

# 37th Canadian Operator Theory Symposium

## Regina, Saskatchewan

### May 26-30, 2009

#### 1. Featured Speaker

**Uffe Haagerup (University of Southern Denmark)**

**Title:** Problems in operator algebras motivated by operator space theory

**1st Lecture:** *Classification of hyperfinite factors up to completely bounded isomorphism of their preduals.* We consider the following problem: When are the preduals of two hyperfinite factors (on separable Hilbert spaces) cb-isomorphic, i.e. isomorphic as operator spaces? We show that if  $M$  is semifinite and  $N$  is Type III, then their preduals are not cb-isomorphic. Moreover we construct a one parameter family of hyperfinite type III<sub>0</sub> factors with mutually non cb-isomorphic preduals, and we give a characterization of those hyperfinite factors  $M$  whose preduals are cb-isomorphic to the predual of the hyperfinite type III<sub>1</sub> factor. In contrast, Christensen and Sinclair proved in 1989 that all infinite dimensional hyperfinite factors with separable preduals are cb-isomorphic and more recently, Rosenthal, Sukochev and the first named author proved that all hyperfinite type III<sub>λ</sub> factors, where  $0 < λ ≤ 1$ , have cb-isomorphic preduals. (Joint work with Magdalena Musat).

**2nd Lecture:** *Solution of the Effros-Ruan Conjecture for bilinear forms on C\*-algebras.* In 1991 Effros and Ruan conjectured that a certain Grothendieck-type inequality for a bilinear form on a pair of C\*-algebras holds if (and only if) the bilinear form is jointly completely bounded. In 2002 Pisier and Shlyakhtenko proved that this inequality holds in the more general setting of operator spaces, provided that the operator spaces in question are exact. Moreover they proved that the Effros-Ruan conjecture holds for pairs of C\*-algebras of which at least one is exact. In a recent joint work with Magdalena Musat we prove the Effros - Ruan conjecture for general C\*-algebras (and with constant one), i.e. for every jointly completely bounded (jcb) bilinear form  $u$  on a pair of C\*-algebras  $A, B$  there exists states  $f_1, f_2$  on  $A$  and  $g_1, g_2$  on  $B$ , such that

$$|u(a, b)| \leq \|u\|_{\text{jcb}} (f_1(aa^*)^{1/2} g_1(b^*b)^{1/2} + f_2(a^*a)^{1/2} g_2(bb^*)^{1/2})$$

While the approach by Pisier and Shlyakhtenko relied on free probability theory, our proof uses more classical operator algebra methods, namely Tomita Takesaki theory and special properties of the Powers factors of Type III<sub>λ</sub>,  $0 < λ < 1$ . (Joint work with Magdalena Musat).

**3rd Lecture:** *Factorization and dilation problems for completely positive maps on von Neumann algebras.* We study factorization and dilation properties for completely positive unital trace preserving maps (for short, cp.u.t. maps) on  $(M, \text{tr})$ , where  $M$  is a von Neumann algebra and  $\text{tr}$  is a normal faithful trace state on  $M$ . The first property is Anantharaman-Delaroche's factorization property from 2004. We provide simple examples of non-factorizable cp.u.t. map on the  $n \times n$  matrices for  $n \geq 3$  as well as an example of a one-parameter semigroup  $(T_t)_{t \geq 0}$  of cp.u.t. maps on the  $n \times n$  matrices such that  $T_t$  fails to have the Markov dilation property for all small values of  $t > 0$ . Moreover we will discuss the connection to Kummer's work on the Markov dilation property from the 80's. The second property is the non-commutative Rota dilation property introduced by Junge, Le Merdy and Xu in 2006. We show that the most natural generalization of Rota's classical dilation theorem to the non-commutative setting does not hold by providing an example of a selfadjoint cp.u.t. map  $T$  on the  $n \times n$  matrices for some large  $n$ , such that  $T^2$  does not have the non-commutative Rota dilation property. (Joint work in progress with Magdalena Musat).

## 2. Invited and Contributed Speakers

### Archev, Dawn (Ben Gurion University)

**Title:** Crossed product  $C^*$ -algebras and the projection free tracial Rokhlin property  
**Abstract:** We will define the projection free tracial Rokhlin property (an analog of the tracial Rokhlin property suitable for algebras with few projections). Then we will present the following theorem: Let  $A$  be an infinite dimensional stably finite simple unital  $C^*$ -algebra such that all 2-quasi-traces are traces, and with only finitely many extreme tracial states. Assume  $A$  has stable rank one and strict comparison of positive elements. Let  $\alpha : G \rightarrow \text{Aut}(A)$  be an action of a finite group with the projection free tracial Rokhlin property. Then  $B = C^*(G, A, \alpha)$  also has stable rank one. We will discuss as many of the following as time allows:  
-The definition of strict comparison of positive elements -A brief summary of the proof -An example to illustrate the theorem (the tensor flip on two copies of the Jiang-Su algebra).

### Boersema, Jeff (Seattle University)

**Title:** Classification of Purely Infinite Real  $C^*$ -Algebras  
**Abstract:** We outline a program to classify real simple purely infinite  $C^*$ -algebras using united  $K$ -theory. As an application, we describe the real forms of complex Cuntz algebras. This is joint work with Peter Stacey and Efren Ruiz.

### Brown, Nate (Penn State)

**Title:** The Cuntz Semigroup  
**Abstract:** I'll give a survey of the Cuntz semigroup.

### Buck, Julian (University of Oregon)

**Title:** Crossed-Product  $C^*$ -algebras by Automorphisms with the Tracial Quasi-Rokhlin Property  
**Abstract:** We introduce the tracial quasi-Rokhlin property for an automorphism  $\alpha$  of a (not-necessarily simple)  $C^*$ -algebra  $A$ , and describe various structure properties of the crossed-product  $C^*$ -algebra  $C^*(\mathbb{Z}, A, \alpha)$ . We also show that under certain assumptions on  $X$  and  $A$ , a class of automorphisms of  $C(X, A)$  (coming from a minimal homeomorphism of  $X$  and an automorphism of  $A$ ) have this property. The question of determining a recursive structure for certain subalgebras of the crossed-product will be discussed.

**Chiumiento, Eduardo (Universidad Nacional de La Plata)**

**Title:** Local minimal curves in homogeneous reductive spaces of the unitary group of a finite von Neumann algebra

**Abstract:** We study the metric geometry of homogeneous reductive spaces of the unitary group of a finite von Neumann algebra with a non complete Riemannian metric. The main result gives an abstract sufficient condition in order that the geodesics of the Levi-Civita connection are locally minimal. Then, we show how this result applies to several examples.

**Choi, Man-Duen (University of Toronto)**

**Title:** Normal dilations

**Abstract:** Which operators can be dilated to normal operators with some prescribed spectra? This turns out to be useful for the illustration of Naimark's theorem about the structure of a unital positive linear map from  $C(X)$  to  $B(H)$ . Even in the finite dimensional cases, there are very hard problems of unknown depth in matrix analysis, related to quantum information.

**Collins, Benoit (University of Ottawa - CNRS)**

**Title:** Some geometric and probabilistic properties of the free quantum groups

**Abstract:** In this talk I will report on recent progresses about the free quantum group  $A_o(n)$ . In particular I will explain a method to compute the probability distribution of its generators and show that they don't have atoms; I will also give estimates for the free entropy dimension of  $A_o(n)$ . Last, I will discuss the Connes Embedding property for  $A_o(n)$  and  $A_s(n)$ .

**Davidson, Ken (University of Waterloo)**

**Title:** Semicrossed products of the non-commutative disc algebra

**Abstract:** The non-commutative disc algebra  $\mathfrak{A}_n$  is the norm closed nonself-adjoint operator algebra generated by  $n$  isometries with pairwise orthogonal ranges. The automorphisms are determined by conformal automorphisms of the unit ball in  $\mathbb{C}^n$ . I will discuss the classification and representation theory of the semicrossed product of  $\mathfrak{A}_n$  by an automorphism.

**Dean, Andrew (Lakehead University )**

**Title:** Classification of  $C^*$ -dynamical systems

**Abstract:** We will discuss recent progress on developing invariants to study  $C^*$ -dynamical systems.

**Elliott, George (University of Toronto)**

**Title:** On the classification of non-simple inductive limits of matrix algebras over one-dimensional spaces

**Abstract:** Simple inductive limits of (sequences of) matrix algebras over one-dimensional (metrizable) locally compact spaces were classified by Liangqing Li, after earlier work by me in the case of circles. It turns out that, as a consequence of the work of Li, the case of circles exhausts the simple inductive limits. The invariants for this class of algebras, say assumed to be stable, may be viewed as just the Cuntz semigroup together with the Banach algebra  $K_1$ -group.

In the non-simple case, if the Banach algebra  $K_1$ -group is assumed to be zero, then it was shown recently by Ciuperca, Robert, Santiago, and me that the Cuntz semigroup is a complete invariant. (Again, in the stable case—otherwise one needs to keep track of the canonical Cuntz class—which is just the largest element of the semigroup in the stable case.) (Ciuperca and I had obtained this result in the case of closed or half-open intervals in the line, building on work of Coward, Ivanescu, and me on the general theory of the Cuntz semigroup, and in particular on the fact that, in a suitable sense, this invariant is preserved under passage to inductive limits—and building also on work of Robert, who obtained essentially the same result (closed or half-open intervals) in an important case (totally ordered ideal lattice for the limit algebra) but without mentioning the Cuntz semigroup; it only became clear in fact after the work of Ciuperca and me, in work of Robert, Santiago, and me, that Robert’s invariant, in the case that he considered, actually was the Cuntz semigroup.

How to calculate the Cuntz semigroup in more general cases, even in the case that the spaces are closed or half-open intervals, is still an interesting problem. The general case, in which one has to consider not only the Banach algebra  $K_1$ -group, but also the algebraic  $K_1$ -group, and not only for the whole algebra, but also inside each hereditary sub- $C^*$ -algebra—equivalently, in the present case (in which the stable rank is one), inside each Cuntz element—seems not quite to be within reach at the moment. The case that the algebra has the so-called ideal property—ideals generated by projections—, dealt with by K. Stevens in the setting of closed intervals, appears to be manageable (work in progress with Ciuperca, Robert, and Santiago).

**Laca, Marcelo (University of Victoria)**

**Title:** Quasi-lattices, boundary quotients, phase transitions, and Cuntz’s  $Q_N$

**Abstract:** Cuntz has recently introduced and studied a  $C^*$ -algebra  $Q_N$  associated to the affine semigroup over the natural numbers. We show that this semigroup is quasi-lattice ordered in the sense of Nica, and that its Toeplitz algebra has Cuntz’s  $Q_N$  as boundary quotient. There is a natural extension of the dynamics considered by Cuntz on  $Q_N$  to the Toeplitz algebra, and the resulting Toeplitz dynamical system has a very interesting phase transition with spontaneous symmetry-breaking. This is joint work with Iain Raeburn.

**MacDonald, Gordon (U. of Prince Edward Island)**

**Title:** Composition Operators on the Newton Space

**Abstract:** We investigate properties of composition operators  $C_\phi$  on the Newton space (the Hilbert space of analytic functions which have the Newton polynomials as an orthonormal basis). We derive a formula for the entries of the matrix of  $C_\phi$  with respect to the basis of Newton polynomials in terms of the value of the symbol  $\phi$  at the non-negative integers. We also establish conditions on the symbol  $\phi$  for boundedness, compactness, and self-adjointness of the induced composition operator  $C_\phi$ . A key technique in obtaining these results is use of an isomorphism between the Newton space and the Hardy space via the Binomial Theorem. (Joint with Peter Rosenthal.)

**Manjegani, S. Mahmoud (Isfahn University of Technology, Iran)**

**Title:** Operator-Valued Bochner Integrable Functions and Jensen's Inequality

**Abstract:** In this talk we introduce a Jensen's type inequality for operator-valued integrable functions which generalizes some of the previous results in this regard. More precisely, if  $(\Omega, \Sigma, \mu)$  is a probability measure space and if  $\nu$  is an operator convex function then, under suitable conditions, we show that  $\nu(\int_\Omega g^*fg \, d\mu) \leq \int_\Omega g^*\nu \circ fg \, d\mu$ , where  $f : \Omega \rightarrow B(H)^{sa}$  is assumed to be Bochner integrable and  $g : \Omega \rightarrow B(H)$  is a measurable function with  $\int_\Omega g^*g \, d\mu = \mathbf{1}$ .<sup>a</sup>

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<sup>a</sup>Key words and phrases: Operator convex function, Jensen's inequality, Bochner integral, Operator-valued function

**Marcoux, Laurent (University of Waterloo)**

**Title:** Triangularizability of operators with increasing spectrum

**Abstract:** We establish finite- and infinite-dimensional versions of the following assertion. If  $M$  is a matrix with the property that whenever  $P$  and  $Q$  are diagonal projections with  $P \leq Q$ , the spectrum of  $PMP$  (considered as an operator on the range of  $P$ ) is contained in that of  $QMQ$  (considered as an operator on the range of  $Q$ ), then there is a permutation matrix  $U$  such that  $U^{-1}MU$  is triangular.

**Massey, Pedro (Universidad Nacional de La Plata)**

**Title:** Non-commutative Schur-Horn theorems

**Abstract:** Let  $A$  be a unital  $*$ -subalgebra of the algebra of all  $n$  by  $n$  complex matrices and let  $E$  denote the trace preserving conditional expectation onto  $A$ . If  $U(b)$  (resp.  $C(b)$ ) denotes the unitary (resp. contractive) orbit of the positive matrix " $b$ " then we give necessary and sufficient conditions for the matrix  $a$  in  $A$  to be in  $E(U(b))$  (resp.  $E(C(b))$ ); these conditions are related with the Horn-Klyachko's equations. We consider these results as non-commutative extensions of the classical Schur-Horn theorem. We use the previous results to introduce what we call extended (sub)-majorization between positive matrices. We study some Jensen's-type inequalities associated with the extended majorization and apply these notions in the context of protocols (general linear reconstruction systems).

**Mathieu, Martin (Queen's University Belfast)**

**Title:** Sheaves of  $C^*$ -Algebras

**Abstract:** I shall report on joint work in progress with Pere Ara (Barcelona) in which we develop the basics of a theory of sheaves of  $C^*$ -algebras and, in particular, compare it to the existing theory of  $C^*$ -bundles. The details of two fundamental examples, the local multiplier sheaf and the injective envelope sheaf, are discussed.

**Mingo, James A. (Queen's University)**

**Title:** Second Order Freeness and Second Order Cumulants

**Abstract:** In free probability the coefficients of Voiculescu's  $R$ -transform form the free cumulants of a random variable. As in ordinary probability, the vanishing of mixed cumulants is a necessary and sufficient condition for a sequence of random variables to be freely independent. In recent papers, we introduced a concept of second order freeness to handle the fluctuations of random matrices and constructed an extension of free cumulants to ones of second order. We show that the classical rule still holds: second order freeness is equivalent to the vanishing of mixed free cumulants of first and second order. This is joint work with Roland Speicher.

**Paulsen, Vern (University of Houston)**

**Title:** Operator Algebras of Functions

**Abstract:** Many operator algebras of current interest, such as the Arveson-Drury and Schur-Agler algebras are really operator algebras of functions on their underlying domains. We prove that any operator algebra algebra of functions satisfying certain conditions is residually finite dimensional, a dual operator algebra and is completely isometrically isomorphic to the multiplier algebra of a RKHS. We show that the Arveson-Drury and Schur-Agler algebras satisfy our conditions.

**Phillips, Chris (University of Oregon)**

**Title:** Minimal actions on prime nonsimple  $C^*$ -algebras

**Abstract:** This is joint work with Eberhard Kirchberg. Let  $G$  be a second countable locally compact but noncompact group. We prove the existence of a prime  $C^*$ -algebra which is not simple and which admits an outer action of  $G$  which is minimal, that is, there are no nontrivial  $G$ -invariant ideals. This answers a question raised by Benjamin Itza-Ortiz. In some cases, the dual actions on the crossed products give “exotic” actions of locally compact abelian groups on simple  $C^*$ -algebras (often the Cuntz algebra  $O_2$ ). The crossed products by these actions are prime but not simple, so that the actions are unlike previously known ones.

**Popov, Alexey (University of Alberta)**

**Title:** Almost invariant half-spaces

**Abstract:** We say that a bounded linear operator  $T$  on a Banach space  $X$  admits an almost invariant half-space if there exists an infinite dimensional and infinite codimensional closed subspace  $Y$  (a half-space) and a finite dimensional subspace  $F$  such that  $T(Y) \subseteq Y + F$ . The question whether every bounded linear operator admits an almost invariant half-space is connected to the invariant subspace problem, but it is not necessarily weaker. In this talk we introduce a promising technique for approaching this question and prove several positive results for weighted shifts operators. In particular we show that Donoghue operators, which do not have invariant half-spaces, admit almost invariant half-spaces with one dimensional



**Putnam, Ian (University of Victoria)**

**Title:** A survey of orbit equivalence for Cantor minimal systems

**Abstract:** Orbit equivalence in the measurable category (and its connections with operator algebras) has been a major development in the subject through the work of Dye, Krieger, Connes, Rudolph and many others. There is also an analogous program in Borel dynamics. In the past twenty years, I, along with Herman, Gioradno, Skau and Matui, have obtained results in the topological category. As the title says, I will give an overview of this, including some more recent results and some new approaches to some of the older results.

**Radjavi, Heydar (University of Waterloo)**

**Title:** When small parts imply small wholes

**Abstract:** A well-known elementary theorem in group representation states that an irreducible group of matrices on which trace takes a finite number of values is itself finite. This is also true if the finiteness condition is replaced with that of boundedness. We discuss various recent results of this general flavour in finite- and infinite-dimensional settings.

**Ramsey, Chris (University of Waterloo)**

**Title:** Operator algebras of dynamical systems

**Abstract:** A dynamical system in this case is a locally compact space along with  $n$  proper continuous self-maps. To such a system there is an associated universal operator algebra such that if two algebras are isomorphic then their corresponding systems are the same up to a natural conjugacy. The converse has been established for  $n = 2, 3$  and  $4$ .

**Redelmeier, Emily (Queen's University)**

**Title:** Asymptotics and fluctuations of real Wishart random matrices

**Abstract:** I will present an Euler characteristic expansion for the real Wishart matrix ensemble, similar to the genus expansion for the complex case. This provides a combinatorial approach for calculating the asymptotic values of the moments of real Wishart matrices, as well as the distribution around this value.

**Rosenthal, Haskell (University of Texas)**

**Title:** The closeability property for algebras of operators on a complex Banach space  
**Abstract:** Let  $A$  be a strongly closed algebra of (bounded linear) operators on a Banach space  $X$ .  $A$  is said to have the closability property (cp) if every densely defined operator which commutes with  $A$  is closable. Also, an operator on  $X$  is said to have the cp if the strongly closed algebra it generates has it. Conjecture: If  $A$  has the cp, either  $A$  is all of  $L(X)$  or  $A$  has (nontrivial closed linear) invariant subspace. Deep work from Arveson from the 60's shows that MASA's on  $B(H)$  have the cp, and also our conjecture holds for any (str closed) algebra containing such. He also established the same for the unilateral shift. However recent work of Bercovici, Douglas and Foias yields that the bilateral shift fails the cp. We further investigate the Conjecture (which would generalize the Arveson MASA result): Let  $N$  be a vNA with finite commutant. Then  $N$  has the cp, and moreover our conjecture holds in the special case where  $A$  contains  $N$ . The cp and partial results concerning the above is joint work with Timur Oikhberg and Vladimir Troitsky.

**Ruiz, Mariano (Universidad Nacional de La Plata)**

**Title:** The structure of minimizers of the frame potential on fusion frames  
**Abstract:** The fusion frame potential is a generalization of the Benedetto-Fickus (vectorial) frame potential to the finite-dimensional fusion frame setting. In this talk, we present some results of the study of the structure of local and global minimizers of this potential, when we restrict it to suitable sets of fusion frames. These minimizers are related to tight fusion frames as in the classical vector frame case. Still, tight fusion frames are not as frequent as tight frames; indeed we show that there are choices of parameters involved in fusion frames for which no tight fusion frame can exist. We exhibit necessary and sufficient conditions for the existence of tight fusion frames with prescribed parameters, involving the so-called Horn-Klyachko's compatibility inequalities. The second part of the talk is devoted to the study of the minimization of the fusion frame potential on a fixed sequence of subspaces, varying the sequence of weights. We related this problem to the index of the Hadamard product by positive matrices and use it to give different characterizations of these minima.

**Runde, Volker (University of Alberta)**

**Title:** Co-representations of Hopf-von Neumann algebras on operator spaces other than column Hilbert space

**Abstract:** Let  $(M, \Gamma)$  be a Hopf-von Neumann algebra. A *co-representation* of  $(M, \Gamma)$  on a Hilbert space  $H$  is a (often unitary) operator  $U \in M \bar{\otimes} \mathcal{B}(H)$  such that

$$(\Gamma \otimes I)(U) = U_{1,3}U_{2,3},$$

(using “leg notation”).

Recently, M. Daws showed that, for abelian  $M$ , the notion of co-representation can be extended beyond Hilbert space to general reflexive Banach spaces, and it requires little effort to extend this to the framework of subhomogeneous  $M$  and to operator spaces.

In this talk, we shall see that this is about as far as we can get: if  $M$  is a von Neumann algebra and  $E$  is a reflexive operator space such that  $M \bar{\otimes} \mathcal{CB}(E)$  is an algebra (in the canonical way), then  $M$  is subhomogeneous or  $E$  is column Hilbert space.

**Samei, Ebrahim (University of Saskatchewan)**

**Title:** Projectivity of modules over Segal algebras

**Abstract:** We will study the projectivity of various natural modules associated to operator Segal algebras of the Fourier algebra of a locally compact group. In particular, we will focus on the question of identifying when such modules will be projective in the category of operator spaces. We will show that projectivity often implies that the underlying group is discrete or even finite. We will also look at projectivity for modules of  $A_{cb}(G)$ , the closure of  $A(G)$  in the space of completely bounded multiplier. Here we give evidence to show that weak amenability of  $G$  plays an important role. This is a joint work with Brian Forrest and Hun Hee Lee.

**Santiago Moreno, Luis (Fields institute, Universitat Autnoma de Barcelona)**

**Title:** Classification of  $*$ -homomorphisms using the Cuntz semigroup functor

**Abstract:** We will show how the homomorphisms from the  $C^*$ -algebra of continuous function on a tree to a  $C^*$ -algebra of stable rank one can be classified by means of the Cuntz functor. In the special case when the tree consists of a single edge we describe a class of codomain  $C^*$ -algebras—not necessarily of stable rank one—for which this classification holds. We will also discuss how in certain cases the classification fails. These results are obtained in a joint work with Alin Ciuperca and George Elliott and in a joint work with Leonel Robert.

**Smith, Roger (Texas A&M University)**

**Title:** Close separable nuclear  $C^*$ -Algebras

**Abstract:** Kadison and Kastler introduced a metric on the set of subalgebras of  $B(H)$  in terms of the Hausdorff distance between unit balls. They conjectured that two sufficiently close  $C^*$ -algebras  $A$  and  $B$  are isomorphic, and further that there is a unitary  $u \in B(H)$  such that  $uAu^* = B$ . Positive resolutions have been obtained for injective von Neumann algebras, and certain classes of separable nuclear  $C^*$ -algebras, notably the AF-algebras and those of continuous trace. In this talk I will describe the full solution of this problem for separable nuclear  $C^*$ -algebras. Separability is essential here since it is known to be false for nonseparable nuclear  $C^*$ -algebras. This is joint work with Erik Christensen, Allan Sinclair, Stuart White and Wilhelm Winter.

**Tikuisis, Aaron (University of Toronto)**

**Title:** The Cuntz Semigroup for Commutative  $C^*$ -algebras

**Abstract:** In the program to classify  $C^*$ -algebras, the Cuntz semigroup has recently emerged as a promising tool in situations where K-theory and traces do not suffice. Yet this ordered semigroup has only been computed in limited situations. In particular, little is known about the Cuntz semigroup for most commutative  $C^*$ -algebras. Attempts to describe it becomes slightly more tractable by using open projections to represent elements of the Cuntz semigroup. I will describe how, using this approach, a picture emerges for algebras  $C(X)$  where  $X$  is a compact metric space of dimension at most three. This is joint work with Leonel Robert.

**Viola, Maria Grazia (Lakehead University)**

**Title:** A simple, separable, exact  $C^*$ -algebra non-isomorphic to its opposite algebra

**Abstract:** We give an example of a simple, separable, exact  $C^*$ -algebra which is not isomorphic to its opposite algebra. The  $C^*$ -algebra  $A$  satisfies a series of nice properties. It is stably finite, approximately divisible, has real rank zero and stable rank one. Also,  $A$  has a unique trace, and the order of projections over  $A$  is determined by traces. We can also explicitly compute the K-theory of  $A$ , namely  $K_0(A) \simeq \mathbb{Z}[\frac{1}{3}]$  and  $K_1(A) = 0$ . Joint work with C. Phillips.

**Wang, Jiun-Chau (Queen's University)**

**Title:** Superconvergence for unbounded free random variables

**Abstract:** We will discuss some new features of the free central limit theorem for identically distributed, unbounded summands. We will present a local estimate for the densities of the distributions in the central limit process and an entropic central limit theorem. The main tool for proving these results is the subordination functions for free additive convolution.

**Yang, Dilian (University of Windsor )**

**Title:** Endomorphisms and Modular Theory of 2-Graph Algebras

**Abstract:** We consider endomorphisms and modular theory of the graph  $C^*$ -algebra of a 2-graph on a single vertex. We prove that there is a semigroup isomorphism between unital endomorphisms of a 2-graph  $C^*$ -algebra and its unitary pairs with a twisted property. We characterize when endomorphisms/automorphisms preserve the fixed point algebra of gauge automorphisms and its canonical masa. As far as the modular theory is concerned, we show that the algebraic  $*$ -algebra generated by the generators of a 2-graph  $C^*$ -algebra with the inner product induced from a distinguished state is a modular Hilbert algebra. We will also discuss the type of the von Neumann algebra generated by its GNS representation.