Submittee: Michael Ward
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Title: Workshop on Pattern Formation
Event Type: Conference-Workshop

Location:
Dalhouse University, Halifax

Dates:
Workshop was held July 18-19, 2015. This was followed by a week long intensive research week July 20-24th.

Topic:
The focus of the workshop was to present latest results and open problems related to partial differential equation models (PDE) whose solutions exhibit complex spatio-temporal dynamics. A fundamental technique to understand these dynamics is to identify essential features which act as the basic building blocks of the solution to the PDE. Using these basic components, a complex PDE system can often be effectively described by a finite dimensional dynamical system of interacting particles. This reduced system of coupled ordinary differential equations (ODEs) is usually more amenable for the study of the existence, stability and dynamics of the different configurations supported by the system. This procedure is continually used with great success in many physical systems. The workshop focussed on bringing together researchers who use these finite dimensional reductions in a wide range of areas of application, including the study of vortex dynamics in Bose Einstein condensates, in models of swarming behavior, in the study of first passage time problems in the presence of localized traps.

Methodology:
Lectures given over a two day period. This was followed by a week long "research week" where many of the participants remained at Dalhousie to seek new collaborations on problems of a common interest. This research week was rather successful in generating new directions and progress on several specific research questions. The workshop featured talks by four advanced graduate students (who have published high quality papers), and three postdocs. This mix of senior and junior researchers was a key feature of this workshop so as to further expose young researchers to important problems related to PDE modeling of pattern formation. Further, many graduate students participating in the month long summer school attended the weekend workshop.

Objectives Achieved:
One of the key objectives was to bring together a diverse group of researchers who focus on the analysis of collective dynamics; limiting dynamical systems or optimization problems of "point-particles" that arise from systematics reductions of complicated PDE models. Examples of such problems include the dynamics and stability of nonlinear vortices in Bose Einstein fiield theories, the dynamics of moving target traps for mean first passage time problems, the dynamics and equilibria of localized particle like solutions to reaction-diffusion systems, the effect of localized defects on vibrational modes of thin plates. There is a similar mathematical structure with many of

the these diverse problems, and this workshop provided a forum where common elements of the mathematical analysis could be discussed. In this way, cross-fertilization of mathematical ideas to different areas of application where these problems arise was initiated.

Scientific Highlights:
Within the field of Applied mathematics, unlike pure mathematics, the field does not progress from the resolution of one conjecture after the next. Instead, it is the development of new mathematical techniques and approaches, and their application and cross-fertilization to different areas of application, that is often the landmark.

There were new collaborations initiated during this workshop and the subsequent research week that holds considerable promise for high quality innovative publications: A few of these include:

i) collaboration between Tzou, Kevrekides, Carretero and Kolokolniov on the study of the nucleation of vortices in Bose Einstein in heterogeous environments that trigger the birth of vortices. The aim here is to theoretically explain recent theoretical findings.

ii) collaboration between Lindsay, Tzou and Ward, on accurately calculating the mean first passage time for brownian motion in spatial environments that include intermittent Brownian motion together with directed transport to the cell nucleus as a result of molecular motors. The analysis of transport across the nuclear boundary via a homogenization approach, when there are nearly 4000 nanopores on the nuclear membrane, is also being considered. We have made good progress in understanding the homogenized limit, and this has been an open problem for many years as suggested originally by Stas Shvartsman (Princeton), David Holcman (U. Pierre et Marie Curie).

Organizers:
Kolokolnikov, Theodore, Dept. of Mathematics, Dalhousie U.
Carretero, Ricardo, Dept. of Mathematics, San Diego State U.,
Ward, Michael, Dept. of Mathematics, UBC

Speakers:
Speaker: Panayotis Kevrekidis, Dept. of Mathematics, U. Mass, Amherst
Title: Existence, Stability and Dynamics of Solitary Waves and Vortices in Bose-Einstein Condensates: From Theory to Experiments

Abstract: In this talk, we will present an overview of some of our recent theoretical, numerical and experimental efforts concerning the static, stability, bifurcation and dynamic properties of coherent structures that can emerge in one- and higher-dimensional settings within Bose-Einstein condensates. We will discuss how this ultracold setting can be approximated at a mean-field level by a deterministic PDE of the nonlinear Schrodinger type and what the fundamental nonlinear waves of the latter are, such as dark solitons and vortices. Then, we will try to go to a further layer of simplified description via nonlinear ODEs encompassing the dynamics of the waves within the traps that confine them, and the interactions between them. Finally, we will attempt to compare the analytical and numerical implementation of these reduced descriptions to recent experimental results and speculate towards a number of interesting possibilities for the future.

Speaker: Ricardo Carreterro, Dept. of Mathematics, San Diego State U.
Title: Vortex pair configurations in Bose-Einstein condensates:
theory and experiments.

Abstract: We study the dynamics and stability of vortex configurations bearing a small number of vortices in harmonically trapped Bose-Einstein condensates. The dynamics is reduced to a system of quasi-particle ODEs describing their positions. Periodic and quasi-periodic solutions, and their stability, are studied and compared favorably with experimental observations. A symmetry-breaking bifurcation for regular polygonal states is identified and matched to experimental observations.

Speaker: Roy Goodman, Dept. of Mathematics, NJIT
Title: Hamiltonian Phenomena in NLS-Like Systems: Some Results, Some Plans, Some Ideas

Abstract: The nonlinear-Schrodinger equation admits a Hamiltonian formulation, as do many finite-dimensional reduced systems based upon it. We show some examples where methods of Hamiltonian reduction allow us to discover and explain features in the dynamics. In a system of coupled optical waveguides, we show how relative periodic orbits and chaotic dynamics are affected by Hamiltonian Hopf bifurcations. We then discuss how techniques of Hamiltonian reduction and symmetry can be used to study vortex interactions in two-dimensional Bose-Einstein condensates. Time permitting, we will discuss some other ideas of additional phenomena we plan to explore and the methods we intend to use.

Speaker: Alanna Hoyer-Leitzel, Dept. of Mathematics, Bowdoin College
Title: Using algebraic geometry to find bifurcations and symmetry in the n-vortex problem.

Abstract: Relative equilibrium solutions of the n-vortex problem are periodic solutions where the vortices maintain a fixed configuration as they rotate around the center of vorticity (analogous to center of mass in a gravitation problem). Under the assumption of one large and n small vortices, and specifically in the case when there are 3 small vortices, computational algebraic geometry can be used to examine the set of relative equilibrium solutions that would be hard to describe otherwise. After writing the equations for equilibrium solutions as polynomial equations, we can find bifurcations in the number of relative equilibria as the relative strengths of the small vortices change, and prove that symmetry in the configurations occurs only when the relative strengths of two of the small vortices are the same.

Speaker: Yana Nec: Dept. of Mathematics, Dalhousie University
Title: Pattern formation with fractional derivatives.

Abstract: The talk will introduce fractional derivatives and explain how the regular calculus and normal diffusion are a special limit in a universe of anomalies. The peculiarities of different types of fractional derivatives and their suitability for particular analytical purposes will be discussed, alongside the classic notions of calculus that fall into abeyance.

Application of fractional derivatives to the generation of spike patterns with sub-diffusion and L\'{e}vy flights will delineated, with emphasis on the benefit to the parameter regime of pattern existence. Both types of anomaly will be compared to the regular diffusion as far as the shape, evolution and stability of the spike are concerned. The eigenvalue loci and their correspondence to enhanced or diminished stability of the system will be used to create a bigger picture regarding the pattern's behaviour.

No prior knowledge on fractional operators is required. Familiarity with PDEs and dynamical systems is of import.

Speaker: David Iron, Dept. of Mathematics, Dalhousie U.
Title: A model of receptor protein aggregation

Abstract: We consider a model of the aggregation of cell receptor molecule aggregation in the cell membrane. We consider two cases. First we allow receptor molecules to diffuse freely on the cell membrane. Secondly we will considerdensity dependent diffusion. In this case large clusters of receptors will diffuse much more slowly than sparsely distributed ones.

Speaker: Alan Lindsay, Dept. of Mathematics, Notre Dame University
Title: Vibrational Patterns of thin plates with clamped patches.

Abstract:
In this talk I will discuss the problem for the modes of vibration of a thin elastic plate with a collection of N small clamped patches. This talk will center on several fourth order eigenvalues problem and analysis of these in the limit of small patch size. These N patches represent defects in the plate and the main goal is to understand the effect of the number and location of these holes on the vibrational modes of the plate. The deviation of the eigenvalues from the patch free case are quantified and certain configurations which maximize this deviation for certain N are identified.

Speaker: Justin Tzou, Dept. of Mathematics, Dalhousie U.
Title: First passage times with mobile traps in one and two dimensions

Abstract: Various problems in nature may be formulated in terms of mean first passage times (MFPT) of Brownian particles in the presence of traps. A typical example in cellular biology involves transport of molecules between the nucleus and cytoplasm of a cell. While most existing works focus on stationary traps, many scenarios may involve traps or targets that are non-stationary (e.g., predator-prey dynamics, search and rescue, diffusion-limited reactions). We discuss here the formulation of one and two-dimensional MFPT problems in the presence of mobile traps. In simple geometries, we exploit symmetries that allow for analysis by means of asymptotic methods. We also highlight some interesting and counter-intuitive results that arise due to the motion of the traps.
Joint works with T. Kolokolnikov, A. Lindsay and S. Xie.

Speaker: Daniel Gomez, Dept. of Mathematics, UBC
Title: Narrow Escape Problems in Non-Spherical Three-Dimensional Domains

Abstract: The narrow escape problem (NEP)
involves the calculation of the expected time, known as the mean first

passage time (MFPT), for a randomly moving particle to escape a domain that is everywhere reflecting except at finitely many small holes through which the particle may escape. The typical approach to these problems involves developing asymptotic expansions for the MFPT and its spatial average that increases in accuracy as the holes become smaller. This talk addresses the NEP in three-dimensions for domains bounded by a level surface of an orthogonal coordinate system. Such domains include for example prolate and oblate spheroids, spheres, and biconcave disks. By using the method of matched asymptotic expansions and the singular behaviour of the surface Neumann Green's function in three-dimensional domains we obtain a two-term asymptotic expression for the average MFPT. The result depends on an assumption that remains an open problem, but is nevertheless supported by comparisons to numerical results.

Speaker: Mary Silber, Department of Applied Mathematics, Northwestern U,
Title: Pattern Formation in the Drylands: Self Organization in Semi-Arid Ecosystems

Abstract: Much of our understanding of spontaneous pattern formation in spatially extended systems was developed in the ?wetlands'' of fluid mechanics. That setting allowed well-controlled table-top laboratory experiments; it came with fundamental equations governing the system; it benefitted from a back-and-forth between theory and experiment. These investigations identified robust mechanisms for spontaneous pattern formation, and inspired the development of equivariant bifurcation theory. Recently, these pattern formation perspectives have been applied to modeling the vegetation in dryland ecosystems, where satellite images have revealed strikingly regular spatial patterns on large scales. Ecologists have proposed that characteristics of vegetation pattern formation in these water-limited ecosystems may serve as an early warning sign of impending desertification. We use the framework of equivariant bifurcation theory to investigate the mathematical robustness of this approach to probing an ecosystems robustness. Additionally, we identify new applied pattern formation research directions in this far-from-pristine setting, where there are no fundamental equations and no controlled laboratory experiments.

Speaker: Yuxin Chen, Dept. of Applied Mathematics, Northwestern U.
Title: Effects of the rate of precipitation changes on vegetation patterns.

Abstract: When faced with decrease in precipitation, vegetation in arid and semi-arid environments are prone to sudden irreversible changes, such as desertification, as the precipitation drops below a tipping point of the system. A possible coping mechanism is the formation of spatial patterns, which allows for concentration of sparse resources and the survival of the species within 'ecological niches' even below the tipping point of the homogeneous vegetation state. However, if the change in precipitation occurs too sudden, the system may not have time to transition to the patterned state and will pass through the tipping point, leading to extinction. We propose that the deciding factors are the rate of precipitation change and the amount of seasonal rainfall variability. We illustrate this phenomenon on a modified simple vegetation model proposed by Klausmeier.

Speaker: Andrew J. Bernoff, Dept. of Mathematics, Harvey Mudd College
Title: Energy Driven Pattern Formation in Nearly Planar Fluid Systems.

Abstract: Nearly planar fluid systems include Langmuir layers, which are molecularly-thin polymer layers on a substrate (generally a quiescent subfluid) and Hele-Shaw systems where a thin fluid layer is confined between two glass plates. These systems are driven by intermolecular forces and damped by viscous dissipation. Their dynamics can often be described as dissipative gradient flows where the solution is driven toward energy minimizers. We describe two such systems. In the first, line tension (the two-dimensional analogue of surface tension) drives the fluid domains to become circular and the rate of relaxation to these circular domains can be used to deduce the magnitude of the line tension forces. The second system models the formation of convoluted fingered domains observed experimentally in ferrofluids for which pattern formation is driven by line tension and dipole-dipole repulsion. We asymptotically obtain an energy minimization problem depending only on a generalized line tension, Λ. Numerical studies yield a few highly symmetric stable shapes, but nothing that resembles the experimentally observed diversity. Adding a weak random background stabilizes a menagerie of domain morphologies recovering the diversity observed experimentally.

Speaker: Alexandria Volkening, Division of Applied Math., Brown U.
Title: Modeling the formation of stripes in zebrafish

Abstract: Zebrafish is a small fish with distinctive black and yellow stripes that form due to the interaction of different pigment cells. Working closely with the biological data, we present an agent-based model for these stripes that accounts for the behavior of all three types of pigment cells. The development of both wild-type and mutated patterns will be discussed, as well as the effects of fish domain growth on the scale of long-range interactions. Joint work with Bjorn Sandstede.

Speaker: Shuangquan Xie, Dept. of Mathematics, Dalhousie U.
Title: Hopf bifurcation for two dimensional Schnakenburg Model.

Abstract: We consider the stability of a single spot solution to the Schnakenburg Model in a two-dimensional disk domain. For large values of the reaction-time constant t, this spot can undergo two different types of Hopf bifurcations. The first Hopf bifurcation induces height oscillation. The second Hopf bifurcation induces a circular motion of the spot position. We use formal asymptotics to delieate the two different regimes. For the rotating spot, we derive a reduced PDE-ODE system to characterize the dynamics and compute the radius and the speed of its rotation by solving the reduced system.

Speaker: Xiaofeng Ren, Dept. of Mathematics, George Washington U.
Title: The impact of the domain boundary on an inhibitory system: boundary half discs in stationary assemblies.

Abstract: The nonlocal geometric variational problem derived from the Ohta-Kawasaki diblock

copolymer theory is an inhibitory system with self-organizing properties. The system can prevent a disc from drifting towards the domain boundary. This raises the question whether a stationary set may have its interface touch the domain boundary. It is proved that a small, perturbed half disc exists as a stable stationary set, where the circular part of its boundary is inside the domain, as the interface, and the almost flat part of its boundary coincides with part of the domain boundary. The location of the half disc depends on two quantities: the curvature of the domain boundary, and a remnant of the Green's function after one removes the fundamental solution and a reflection of the fundamental solution. The notion of reflection here is an interesting new concept that generalizes the familiar notions of mirror image and circle inversion. Our analysis of a boundary half disc leads to constructions of stationary assemblies with both interior discs and boundary half discs.

Speaker: Matthias Winter, Dept. of Mathematics, Brunel U.
Title: Existence and Stability of Spike Clusters for Biological Reaction-Diffusion Systems

Abstract: We study the existence and stability of spike clusters for biological reaction-diffusion systems with two small diffusion constants. In particular we consider the Gierer-Meinhardt system with a precursor gradient. In a spike cluster the spikes converge to the same limiting point. We will present results on the asymptotic behaviour of the spikes including their shapes, positions, and amplitudes. We will also compute the asymptotic behaviour of the eigenvalues. Such systems and their solutions play an important role in biological modeling to account for the bridging of lengthscales, e.g. between genetic, nuclear, intra-cellular, cellular and tissue levels, or for the hierarchy of biological processes, e.g. first a large-scale structure appears and then it induces patterns on a smaller scale. This joint work with Juncheng Wei.

Speaker: Juncheng Wei, Dept, of Mathematics, UBC
Title: On Type II blow up solutions for harmonic map flow

Abstract: We consider the following harmonic map flow equation:
$$ u_t=\Delta u + |\nabla u|^2 u$$
where $ u: \Omega \to S^2, |u|=1$ and Ω is a general two-dimensional domain. We construct in general (non-symmetric) domain and general (non-symmetric) initial condition a Type II blow-up with blowing rate at $ (T-t)/ (\mbox{log}^2 (T-t))$. Furthermore we show that this blow-up is generic, universal and stable. We will also discuss other types of solutions: infinite-time blow-ups, bubble-trees and reverse bubbles. (joint work with M. del Pino and J. Davila)

Links:
http://mathstat.dal.ca/~tkolokol/summer/workshop-patterns.pdf

http://mathstat.dal.ca/~tkolokol/summer/#patterns

File Uploads:
Additional Upload 1: http://www.pims.math.ca/files/final_report/schedule_1.pdf