COUNTING POINTS OF HOMOGENEOUS VARIETIES OVER FINITE FIELDS

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This talk is based on joint work with Emmanuel Peyre. We consider an algebraic variety X, homogeneous under an algebraic group G. Assuming that X and G are defined over a finite field $\mathbb{F}_q$, we study the number of points of X over finite extensions $\mathbb{F}_{q^n}$.

When G is linear, we show that $|X(\mathbb{F}_{q^n})|$ is a periodic polynomial function of q^n with integer coefficients. Moreover, the "shifted" periodic polynomial function, where q^n is formally replaced with q^n+1, has non-negative coefficients.

For an arbitary G, we obtain a factorization

$$|X(\mathbb{F}_{q^n})| = |A(\mathbb{F}_{q^n})|\,|Y(\mathbb{F}_{q^n})|$$

where A is an abelian variety, and Y is homogeneous under a linear algebraic group.