Quantification of uncertainties in atmospheric analyses and forecasts by using normal modes

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- Global energy spectra

How large part of the global energetics is represented by the inertio-gravity (IG) motions?

In particular, how much of the large-scale tropical variability is associated with the Kelvin wave, mixed Rossby-gravity (MRG) wave, other IG waves?

How these percentage vary between the analysis datasets?

- Related data assimilation issues

What part of the short-range forecast errors in the tropics belong to the IG motions? How are the tropical errors spread across the scales, time and motion types ?

What modes are picked by model biases?

How important are large-scale tropical waves for the data assimilation?

What is the real potential of the EnKF in the tropics due to flow-dependent background-error covariances in comparison to 4D-Var?

Outline: certainty and uncertainty

- Motivation for the revival of normal mode expansion with emphasis on large-scale tropical motions
- Derivation of normal modes for various datasets
- Quantification of energy in various analysis datasets: DART/CAM, ECMWF and NCEP
- Analysis of time averaged analysis increments in terms of various divergent and non-divergent motions
- Quantification of time-dependency of the short-range forecast uncertainties in the ensemble system DART/CAM
- Conclusions

Normal mode functions

Kasahara and Puri, Mon.Wea.Rev. 1981

- Linearization around the mean state (vertically stratified in N_z σ levels and at rest)
- New mass variable P $P = gz + RT_0(\sigma)q$ $q=\ln(p_s)$

$$\frac{\partial u'}{\partial t} - 2\Omega v' \sin\phi = -\frac{\partial P'}{a\cos\phi\partial\lambda},$$
$$\frac{\partial v'}{\partial t} + 2\Omega u' \sin\phi = -\frac{\partial P'}{a\partial\phi},$$
$$\frac{\partial}{\partial t}\left[\frac{\partial}{\partial\sigma}\left(\frac{\sigma}{R\Gamma_0}\frac{\partial P'}{\partial\sigma}\right)\right] - \nabla\cdot\mathbf{V}' = 0.$$

→

$$\frac{\partial\tilde{u}}{\partial t} - 2\Omega\sin\phi\tilde{v} = -\frac{g}{a\cos\phi}\frac{\partial\tilde{h}}{\partial\lambda},$$
$$\frac{\partial\tilde{v}}{\partial t} + 2\Omega\sin\phi\tilde{u} = -\frac{g}{a}\frac{\partial\tilde{h}}{\partial\phi},$$
$$\frac{\partial\tilde{h}}{\partial t} + D\nabla\cdot\tilde{\mathbf{V}} = 0,$$

assume separation of variables by new vertical dependence function Ψ

$$\left.\begin{aligned} u' &= \tilde{u}\Psi(\sigma)\\ v' &= \tilde{v}\Psi(\sigma)\\ P' &= g\tilde{h}\Psi(\sigma)\end{aligned}\right\}.$$

$$\int_0^1 \Psi_i(\sigma)\Psi_j(\sigma)d\sigma = \delta_{ij},$$

$$\frac{d}{d\sigma}\left(\frac{\sigma g}{R\Gamma_0}\frac{d\Psi}{d\sigma}\right) + \frac{1}{D}\Psi = 0.$$

$$\Gamma_0 = \frac{\kappa T_0}{\sigma} - \frac{dT_0}{d\sigma}$$ Stability parameter

D - separation constant of dimension length - “equivalent depth”

Normal mode functions

Beauty of physical significance

System of equations for the horizontal structure of modes

$$\frac{\partial \bar{u}}{\partial t} - 2\Omega \sin\phi \bar{v} = -\frac{g}{a\cos\phi}\frac{\partial \bar{h}}{\partial \lambda},$$

$$\frac{\partial \bar{v}}{\partial t} + 2\Omega \sin\phi \bar{u} = -\frac{g}{a}\frac{\partial \bar{h}}{\partial \phi}, \qquad (*)$$

$$\frac{\partial \bar{h}}{\partial t} + D\nabla\cdot\bar{\mathbf{V}} = 0,$$

$$(\bar{u},\bar{v},\bar{h})^{\mathrm{T}} = \mathbf{S}_n \mathbf{H}_r^s(\lambda,\phi;n)\exp(-i\sigma_r^s t),$$

$$\mathbf{S}_n = \begin{pmatrix} (gD_n)^{1/2} & 0 & 0 \\ 0 & (gD_n)^{1/2} & 0 \\ 0 & 0 & D_n \end{pmatrix}$$

Hough functions

$$\mathbf{H}_r^s(\lambda,\phi;n) = \mathbf{H}_r^s(\phi;n)e^{is\lambda}$$

$$\frac{1}{2\pi}\int_0^{2\pi}\int_{-1}^{1}\mathbf{H}_{r'}^{s'}\cdot(\mathbf{H}_r^s)^* d\mu d\lambda = \delta_{rr'}\delta_{ss'},$$

n - vertical mode index

s - zonal mode index

r - meridional mode index

σ – eigen frequency

Energy partitioned into rotational (ROT) and inertio-gravity (IG) motions (eastward-EIG and westward-WIG) for each vertical mode

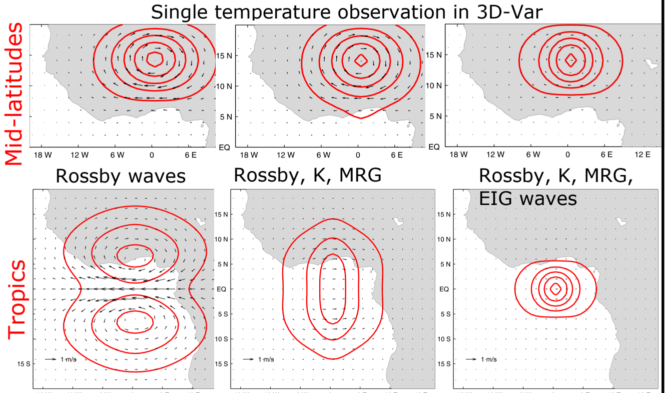
Tropics

Region with largest uncertainties in the existing (re)analysis datasets, because of

- Lack of direct observations of the wind field, especially wind profiles
- Difficult task of the tropical data assimilation due to balance issue

Uncertainty concerning the role of divergent motions: static bkg-error covariance

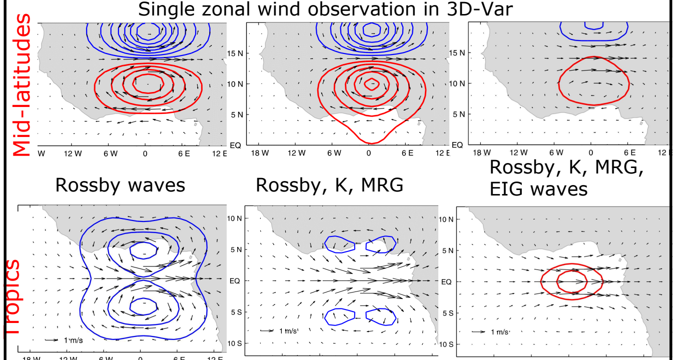
Single temperature observation in 3D-Var



Background–error spectra derived for ECMWF model level ~500 hPa: 43% equatorial Rossby, 8% Kelvin, 10% MRG, 39% other equatorial IG waves.

Uncertainty concerning the role of divergent motions: static bkg-error covariance

Single zonal wind observation in 3D-Var



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Remedies

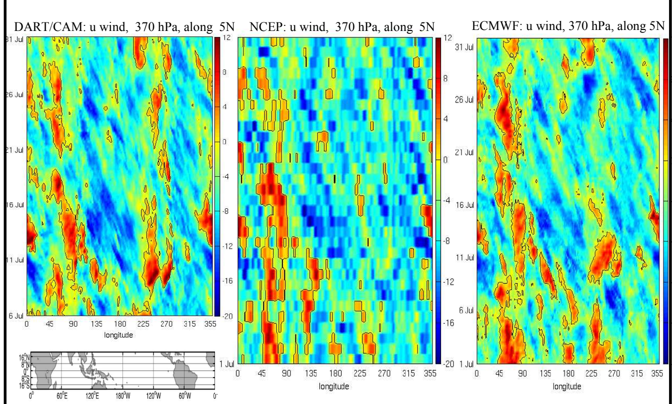
- Improved global observing system
- More advanced data assimilation procedures
- Improvements of the models, especially convective parameterizations and resolution

Application of normal modes to DART/CAM, NCEP and ECMWF datasets

Four analysis dataset for July 2007, global fields every 6 hours

- DART/CAM: ensemble mean from the DART system, version 3.1, T85 horizontal resolution, 26 vertical levels up to 3.5 hPa. Limited number of observations (conventional observations and AMVs).
- ECMWF: operational analyses, 12-hour 4D-Var system, T799 truncation interpolated to N64 grid, 91 vertical level up to 1 Pa. Large amounts of satellite observations.
- NCEP: operational analyses, 3D-Var system, T382 truncation interpolated to N64 grid, 64 vertical level up to 3.3 Pa. Large amounts of satellite observations.
- NCEP/NCAR reanalyses from NCAR mass archive: 3D-Var system, T62 horizontal resolution interpolated to N47 grid, 28 vertical levels up to 2.7 hPa. The assimilation system is not the recent one and it assimilates retrievals.

Tropical winds along 5° N in July 2007 at 370 hPa



Normal mode expansion

Basic idea: select the expansion basis which provides the best fit (best correlation and variance fit to the input grid-point fields) ⇔ tuning of the truncation parameters N_k, N_n, N_m

$$\mathbf{X}(\lambda,\varphi,z,t)=\sum_{m=1}^{N_m}\sum_{n_{i=1,2,3}=0}^{N_n-1}\sum_{k=-N_k}^{N_k}\chi_{knm}(t)\mathbf{S_m}\Pi_{knm}(\lambda,\varphi,z)$$

common expansion coefficient ($\chi_{knm}(t)$); 3D normal mode functions (Π_{knm}); normalization matrix ($\mathbf{S_m}$)

input data vector $\mathbf{X}=(u,v,P/g)^T$

N_m – no. vertical modes, index m

N_n – no. meridional modes per wave type, index n

N_k – no. zonal waves, index k

$$\Pi_{knm}(\lambda,\varphi,z)=\Phi_m(z)\cdot\mathrm{H}_{knm}$$

vertical normal modes ($\Phi_m(z)$); Hough functions (H_{knm})

$$\mathbf{S_m}=\begin{pmatrix}(gH_{eq})^{1/2} & 0 & 0\\ 0 & (gH_{eq})^{1/2} & 0\\ 0 & 0 & gH_{eq}\end{pmatrix}$$

$$\langle\Pi_{knm},\Pi_{k'n'm'}\rangle=\delta_{kk'}\delta_{nn'}\delta_{mm'}$$ orthogonal 3D expansion basis

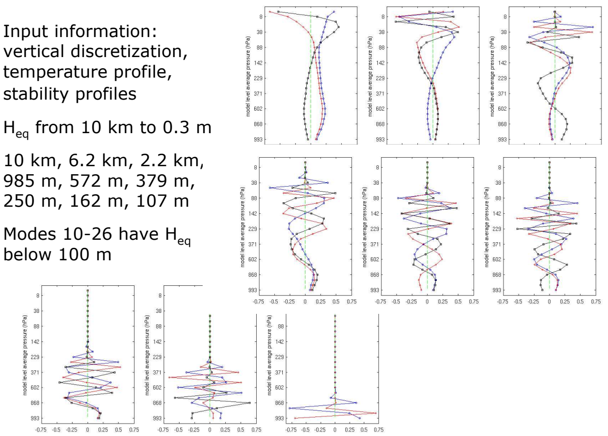
Vertical eigenstructures for DART/CAM

Input information: vertical discretization, temperature profile, stability profiles

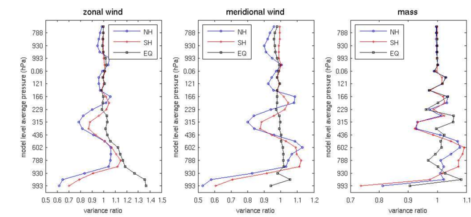
H_{eq} from 10 km to 0.3 m

10 km, 6.2 km, 2.2 km, 985 m, 572 m, 379 m, 250 m, 162 m, 107 m

Modes 10-26 have H_{eq} below 100 m



Verification of the expansion quality for DART/CAM



$N_k = 80$

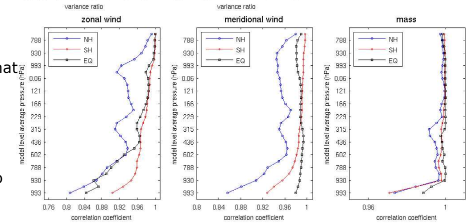
$N_n = 25$

$N_m = 25$

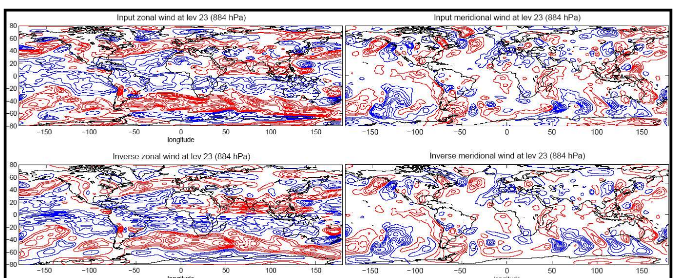
Variance ratio

Below 900 hPa zonal wind variance somewhat overestimated in the tropics, and underestimated in the mid-latitudes

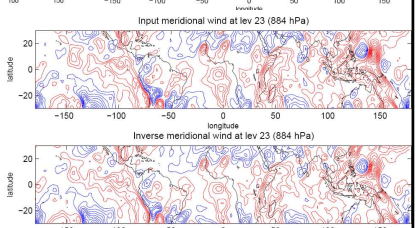
Mass-field variance relatively poor close to the surface due to orography



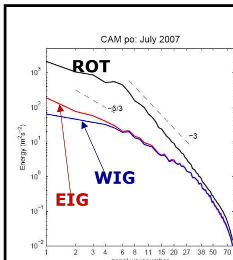
Correlation coefficient



Example of the projection quality for NCEP/NCAR wind field at 884 hPa level



Energy distribution in DART/CAM

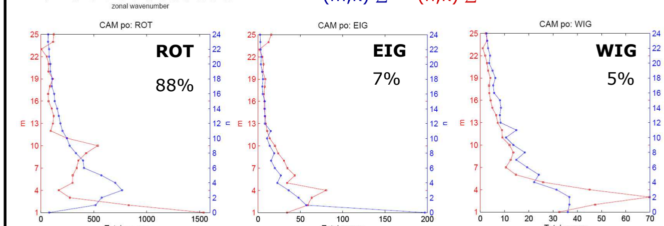


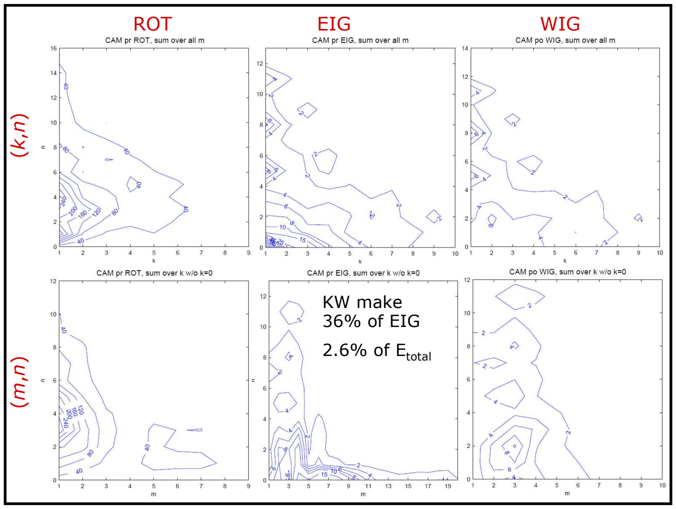
(m,n) Σ

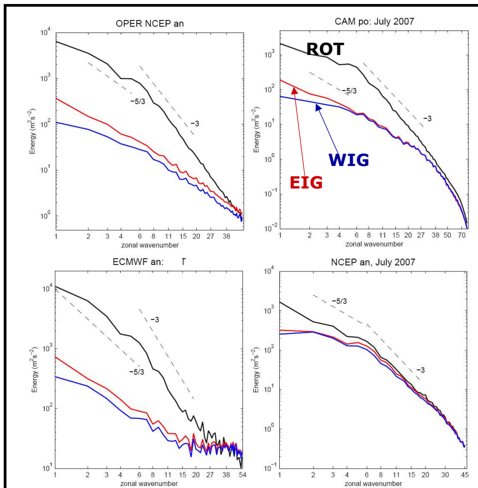
Posterior ensemble mean, average over 25-day period 6-31 July 2007

$$\sum_k \sum_n \sum_m g H_{eq} \left|\chi_{knm}\right|^2$$

(m,k) Σ (n,k) Σ







Analyses inter-comparison

CAM

On average, smallest % in IG among 4 datasets

ECMWF and NCEP

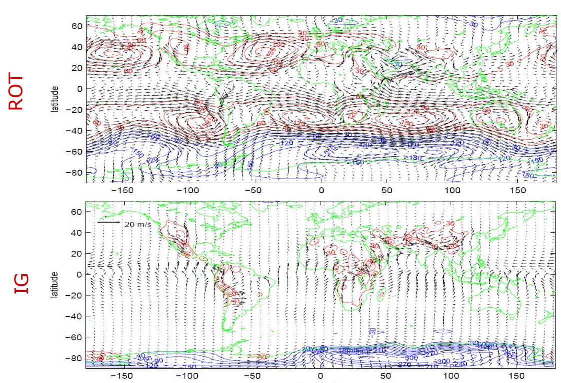
n-mode symmetry in EIG-WIG,

Lowest vertical mode dominant

NCEP/NCAR

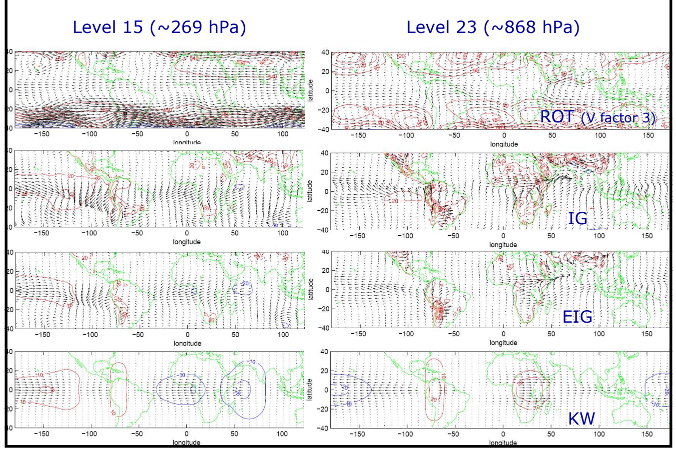
Significant % of IG energy also in SH mid-latitudes

Mean low-level July circulation in DART/CAM

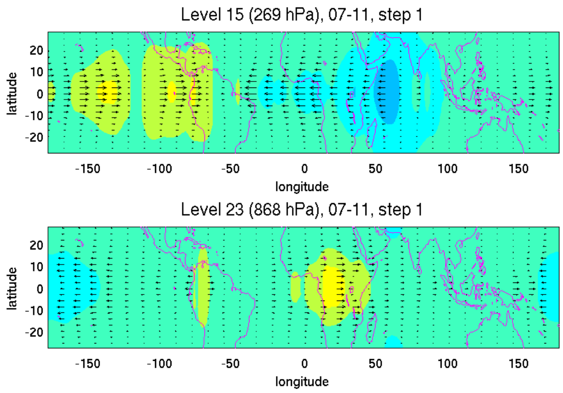


Model level 24 (~929 hPa)

Tropics as envisaged by A. Gill (1980)



Kelvin wave evolution in DART/CAM in July 2007



Kelvin wave evolution: summary

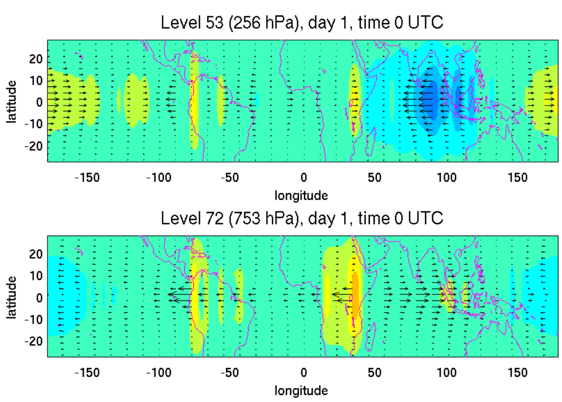
- Reversed flow in the lower and upper troposphere
- Spatial discontinuity of the k=1 signal
- Stronger signal developed by the end of month, especially in the Pacific
- Oscillations on daily basis due to tidal signal and possibly also due to observation coverage

How reliable is this Kelvin wave evolution?

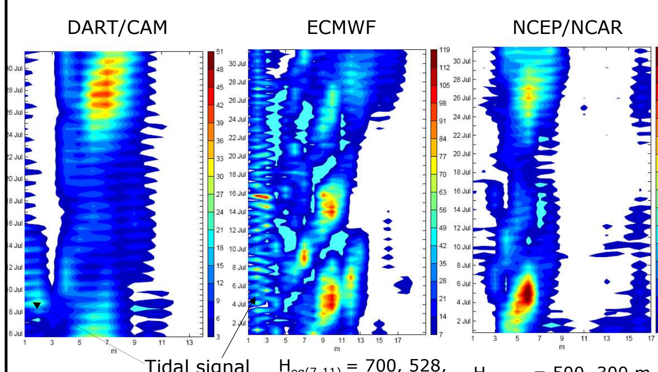
DART/CAM uses few observations in the tropics. The assimilation uses flow-derived (multivariate) background-error covariances

- Inter-comparison with other analyses
- Impact of models' biases
- Estimate of the analysis uncertainty

Kelvin wave evolution in July 2007 by ECMWF



Temporal evolution of the KW, *k*=1 signal

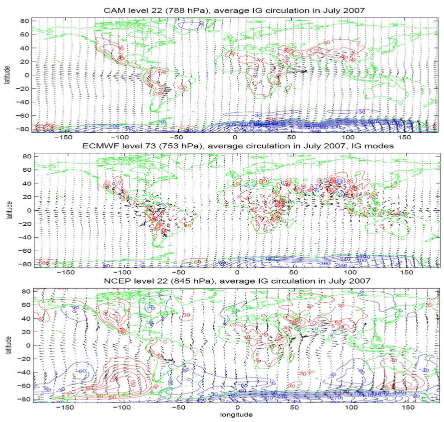


$H_{eq(5-7)}$ = 570, 370, 250 m

$H_{eq(7-11)}$ = 700, 528, 413, 332, 271

$H_{eq(5-6)}$ = 500, 300 m

Average IG motions in July 2007 in the lower troposphere

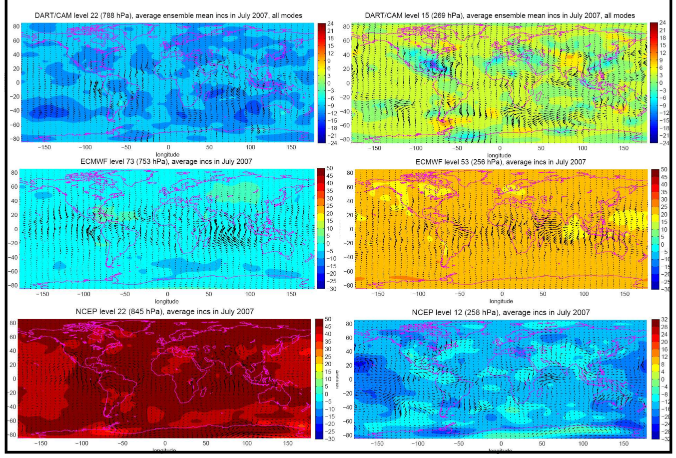


DART/CAM:
lev 22 (~788 hPa)

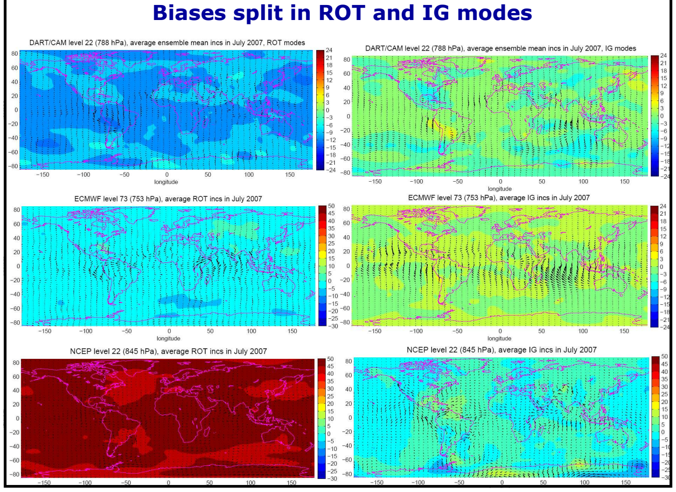
ECMWF:
lev 73 (~753 hPa)

NCEP/NCAR:
lev 22 (~845 hPa)

Time-averaged analysis increments ~ biases

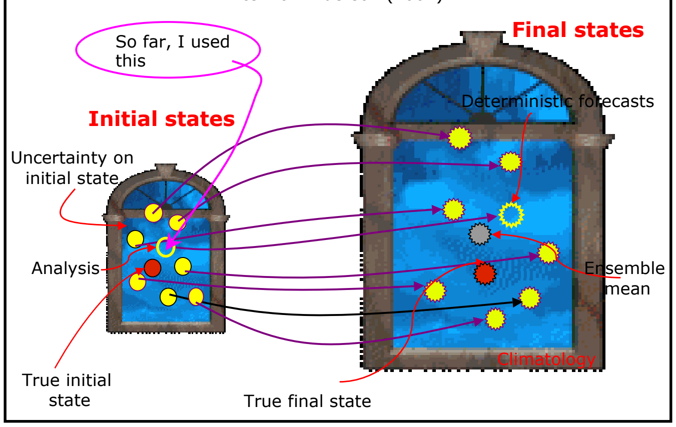


Biases split in ROT and IG modes



Ensemble assimilation:

for DART/CAM solved within the "ensemble adjustment Kalman filter" of Anderson (2001)



Quantifying uncertainties in CAM analyses

To analyse the uncertainty, each prior and posterior ensemble member projected.

To analyse equivalents of 6-hr forecast errors, departures from the ensemble mean fields projected.

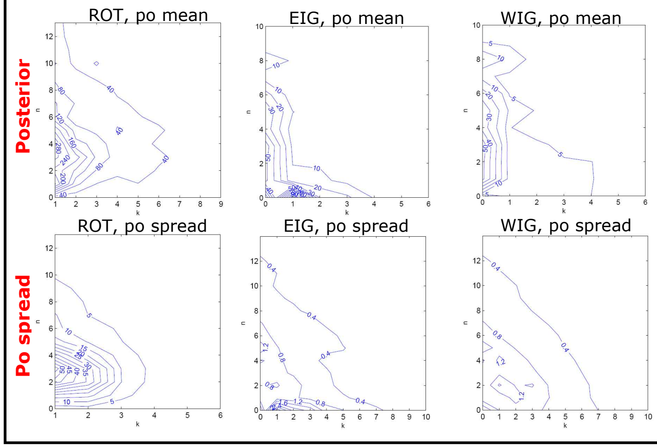
$$\mathbf{X}(\lambda,\varphi,z,t)=(u,v,P)^T \qquad \mathbf{X}(\lambda,\varphi,z,t)=(u-\overline{u},v-\overline{v},P-\overline{P})^T$$

$$\mathbf{X}(\lambda,\varphi,z,t)=\sum_{m=1}^{N_m}\sum_{n_{i=1,2,3}=0}^{N_n-1}\sum_{k=-N_k}^{N_k}\chi_{knm}(t)\mathbf{S}_{\mathbf{m}}\Pi_{knm}(\lambda,\varphi,z)$$

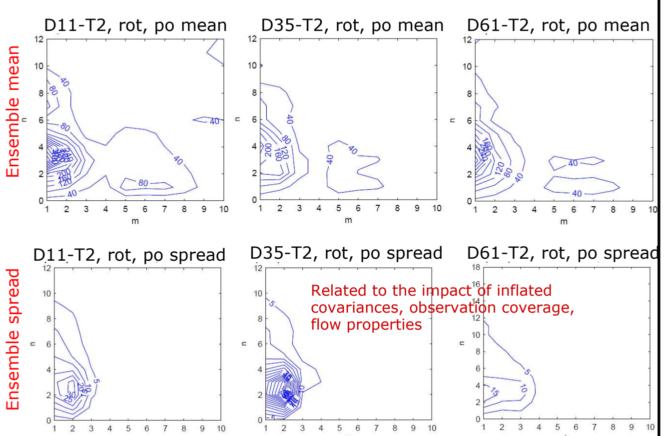
Ensemble size problem accounted for by:

- Covariance localization – reduces the impact of an observation on a state variable by a factor which is a function of their physical distance.
- Covariance inflation – increases the prior ensemble spread leaving the mean and correlations between the variables unchanged (here used is a time constant, spatially varying inflation applied on posterior)

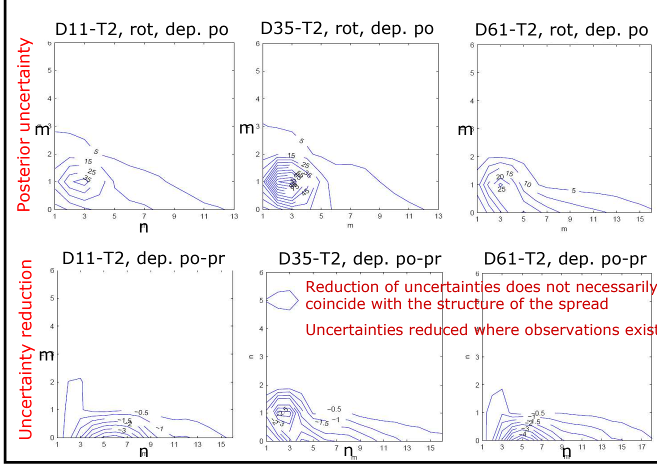
Averaged ens mean and its uncertainty



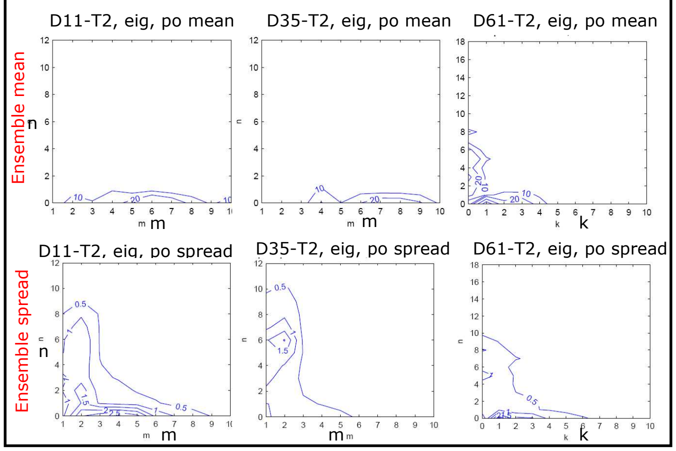
Analysis and its uncertainties in time: ROT modes



Uncertainty reduction in time



Mean and uncertainties in EIG modes



Summary

• Tropics are the area with largest uncertainties in existing analysis datasets. Tropics are also the area with largest biases.

• Normal mode expansion allows to quantify energy in various motions and to modify traditional view of inertio-gravity motions as junk. With normal modes it is possible to quantify variance in various tropical divergent motions and its relevance for data assimilation.

• Application of normal modes offers a physically attractive approach to quantification of uncertainties in analyses and forecasts. It points out the scales and motion types most affected by the inflation, localization, observations and model biases.

• Uncertainties vary in time and space, thus an argument for a flow-dependent covariance matrix for the forecast errors. The normal mode application may also help to address modeling aspects such as model-error covariances and initialization.