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On the behaviour of Liouville function on Polynomials

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Let $\lambda(n)$ denote the Liouville function. Complementary to the prime number theorem, Chowla conjectured that

$$\sum_{n \leq x} \lambda(f(n)) = o(x)$$

for any polynomial $f(x)$ with integer coefficients, not in the form of $bg(x)^2$.

Chowla's conjecture is proved for linear functions but for the degree greater than 1, the conjecture seems to be extremely hard and still remains wide open. One can consider a weaker form of Chowla's conjecture, namely, if $f(x) \in \mathbb{Z}[x]$ and is not in the form of $bg^2(x)$ for some $g(x) \in \mathbb{Z}[x]$, then $\lambda(f(n))$ changes signs infinitely often.

Although it is weaker, Conjecture 1 is still wide open for polynomials of degree > 1. In this talk, I will describe some recent progress made while studying Conjecture 1 for the quadratic polynomials. One can also extend the definition of Liouville function on rationals and we give a analogous result for quadratic polynomials with rational coefficients. This is joint work with Peter Borwein, Stephen Choi and Jonas Jankauskas