



Pacific Institute for the  
Mathematical Sciences

## PIMS Distinguished Speakers Series

# YOUNESS LAMZOURI

## Professor

### Institut Élie Cartan de Lorraine



## Title: A walk on Legendre paths

Abstract: The Legendre symbol is one of the most basic, mysterious and extensively studied objects in number theory. It is a multiplicative function that encodes information about whether an integer is a square modulo an odd prime  $p$ . The Legendre symbol was introduced by Adrien-Marie Legendre in 1798, and has since found countless applications in various areas of mathematics as well as in other fields including cryptography. In this talk, we shall explore what we call "Legendre paths", which encode information about the values of the Legendre symbol. The Legendre path modulo  $p$  is defined as the polygonal path in the plane formed by joining the partial sums of the Legendre symbol modulo  $p$ . In particular, we will attempt to answer the following questions as we vary over the primes  $p$ : how are these paths distributed? how do their maximums behave? and what proportion of the path is above the real axis? Among our results, we prove that these paths converge in law, in the space of continuous functions, to a certain random Fourier series constructed using Rademacher random multiplicative functions. Part of this work is joint with Ayesha Hussain.

Dr. Youness Lamzouri is a Professor of Mathematics at Institut Élie Cartan de Lorraine (IECL) of the Université de Lorraine in Nancy, France. Previously he was an associate professor at York University. He held postdocs at UIUC (Urbana) and IAS (Princeton). He received his Ph.D. at Université de Montréal under the supervision of Andrew Granville. He was awarded the 2011 CMS Blair Spearman Doctoral Prize for this work. Dr. Lamzouri is an expert in analytic and probabilistic number theory with a focus on prime numbers, L-functions, character sums and exponential sums, additive and multiplicative functions, and class numbers of quadratic fields.

**MONDAY, JANUARY 9<sup>TH</sup>, 2023**  
**12:00 PM – 12:50 PM**  
**M1040**  
**SNACKS & COFFEE PROVIDED**