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Fractional calculus and connection to diffusion anomalies

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Most operators are commonly reckoned to exist for integer orders only. In fact, a continuum of operators of any real and even complex orders can be defined. This talk will introduce fractional derivatives and their connection to modelling such physical phenomena as sub-diffusion ($\langle r^2 \rangle \sim t^\gamma,\ 0 < \mathfrak{g} < 1$) and super-diffusion (diverging within finite time mean square displacement).

The simplest way to introduce sub-diffusion is with the aid of a time-fractional operator $\partial_t^\gamma - \nabla^2$, with the fractional derivative ∂_t^γ being a non-local and asymmetric operator of order $0 < \gamma < 1$.

Super-diffusion is usually introduced with a space-fractional operator $\partial_t + (-\Delta)^\gamma$. Here $-(-\Delta)^\mathfrak{g}$ is the fractional Laplacian, a non-local and symmetric operator of order $1 < \gamma < 2$.

The conceptual changes inflicted by these operators to basic theorems of calculus and complex analysis are quite stunning, yet rarely introduced at the level of graduate studies. In this talk the collapse of some fundamental analysis tools like the chain rule, residue theorem, linearisation about a constant ground state other than zero and existence of a spectrum, will be shown. Limitations of the alternative approaches will be discussed.