(T)$ on N, which as we shall show has a surprisingly simple answer.