

Contributed Talks

Pure Mathematics

- SERBAN BELINSCHI (University of Saskatchewan)

Evolution towards free infinite divisibility

The field of noncommutative probability can be viewed as a probabilistic approach to the study of operator algebras. In particular, free probability is a powerful tool for exploring properties of finite algebras arising from representations of free products of groups. The probabilistic aspect of this theory involves naturally limit theorems (like the well-known Central Limit Theorem). The “most general” type of limits, namely limits of triangular arrays, are described according to Hincin’s Theorem, and its free analogue, as the set of infinitely divisible probabilities with respect to the corresponding convolution (classical or free). We will start this talk with a very brief overview of the free probability theory and the convolution it generates. We will define infinite divisibility and explain its importance. Next, we discuss a semigroup of operators B_t on the space of probability measures on the real line that evolve towards infinite divisibility. It is remarkable that for this semigroup infinite divisibility is always reached by time $t = 1$, in the sense that $B_1(\mu)$ is always infinitely divisible. We will conclude with a surprising characterization of B_t in terms of the free Brownian motion. This is joint work with Alexandru Nica (University of Waterloo).

- JAYDEEP CHIPALKATTI (University of Manitoba)

On equations defining general binary orbits

We will identify a complex binary form $\sum_{i=0}^d a_i x_1^{d-i} x_2^i$ with the point $[a_0, \dots, a_d]$ in the projective space \mathbf{P}^d . The latter admits an action of the special linear group $SL(2, \mathbf{C})$ via a change of variables. Now let $\Omega_A \subseteq \mathbf{P}^d$ denote the Zariski closure of the orbit of a *general* binary form A . One should like to find the equivariant minimal generators of the defining ideal of the variety Ω_A . I will present a computational answer to this question for $d \leq 10$. The calculation reveals a curious phenomenon, namely the possible existence of what may be called ‘invisible’ generators in the ideal. This imposes a dichotomy on the set of integers d , dividing them into ‘prosaic’ and ‘erratic’. Hitherto, only the cases $d = 7, 10$ are known to be erratic, but it is anyone’s guess how many remain to be discovered.

- ANDREW DOUGLAS (New York City College of Technology)

Lie-Yamaguti structures on the $\mathfrak{sl}(3, \mathbb{C})$ -modules $V(n, n)$

The Lie algebra $L = \mathfrak{sl}(3, \mathbb{C})$ of dimension 8 has an irreducible representation $V = V(2, 2)$ of dimension 27, and L and V each occur with multiplicity 1 in the exterior square $\Lambda^2 V$. The projections $\alpha: \Lambda^2 V \rightarrow L$ and $\beta: \Lambda^2 V \rightarrow V$ define a binary-ternary algebra structure on V by $[x, y, z] = \alpha(x \wedge y) \cdot z$ and $[x, y] = \beta(x \wedge y)$. We will describe how computer algebra was implemented to determine the polynomial identities satisfied by this structure in degrees ≤ 6 , and see that it precisely satisfies the defining identities of Lie-Yamaguti algebras. We will also discuss the extension of the computations to $V(3, 3)$ and $V(4, 4)$ whose structures are shown to be generalizations of Lie-Yamaguti algebras. This is joint work with Murray Bremner (University of Saskatchewan).

- HADER ELGENDY (University of Saskatchewan)

Algebra structures on the fourth exterior power of a simple $\mathrm{sl}_2(\mathbb{C})$ -module

We discuss how the Polya's enumeration theorem can be used to find a formula for the multiplicity of the irreducible representation $V(n)$ of $\mathrm{sl}_2(\mathbb{C})$ as a direct summand of its own m -th exterior power $\Lambda^m V(n)$, where m is any positive integer. We study the situation in the case $m = 4$ (the fourth exterior power). We find that the multiplicity is 1 if and only $n \in \{4, 6\}$. For these values of n , the module $V(n)$ admits a unique $\mathrm{sl}_2(\mathbb{C})$ -invariant quaternary algebra structure obtained from the projection $\Lambda^4 V(n) \rightarrow V(n)$. It also follows that the multiplicity is 2 if and only $n \in \{8, 10\}$. For these values of n , the module $V(n)$ admits a one-parameter family of quaternary algebra structures. In these four cases, we determine the polynomial identities of degree ≤ 7 satisfied by these quaternary algebra structures. We use computer algebra to represent the identities as the nullspace of a large matrix, find the row canonical form, and extract a nullspace basis. This is joint work with Murray Bremner (University of Saskatchewan).

- CLINT ENNS (University of Manitoba)

Product and co-product for topological modules

A significant property of the direct sum of modules is that it can be endowed with a topology that makes it a co-product in the category of topological modules. In this talk, a description of this co-product topology and some of its properties will be given. It will also be shown that this is indeed a co-product in the category of topological modules.

- DOUG FARENICK (University of Regina)

On the homotopy groups of real, complex, and quaternionic pseudospheres

Consider the right vector space \mathbb{K}^n , where \mathbb{K} denotes one of \mathbb{R} , \mathbb{C} , or \mathbb{H} (quaternions), and let $\lambda \mapsto \lambda^*$ denote the canonical involution on \mathbb{K} . If p, q are nonnegative integers with $p + q = n$, then the function $[\cdot, \cdot] : \mathbb{K}^n \times \mathbb{K}^n \rightarrow \mathbb{K}$ defined by

$$[\xi, \eta] = \xi_1^* \eta_1 + \cdots + \xi_p^* \eta_p - \xi_{p+1}^* \eta_{p+1} - \cdots - \xi_n^* \eta_n$$

is a nondegenerate hermitian form on \mathbb{K}^n of signature (p, q) . The (p, q) -pseudosphere over \mathbb{K} is the set

$$S_{p,q}(\mathbb{K}) = \{\xi \in \mathbb{K}^n \mid [\xi, \xi] = 1\},$$

a closed C^∞ -manifold in the standard topology of \mathbb{K}^n . Such manifolds arise in a variety of settings: $S_{p,0}(\mathbb{K})$ are Euclidean spheres, $S_{3,1}(\mathbb{R})$ is the standard Lorentz manifold, and $S_{n-1,1}(\mathbb{R})$ and $S_{2,n-2}(\mathbb{R})$ are, respectively, the standard de Sitter and anti de Sitter manifolds. In this lecture, I will determine precisely the homotopy groups π_1 and π_2 of the pseudospheres $S_{p,q}(\mathbb{K})$. This is joint work with Chris Ramsey (University of Waterloo).

- ALLEN HERMAN (University of Regina)

Examples of representations of Hecke algebras over \mathbb{Q} that produce division algebras

A Hecke algebra over \mathbb{Q} is a subalgebra of the rational group algebra $\mathbb{Q}G$ of the form $e\mathbb{Q}Ge$, where G is a finite group and e is an idempotent of $\mathbb{Q}H$ for some (usually nonnormal) subgroup H of G . By an analog of Maschke's theorem, these Hecke algebras are semisimple \mathbb{Q} -algebras, and so we know that their simple components (or blocks) will be isomorphic to matrix rings over division algebras. We have recently shown that the possible division algebras that can occur as the division algebra part of a simple component of one of these Hecke algebras over \mathbb{Q} are precisely those that occur among simple components of rational group algebras of finite groups. But very few nontrivial examples are available. I will present

13 small examples of division algebras occurring as simple components of Hecke algebras over \mathbb{Q} . In all of these, $[G : H] = 16$, and the Hecke algebra is not a rational group algebra.

- JIAXIONG HU (University of Saskatchewan)

Lie invariants in the natural representations of $sl(2)$ and $sl(3)$

Wever (1949) observed that the degree d of a Lie invariant must be a multiple of q which is the number of generators of the free Lie algebra. He also gave a formula for the number of invariants for given d and q in the natural representation of $sl(n)$. The free Lie algebra is spanned by a Hall set which is a subset of the free magma. We study the invariant Lie polynomials in the free Lie algebras L generated by the natural representations $sl(2)$ and $sl(3)$; so L is the free Lie algebra on 2 or 3 generators. We find new explicit Lie invariants in higher degrees than were previously computed. The number of Lie invariants in each degree agrees with Wever's observation. This is joint work with Murray Bremner (University of Saskatchewan).

- DELARAM KAHROBAEI (New York City Tech and CUNY Graduate Center)

Applications of non-abelian group theory in cryptography

As computers become faster and faster, and with the advent of quantum computers on the horizon, the question of protecting information transmitted over the World Wide Web becomes ever more important. Both academic and industrial establishments are concerned with developing unbreakable cryptosystems. In recent years it has been proposed to use non-commutative groups as a platform on which to build cryptosystems. This work was initiated in 1984 by Wagner et al who proposed an approach to design public-key cryptosystems based on the undecidable word problem for groups and semigroups. In 1999, Anshel-Anshel-Goldfeld proposed a compact algebraic key establishment protocol. The foundation of their method lies in the difficulty of solving equations over algebraic structures, in particular non-commutative groups. In joint work with Eick, I proposed a new cryptosystem based on polycyclic groups. In joint work with Khan, I proposed non-commutative key-exchange schemes which generalize the classical ElGamal Cipher to polycyclic groups. Recently in a joint work with Anshel, I proposed a problem concerning decision versus search arising in Group-Based Cryptography. I will give an overview on the recent development of this field and discuss interesting open problems which arise naturally from this path of research.

- DAVID KLEIN (Brandon University)

Fundamental groupoids and moduli spaces of flat connections

Starting with the fundamental groupoid of a Riemann surface, we show how the moduli space of gauge equivalence classes of flat connections on the surface may be identified with a much simpler space.

- SALMA KUHLMANN (University of Saskatchewan)

An uncountable family of logarithmic functions with distinct growth rates

We consider a totally ordered set Γ of cardinality \aleph_1 , of which elements are germs at $+\infty$ of real valued functions of a real variable. We show that the order type of Γ is that of a lexicographic ordering which admits 2^{\aleph_1} automorphisms of pairwise distinct orbital growth. We associate to each such automorphism a well defined logarithmic function on the field $\mathbb{R}((G))_{\aleph_1}$, where $\mathbb{R}((G))_{\aleph_1}$ is the field of generalized series with countable support, real coefficients and exponents in the group G of transmonomials at $+\infty$ defined by Γ . We show that distinct automorphisms induce logarithmic functions of distinct growth rates. This is joint work with J.-P. Rolin (Université de Bourgogne).

- CHENKUAN LI (Brandon University)

The distributional products on spheres and Pizetti's formula

The distribution $\delta^{(k)}(r - a)$ concentrated on the sphere O_a of $r - a = 0$ is defined as

$$(\delta^{(k)}(r - a), \phi) = \frac{(-1)^k}{a^{n-1}} \int_{O_a} \frac{\partial^k}{\partial r^k} (\phi r^{n-1}) d\sigma.$$

Applying the Fourier transform of distribution and the integral representation of the Bessel function, we obtain an infinite expansion of $\delta^{(k)}(r - a)$ in terms of $\Delta^j \delta(x_1, \dots, x_n)$ to show that the well-known Pizetti's formula is a convergent series in the Schwartz space. Furthermore, we derive the generalized product $\phi(x_1, \dots, x_n) \delta^{(k)}(r - a)$, where ϕ is an infinitely differential function, based on the interesting formula of $\Delta^m(\phi\psi)$, and hence we are able to characterize the distributions focused on spheres, which can be written as the sum of multiplet layers in the Gel'fand sense.

- MURRAY MARSHALL (University of Saskatchewan)

Positive polynomials and sums of squares

The talk will be introductory in nature. I will talk about the beginnings of the subject as I understand them: finding certificates for positivity in terms of sums of squares, work of David Hilbert, Emil Artin and others, and how this led eventually to the formulation and proof of the famous Positivstellensatz. I will also talk about the more recent history, how Konrad Schmuedgen changed the subject in 1991, merging ideas from algebra and model theory with ideas from functional analysis, the application of semidefinite programming to polynomial optimization introduced by Jean Bertrand Lasserre and others, and where the subject has been going in the past 18 years.

- TAGREED MOHAMMED (University of Manitoba)

Equivariant projection formulae for Specht modules

Let d be a positive integer, and let λ denote a partition of d . The Specht module V_λ is a finite-dimensional irreducible representation of the symmetric group \mathfrak{S}_d . It has a basis \mathcal{B}_λ indexed by the standard tableaux of shape λ filled with the entries $\{1, 2, \dots, d\}$. Given two partitions λ and μ , the tensor product $V_\lambda \otimes V_\mu$ splits into a direct sum $\bigoplus_\nu (V_\nu)^{m_\nu}$. This defines \mathfrak{S}_d -equivariant projection morphisms $V_\lambda \otimes V_\mu \longrightarrow V_\nu$, but in general it seems difficult to describe them explicitly in terms of the tableaux bases. In this talk, I will give such descriptions in a few cases, e.g., when

- (1) $\lambda = \mu = \nu = (d-1, 1)$,
- (2) $\lambda = \mu = (d-2, 1, 1)$ and $\nu = (d)$.

I will present a conjecture for the case $\lambda = \mu = (d-r, 1, \dots, 1)$ and $\nu = (d)$.

- EBRAHIM SAMEI (University of Saskatchewan)

Operator amenability and Arens regularity of Beurling-Fourier algebras of $SU(2)$

We first give a brief introduction on the cohomological properties of Fourier algebras of compact groups. We then introduce the concept of Beurling-Fourier algebras which is the non-commutative generalization of classical Beurling algebras on compact abelian groups. We give some general results with regard to the operator amenability and Arens regularity of these algebras. We then apply them to study explicitly the Beurling-Fourier algebras over 2×2 unitary group $SU(2)$. This, in particular, allows us to construct families of closed subalgebras of the Fourier algebra of certain products of $SU(2)$ which are Arens regular, a

result which does not happen in the commutative case. This is joint work with Hun Hee Lee (Chungbuk National University).

- ERIC SCHIPPERS (University of Manitoba)

Interactions between conformal field theory and complex function theory

One of the models of two-dimensional conformal field theory involves a moduli space of Riemann surfaces endowed with analytic maps near specified points. The “sewing operation” in this moduli space is a kind of product which joins two surfaces according to the data specified by the analytic maps. In this talk I discuss how complex function theory sheds light on this moduli space and the sewing operation. In particular it is possible to show that the sewing operation is complex analytic, which was one of countless outstanding issues in the formulation of conformal field theory.

- MARINA TVALAVADZE (University of Saskatchewan)

Universal non-associative enveloping algebras of low-dimensional Malcev algebras

The purpose of this talk is to provide a brief review on the progress we have made on universal enveloping algebras of low-dimensional Malcev algebras. In our work, we were particularly interested in the description of structural constants of the universal enveloping algebra of the non-solvable 5-dimensional Malcev algebra. Malcev algebras are the natural generalizations of Lie algebras. They satisfy *anticommutativity*, and the *Malcev identity*,

$$[a, a] = 0, \quad [[a, c], [b, d]] = [[[a, b], c], d] + [[[b, c], d], a] + [[[c, d], a], b] + [[[d, a], b], c],$$

for all a, b, c, d . Recently, Shestakov and Pérez-Izquierdo extended the PBW-theorem to Malcev algebras and provided the general construction of a universal non-associative enveloping algebra for any Malcev algebra. We use this construction together with known results about derivations and differential operators to determine explicit structure constants of the enveloping algebra. This is joint work with Murray Bremner (University of Saskatchewan).

Applied Mathematics and Mathematical Physics

- MOTASSEM AL-ARYDAH (University of Ottawa)

Proving the existence of a positive bounded weak solution for a system of nonlinear PDEs in a domain with a triple phase boundary

We consider a nonlinear PDEs system describing the reaction-diffusion dynamics near a triple-phase boundary in the catalyst layer of hydrogen fuel cells. The system describes bulk and surface reaction-diffusion processes and is an approximation of a model with a thin layer. The coupling of surface and bulk diffusion involves a singular boundary condition. Using certain a priori estimates, variational methods and fixed point theorem, we prove the existence of a positive weak solution; moreover we prove that the solution is small if the data are small. Finally, we support our analysis results by some numerical results.

- VAHID ANVARI (York University)

A mathematical model to investigate the treatment policy for fluroquinolone drug resistance of gonorrhea

New research shows that Quinolone-Resistant Neisseria Gonorrhea (QRNG) is now widespread among heterosexuals and men who have sex with men. It has gone beyond the 5 per cent threshold in heterosexuals in the United States so the Center for Disease Control and Prevention (CDC) announced it no longer recommends quinolones as a treatment for

gonorrhea. I will talk about a Deterministic Compartmental SIR Model (Susceptible, Infectious and Removed Compartments) which calculates the Control Reproduction Number for studying QRNG problem and the threshold in Ontario. In fact, continued use of quinolones may result in QRNG becoming epidemic in the Ontario population. The model implicates this fact and also facilitates to answer: 1) What proportion of prescribing physicians must follow guidelines for the incidence of QRNG strains to decrease? 2) What is the impact of transition to cephalosporins as treatment by prescribing physicians?

- SAMUEL BUTLER (University of Saskatchewan)

Modelling fluid flow in compacting porous media: application to melt transport in earth's mantle

Earth's oceanic crust is produced from melt that makes its way to the surface at volcanoes on mid-ocean ridges. This melt is produced in the top 60 km of the mantle over a lateral distance of roughly 100 km. The transport of this melt through a solid porous matrix can be modelled as the flow of two interpenetrating fluids with very different viscosities. If the viscosity of the solid matrix decreases with porosity, an instability occurs when an external stress is imposed that localizes the porosity. These regions of high and low porosity are oriented in bands at well defined orientations relative to the stress axes and it has been argued that these may serve as conduits that channel mantle melt towards the axes of mid-ocean ridges. In this presentation, I will outline the theory of flow in compacting porous layers and I will present a linearized theory that can be used to predict the orientation of the porosity bands and their growth rate. I will also present the results of numerical simulations with different dependencies of the viscosity of the solid matrix on porosity and strain-rate. It will be demonstrated that the orientation of the melt bands may not be optimal for channelling melt towards ridge axes.

- ALEXEI CHEVIAKOV (University of Saskatchewan)

Computation and applications of local conservation laws

Many PDE systems that arise in applications have local conservation laws, such as conservation of mass, energy, momentum, charge, etc. Knowledge of conservation laws admitted by a given model is beneficial in many ways, in particular, for numerical analysis. I will discuss available methods of systematic construction of conservation laws of PDEs (and similarly, first integrals of ODEs), with and without Noether's theorem, and present some examples.

- RAINER DICK (University of Saskatchewan)

Dimensionally hybrid Green's functions, density of states, and impurity scattering in low-dimensional systems

Two-dimensional Hamiltonians are often employed for the modeling of particle properties in surfaces or interfaces, but the transition between two-dimensional and three-dimensional distance laws for correlations or scattering amplitudes of particles near surfaces or interfaces is not well understood. In the talk, I will point out that the concept of linear superposition of two-dimensional and three-dimensional Hamiltonians for particle propagation in the presence of interfaces yields systems which interpolate between two-dimensional behavior at short distances and three-dimensional behavior at large distances. The intermediate cross over length scale between two-dimensional and three-dimensional behavior of correlation functions, densities of states, and electrostatic potentials is set by ratios between bulk and surface effective masses, or by the ratio of interface and bulk permittivities, respectively.

Green's functions and related observables in these systems can often be calculated analytically. Dimensionally hybrid Hamiltonians therefore provide interesting mathematical models for the transition between two-dimensional and three-dimensional behavior of particles in the presence of surfaces or interfaces.

- DARYL GELLER (Stony Brook University)

Spin wavelets on the sphere

Wavelet expansions are extremely useful means of decomposing a function into pieces which are well-localized in space and in frequency. The theory was originally formulated on the real line, but now there is a well-developed theory of wavelets on any smooth compact oriented Riemannian manifold \mathbf{M} . The methods we developed jointly with Azita Mayeli are related to those developed earlier by Narcowich, Petrushev, and Ward for the sphere; but our methods apply to general \mathbf{M} . In addition, in joint works with Domenico Marinucci and with Mayeli, we are able to obtain wavelet expansions of sections of certain line bundles on the sphere, as opposed to the ordinary scalar-valued functions usually considered. Thus we introduce *spin wavelets* on the sphere. We obtain localization properties for them in space and frequency, and investigate stochastic properties for the analysis of spin random fields. Our results are strongly motivated by cosmological applications, in particular in connection to the analysis of Cosmic Microwave Background polarization data.

- MASOUD GHEZELBASH (University of Saskatchewan)

Kerr/CFT correspondence

We investigate the recently proposed Kerr/CFT correspondence in the context of heterotic string theory. The Kerr/CFT correspondence states that the near-horizon states of an extremal four- (or higher-) dimensional black hole could be identified with a certain chiral conformal field theory. The corresponding Virasoro algebra is generated with a class of diffeomorphisms which preserves an appropriate boundary condition on the near-horizon geometry. To understand the chiral conformal field theory, we consider the class of extremal Kerr-Sen black hole as a class of solutions in the low energy limit (effective field theory) of heterotic string theory. We obtain the entropy for the extremal black hole microscopically; in agreement with the macroscopic Bekenstein-Hawking entropy of the extremal black hole.

- DESSALEGN YIZENGAW MELESSE (University of Manitoba)

Lyapunov functions

Since their introduction by Aleksandr M. Lyapunov (1857–1918), Lyapunov functions have been widely used in establishing the asymptotic stability of solutions of nonlinear dynamical systems. In general, these functions are very difficult to construct. In this talk, I will discuss some basic properties of Lyapunov functions and show how they can be applied to completely characterize the global dynamics of the equilibria of a relatively large dynamical system arising in disease transmission dynamics.

- ASHRAFI M. NIGER (University of Manitoba)

Mathematical analysis of the role of repeated exposure on malaria transmission dynamics

This paper presents a deterministic model for assessing the role of repeated exposure on the transmission dynamics of malaria in a human population. Rigorous qualitative analysis of the model, which incorporates three immunity stages, reveals the presence of the phenomenon of backward bifurcation, where a stable disease-free equilibrium co-exists with a stable endemic equilibrium when the associated reproduction threshold is less than unity. This phenomenon persists regardless of whether the standard or mass action incidence is used to model the

transmission dynamics. It is further shown that the region for backward bifurcation increases with decreasing average life span of mosquitoes. Numerical simulations suggest that this region increases with increasing rate of re-infection of first-time infected individuals. In the absence of repeated exposure (re-infection) and loss of infection-acquired immunity, it is shown, using a nonlinear Lyapunov function, that the resulting model with mass action incidence has a globally-asymptotically stable endemic equilibrium when the reproduction threshold exceeds unity. This is joint work with Abba B. Gumel (University of Manitoba).

• HUGO RODRÍGUEZ ORDÓÑEZ (University of Regina)

Hopf invariants and the Lusternik-Schnirelmann category

H. Hopf was first to find a special kind of topological invariant in fibrations where all the spaces involved are spheres. An extended definition of this invariant, which is nowadays common knowledge, will be discussed. In 1967, T. Ganea conjectured that for any finite CW-complex and $r \geq 1$ it ought to hold that $cat(X \times S^r) = cat X + 1$, where cat is the Lusternik-Schnirelmann category. This conjecture has been readily disproved by N. Iwase. A discussion of the role of Hopf invariants in finding a minimum dimensional counterexample to Ganea's conjecture will follow. This is joint work with Don Stanley (University of Regina).

• GEORGE PATRICK (University of Saskatchewan)

Towards better understanding the behaviours of rattlebacks

Rattlebacks are rigid bodies which, when rolled on a table, exhibit a unintuitive spin bias, due to small asymmetries either of the body shape or of the mass distribution. The nonholonomic model consists of a convex body, quite a bit longer than it is wide, rolling without slipping on a plane. It may be possible to understanding better the motion of rattlebacks, by viewing the system as a (singular) perturbation from a nonregular nonholonomic system. I will sketch some preliminary work along these lines.

• CHANDRA NATH PODDER (University of Manitoba)

Mathematical study of the role of gametocytes and an imperfect vaccine on malaria transmission dynamics

A mathematical model is developed to assess the role of gametocytes (the infectious sexual stage of the malaria parasite) in malaria transmission dynamics in a community. The model is rigorously analysed to gain insights into its dynamical features. It is shown that, in the absence of disease-induced mortality, the model has a globally-asymptotically stable disease-free equilibrium whenever a certain epidemiological threshold, known as the *basic reproduction number* (denoted by \mathcal{R}_0), is less than unity. Further, it has a unique endemic equilibrium if $\mathcal{R}_0 > 1$. The model is extended to incorporate an imperfect vaccine with some assumed therapeutic characteristics. Theoretical analyses of the model with vaccination show that an imperfect malaria vaccine could have negative or positive impact (in reducing disease burden) depending on whether or not a certain threshold (denoted by ∇) is less than unity. Numerical simulations of the vaccination model show that such an imperfect anti-malaria vaccine (with a modest efficacy and coverage rate) can lead to effective disease control if the reproduction threshold (denoted by \mathcal{R}_{vac}) of the disease is reasonably small. On the other hand, the disease cannot be effectively controlled using such a vaccine if \mathcal{R}_{vac} is high. Finally, it is shown that the average number of days spent in the class of infectious individuals with higher level of gametocyte is critically important to the malaria burden in the community.

- CHARY RANGACHARYULU (University of Saskatchewan)

Some symmetries and invariances in nuclear and particle physics

Like in many fields of human endeavor, symmetries play invaluable roles in nuclear and particle physics. A foundational formulation was due to Emmy Noether, who provided the connection between the symmetries and corresponding invariance principles in a simple, elegant form. Over several decades of research, nuclear and particle physicists made extensive use of this formulation in discovering and classifying physical phenomena and structural features. Time reversal symmetry, with origins in the microscopic reversibility, plays an important role despite the fact that there is no associated invariance principle. Our group at the National High Energy Physics Laboratory, Tsukuba, Japan, has been involved in a dedicated test of the symmetry. My talk will address this topic in some detail and I will also address the isospin symmetries and its implications.

- OLUWASEUN Y. SHAROMI (University of Manitoba)

*Re-infection-induced backward bifurcation in the transmission dynamics of *Chlamydia trachomatis**

A new two-group deterministic model for *Chlamydia trachomatis* is designed and analyzed to gain insights into its transmission dynamics. The model is shown to exhibit the phenomenon of backward bifurcation, where a stable disease-free equilibrium (DFE) co-exists with one or more stable endemic equilibria when the associated reproduction number is less than unity. It is further shown that the backward bifurcation dynamic is caused by the re-infection of individuals who recovered from the disease. The epidemiological implication of this result is that the classical requirement of the reproduction number being less than unity becomes only a necessary, but not sufficient, for disease elimination. The basic model is extended to incorporate the use of treatment for infectious individuals (include those who show symptoms and those who do not). Rigorous analysis of the treatment model reveals that the use of treatment could have positive or negative population-level impact, depending on the sign of a certain epidemiological threshold. The treatment model is used to evaluate various treatment strategies, namely treating every infected individual showing symptoms of Chlamydia (universal strategy), treating only infectious males showing Chlamydia symptoms (male-only strategy) and treating only infectious females showing symptoms of Chlamydia (female-only strategy). Numerical simulations show that the implementation of the male-only or female-only strategy can induce an indirect benefit of saving new cases of Chlamydia infection in the opposite sex. Further, the universal strategy gives the highest reduction in the cumulative number of new cases of infection.

- DINESH SINGH (University of Regina)

Effects of space-time curvature on spin-1/2 particle Zitterbewegung

This paper investigates the properties of spin-1/2 particle Zitterbewegung in the presence of a general curved space-time background described in terms of Fermi normal co-ordinates, where the spatial part is expressed using general curvilinear co-ordinates. Adopting the approach first introduced by Barut and Bracken for Zitterbewegung in the local rest frame of the particle, it is shown that non-trivial gravitational contributions to the relative position and momentum operators appear due to the coupling of Zitterbewegung frequency terms with the Ricci curvature tensor in the Fermi frame, indicating a formal violation of the weak equivalence principle. Explicit expressions for these contributions are shown for the case of quasi-circular orbital motion of a spin-1/2 particle in a Vaidya background. Formal

expressions also appear for the time-derivative of the Pauli-Lubanski vector due to space-time curvature effects coupled to the Zitterbewegung frequency. As well, the choice of curvilinear co-ordinates results in non-inertial contributions in the time evolution of the canonical momentum for the spin-1/2 particle, where Zitterbewegung effects lead to stability considerations for its propagation, based on the Floquet theory of differential equations.

- ARTUR SOWA (University of Saskatchewan)

On the noncommutative Schrödinger equation

I will discuss a model of nonlocal dynamics describing non-dissipative interaction of quantum systems. Within this framework, the evolution of a composite quantum system is governed by an operator equation

$$-i\hbar\dot{K} = KH + \hat{H}K + \beta Kf(K^*K).$$

Here H and \hat{H} are fixed self-adjoint Hamiltonians, $z \rightarrow f(z)$ is an analytic function, such as entropy, and β is a real parameter. The equation may be viewed as a noncommutative analogue of the nonlinear Schrödinger equation. I will discuss the main properties of this equation, and outline its physical origin, interpretation, and applications. The research results to be presented extend those reported in *A. Sowa, Quantum entanglement in composite systems* (Theoretical and Mathematical Physics, to appear) and elsewhere.

- JACEK SZMIGIELSKI (University of Saskatchewan)

The story of peakons

Peakons are non-smooth solutions to certain nonlinear equations, most notably the Camassa-Holm equation (CH) and the Degasperis-Procesi (DP) equation arising as approximate equations to water wave equations. Both CH and DP are Lax integrable, which means that there is certain linear structure which can be used to analyze these equations and their solutions. I will explain the special role of peakons for Lax integrability, how they are related to many areas of classical analysis, like continued fractions, orthogonal polynomials, and at the same time how these special solutions are linked to modern problems of analysis centered around the multiplication problem for L. Schwartz distributions. The talk is for a general mathematical audience.

Statistics

- MAHSHID ATAPOUR (York University)

Exponential growth of the number of n -edge linked clusters in \mathbb{Z}^3 and the consequences in entanglement percolation

An animal in the simple cubic lattice is a finite connected subgraph of \mathbb{Z}^3 . Let a_n be the number (up to translation) of n -edge animals in \mathbb{Z}^3 . In 1967, Klarner proved that a_n grows exponentially. Let e_n be the number (up to translation) of all n -edge linked clusters, i.e. subgraphs of \mathbb{Z}^3 in which the connected components (animals) are (topologically) non-splittable. In this talk, I will briefly explain how it can be proved that e_n also has a finite exponential growth rate. I will then mention some of the important consequences of this result in entanglement percolation. This is joint work with Neil Madras (York University).

- MIKELIS BICKIS (University of Saskatchewan)

On the distribution of frequencies of oligopeptides

In the collection of all known proteins, certain short sequences of amino acids are very common, whereas some are quite rare, and a small number don't appear at all. The empirical distribution of frequencies is found to be approximately log-normal, although there are serious deviations from this shape in both the upper and lower tails. The empirical distribution is compared with what is expected from several models of generating peptides randomly. This is joint work with Tony Kusalik, Brett Trost and Darja Kanduc (University of Saskatchewan).

- RICHARD K. BOWLES (University of Saskatchewan)

The jamming landscape for confined hard discs

The characterization and enumeration of jammed packings in hard core particle systems, such as hard discs and hard spheres, is a long-standing problem that holds the key to understanding the properties of a wide variety of materials including liquids, glasses, crystals and granular materials. A simple model consisting of hard discs of diameter σ , trapped between two hard lines separated by a distance H , exhibits slow relaxation and heterogeneous dynamics at high densities, which are characteristic of glassy systems. We show that for $H < 2\sigma$, all the collectively jammed packings of the system can be constructed from a small set of locally jammed structures which can be represented as tiles. As a result, we can exactly enumerate the entire jamming landscape of the system and explore how this landscape is connected to the thermodynamics and dynamics of the glassy system. This is joint work with S. S. Ashwin (University of Saskatchewan).

- MELODY GHAHRAMANI (University of Winnipeg)

On combining estimating functions for volatility models

In this talk, we discuss combining estimating functions for the parameter of discrete time stochastic processes where the conditional mean and the conditional variance depend on the same parameter with applications to volatility models such as autoregressive processes with generalized autoregressive conditional heteroscedastic (GARCH) errors and random coefficient autoregressive processes (RCA). The optimal combined estimating function is shown to contain more Godambe information than each of the component estimating functions. As an application of the combined estimating function method, the problem of GARCH model identification is discussed. Recursive estimation of the parameter is of interest when data become available successively over time. Often, recursive estimators that are optimal in the minimum mean square error (MMSE) sense, that is linear estimators, are used. As another application, non-linear recursive estimation of the parameter in some volatility models, based on the optimal combined estimating function is derived. Pre-filtered optimal estimation of the parameter of doubly stochastic time series based on the optimal combined estimating function will also be discussed. This is joint work with A. Thavaneswaran (University of Manitoba).

- WINFRIED K. GRASSMANN (University of Saskatchewan)

Infinite dimensional banded stochastic matrices

When going from finite to infinite stochastic matrices, it is known that matrix multiplication is no longer associative, and that the inverse is no longer unique. We explore this situation and its effect when determining the eigenvector corresponding to the largest eigenvalue. Though this eigenvector is no longer unique either, one can show that in banded stochastic

matrices, there are exactly two solutions that make probabilistic sense. This is joint work with Javad Tavakoli (University of British Columbia).

- XULIN GUO (University of Saskatchewan)

Statistics on remote sensing monitoring program

Remote sensing has been used as a major tool to monitoring prairie ecosystems as remote sensing provides multi-spatial and multi-temporal satellite images. However, statistics is the key linkage between remote sensing and ecosystem biophysical properties. As an application, prairie grassland heterogeneity was measured and monitored with different satellite platforms at different resolutions. Statistics methods applied include simple descriptive analysis, multivariate statistics, geostatistics, and modeling. Biophysical parameters were biomass, canopy cover, vegetation height, and grassland heterogeneity. Most popular used correlation and regression analyses indicated the relationship between these variables and remote sensing signals. Semivariogram and wavelet analyses revealed the spatial scales of each biophysical parameter and in addition factors to control these spatial scales were investigated. As a result, spatial based grassland condition was monitored through continuous remote sensing data at different spatial and temporal scales.

- STEVE KIRKLAND (University of Regina)

Eigenvalues and the scrambling index for stochastic matrices

A square, entrywise nonnegative matrix with every row sum equal to 1 is known as a *stochastic matrix*. Any stochastic matrix T has 1 as an eigenvalue of largest complex modulus, and the size of the second largest (in modulus) eigenvalue of T is of interest because it determines the asymptotic rate of convergence of the sequence of powers of T . In this talk, we introduce the notion of the scrambling index of a directed graph, and discuss some of its properties. We then use the scrambling index of the directed graph of a stochastic matrix T in order to produce an attainable upper bound on the size of the second largest eigenvalue of T . This is joint work with Mahmud Akelbek (Weber State University).

- CHEL HEE LEE (University of Manitoba)

Efficient Monte Carlo random variates generation through discretization

We propose an efficient Monte Carlo method for random variates generation from high dimensional distributions of complex structures. This method is based on the discretization of the sample space and efficient numerical inversion of a multivariate cumulative distribution function. It is non-iterative, easy to implement and overcomes many typical drawbacks of the Markov chain Monte Carlo (MCMC) methods. This method improves the discretization-based algorithm of Fu and Wang (2002). Numerical examples will be presented to illustrate the new method and algorithm. This is joint work with Liqun Wang (University of Manitoba).

- JUXIN LIU (University of Saskatchewan)

Bayesian analysis of a matched case-control study with misclassified binary data

We propose a Bayesian adjustment for the misclassification of a binary exposure variable in a matched case-control study. The method admits a priori knowledge about both the misclassification parameters and the exposure-disease association. The standard Dirichlet prior distribution for a multinomial model is extended to allow separation of prior assertions about the exposure-disease association from assertions about other parameters. The method is applied to a study of occupational risk factors for new-onset adult asthma.

- LISA LIX (University of Saskatchewan)

Quality of administrative health data: a statistical perspective

Administrative health data are data collected for purposes of health system monitoring and financial management. These data are derived from multiple sectors including hospitals, emergency departments, physician offices, vital statistics registries, and prescription drug dispensation systems. Many epidemiologic and health services research studies are conducted using administrative health data. A key concern when using these data for research is their quality, particularly their accuracy and completeness. This session will provide an overview of data quality concerns from a statistical perspective and present case studies that demonstrate the use of capture-recapture techniques and measurement error models to address the limitations of the data.

- TOLUPE T. SAJOBI (University of Saskatchewan)

Multiple testing procedures in the presence of non-normality and covariance heterogeneity

Exact simultaneous multiple testing procedures for multivariate independent group designs are based on the joint distribution of the test statistics. However, these procedures are sensitive to departures from covariance homogeneity and multivariate normality assumptions. Resampling-based procedures which use empirical methods to approximate the joint distribution of the multiple test statistics have been proposed to account for the correlation among the multiple outcomes. This study investigates several procedures for testing differences between two independent groups on multiple outcome variables when covariances are heterogeneous and the data non-normal. Procedures based on empirical permutation and bootstrap methods and least-squares or robust (i.e. trimmed) estimators are considered. A Monte Carlo simulation study is undertaken in which the effects of non-normality, covariance heterogeneity, correlation structure, sample size, and mean configuration on the familywise error rate (FWER), all-variable power, and any-variable power are investigated. The bootstrap procedures based on robust estimators are more powerful than the bootstrap procedures based on least square estimators when the data are non-normal. This is joint work with Lisa M. Lix (University of Saskatchewan).

- MICHAEL SZAFRON (University of Saskatchewan)

Knotting probabilities resulting from a local strand passage in a knot-type K SAP

DNA is prone to several topological entanglement problems. One such problem is that knotted DNA cannot be successfully replicated. This problem is resolved via an interaction between the DNA and the topoisomerase enzymes. These enzymes interact locally with the DNA and allow one strand of the DNA to pass through itself. Because these local strand passages can potentially change the knot-type of the DNA, the frequency of the knots produced can be used to characterize the topoisomerase action on the DNA topology. A self-avoiding polygon (SAP) model was designed to investigate whether these local strand passages are implemented at random locations in the DNA. Some knotting probability estimates determined from a Monte Carlo simulation of this model are to be presented.

- XIKUI WANG (University of Manitoba)

Statistics applied in finance and economics

We explore the application of statistical decision models (such as the bandit processes and regime switching models) and stochastic dynamic programming to understand the dynamic pricing problems as well as investment and consumption problems. We assume unknown

demand functions for the dynamic pricing model, and risky assets for the investment and consumption problem.

- MARK WURTZ (University of Saskatchewan)

Effect of multiple Higgs fields on the phase structure of the $SU(2)$ -Higgs model

The $SU(2)$ -Higgs model, with a single Higgs field in the fundamental representation and a quartic self-interaction, has a Higgs region and a confinement region which are analytically connected in the parameter space of the theory; these regions thus represent a single phase. The effect of multiple Higgs fields on this phase structure is examined via Monte Carlo lattice simulations. For the case of $N \geq 2$ identical Higgs fields, there is no remaining analytic connection between the Higgs and confinement regions, at least when Lagrangian terms that directly couple different Higgs flavours are omitted. An explanation of this result in terms of enhancement from overlapping phase transitions is explored for $N = 2$ by introducing an asymmetry in the hopping parameters of the Higgs fields. It is found that an enhancement of the phase transitions can still occur for a moderate (10%) asymmetry in the resulting hopping parameters.

- GUICHANG ZHANG (University of Saskatchewan)

Some properties on non-homogeneous Poisson process and their applications

In this paper, we get some results about Non-homogeneous Poisson Process (NHPP for short). It is well known that homogeneous Poisson Process has the memoryless property and its inter-arrival time random variables are i.i.d and exponentially distributed. What will it be for NHPP? Our results show that NHPP has the property of conditional identical distribution for its inter-arrival times and quasi-memoryless property. In the second part, such properties of NHPP are used to queues with an infinite number of servers.

- RONG ZHU (McMaster University)

The generalized Poisson-inverse Gaussian family and its application to heavy-tailed count data

We propose a (new) three-parameter discrete distribution family, called the GPIG distribution, which not only extends the over-dispersed Poisson-inverse Gaussian (PIG), but also includes the equally-dispersed Poisson and the extremely heavy-tailed discrete stable. From the modeling perspective, this family is flexible in handling various situations in count data, especially when the heavy-tailed phenomenon occurs. The distribution is defined through the probability generating function, and in general, has no nice explicit form of probability mass function. A recursive method is developed to calculate the probability mass function. Statistical inferences based on likelihood are then feasible. The GPIG family is applied to citation counts of published articles in 1990 in JASA and JSPI respectively. This is joint work with Harry Joe (University of British Columbia).

Industrial and Computational Mathematics

In place of contributed talks, the Industrial and Computational Mathematics session will feature a workshop on programming the Cell Broadband Engine.

ABSTRACT:

The Cell Broadband Engine processor, originally developed for the Sony PlayStation3, is now the computational workhorse of Roadrunner, the world's fastest supercomputer as measured

by the Top500 List. This workshop will explore the potential of the Cell processor for high-performance computing and teach you the details of its architecture and its programming models. You will be given lots of hands-on experience so that after attending this workshop, you should be able to write, compile and run parallel programs on the Cell processor.

AGENDA:

- Session 1: Introduction to the Cell architecture and SDK, basic programming techniques and hands-on system check
- Session 2: Hands-on hello world program, thread parallelization, data movement and communication between threads
- Session 3: Hands-on static timing analysis, dynamic profiling and SIMD programming
- Session 4: Programming tips & techniques, porting tips & techniques, additional hands-on play