Pacific Institute for the Mathematical Sciences

West Coast Algebraic Topology Summer School
Syllabus

Pacific Institute for the Mathematical Sciences
Earth Sciences Building
2207 Mail Mall- UBC

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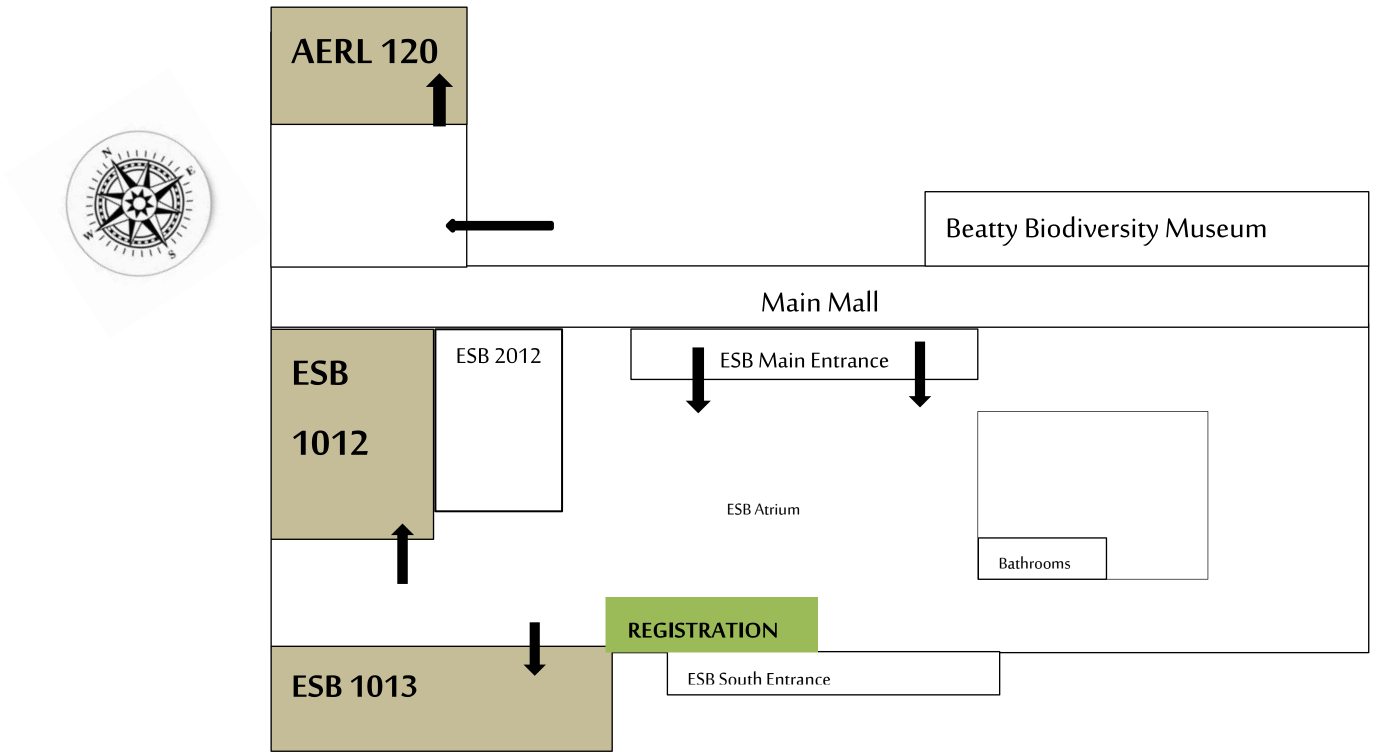
Building Floor Plan:



Mon July 7th :

- 8:30AM: Registration and Check in located at the **ESB Atrium**
- 9:00AM - 12:30PM **ESB 1012**
- 12:30PM- 4:45PM **ESB 1013**

Tue July 8th :

- All day: **AERL 120**

Wed July 9th to **Friday July 11th**:

- 8:30AM - 12:30PM: **ESB 1012**
- 12:30pm - 5:00pm: **ESB 1013**

Day Schedule:

Time	Mon July 7* **AM: ESB 1012** **PM: ESB 1013**	Tue July 8 **AERI 120**	Wed July 9 **AM: ESB 1012**	Thur July 10 **AM: AM: ESB 1012** **PM: ESB 1013**	Fri July 11 **AM: ESB 1012** **PM: ESB 1013**
8:30am-8:55am	Registration and Check in (ESB Atrium)				
9:00am - 10:30am	**Lecture 1.1** Classical actions; path integrals	**Lecture 1.2** 2-dimensional field theories	**Lecture 1.3** Finite gauge theories	**Lecture 1.4** Quantum Chern-Simons theory	**Lecture 1.5** The Verlinde Ring and K-theory
Coffee Break (Lobby)					
11:00am-12:30pm	**Lecture 2.1** Bordism groups and Categories	**Lecture 2.2** Duality	**Lecture 2.3** Extended field Theories	**Lecture 2.4** String topology	**Lecture 2.5** Factorization homology
Lunch (Self Catered)					
2:00pm-3:30pm	**Lecture 3.1** Morse field theory	**Lecture 3.2** The Jones polynomial	Afternoon Excursion	**Lecture 3.3** Khovanov homology	**Lecture 3.4** Heegaard-Floer homology
Coffee Break (Lobby)				Coffee Break (Lobby)	
4:00pm - 5:00pm	Discussion Session‡	Discussion Session		Discussion Session	Wrap Up
5:00pm - 6:00pm	PIMS Public Lecture (ESB 1012)				
6:00pm-7:00pm	Evening Reception** (ESB Atrium)				

*Please note daily room changes

‡ Monday Discussion sessions conclude at 4:45pm

** Please RSVP for the Evening Reception through the PIMS Website : http://www.pims.math.ca/scientific-event/140707-pplfslspd

2014 WCATSS SYLLABUS: TOPOLOGICAL FIELD THEORY

THE ORGANIZERS

There are 3 courses, each consisting of 5 lectures. This syllabus contains a detailed summary of each lecture and suggested literature. Two students will be assigned to prepare each lecture. The lectures are one hour, and in many cases there is more material in the summary than can reasonably fit in an hour, so you will have to make choices. When possible, present explicit examples.

For inspiration and an overview of the subject, we recommend that you read in advance Teleman's lectures on TQFT's (at `http://math.berkeley.edu/~teleman/math/barclect.pdf`) and Freed's expository article on the cobordism hypothesis in the AMS Bulletin (at `http://www.ams.org/journals/bull/2013-50-01/S0273-0979-2012-01393-9/home.html`).

1. Lecture Series I: Gauge theories

1.1. Lecture 1: Classical actions; path integrals

Begin with the action principle in quantum mechanics in the simplest case of a particle moving on the real line $\mathbb{R}$ in a potential $V\colon \mathbb{R}\to\mathbb{R}$. So if the path is $x\colon [0,T]\to\mathbb{R}$, then the action is

$$S(x)=\int_0^T \bigl\{\frac{1}{2}m\dot{x}(t)^2 - V\bigl(x(t)\bigr)\bigr\}\,dt, \tag{1}$$

the kinetic minus potential energy. Show how Newton's law follows. That is the classical picture. Feynman's idea for quantum mechanics is to integrate the *exponentiated* action $e^{iS/\hbar}$ over the space of paths. (Note the inclusion of the constant $\hbar$, which makes the expression in the exponential unitless, which it must be if it is to make sense.)

Now move on to geometric/topological analogs in higher dimension, say spacetime dimension n. *Gauge theory* refers to physical theories which include a connection on a principal G-bundle for some Lie group G. This is a *field*; the field in the mechanics example is the function x. Start with G a finite group, so there is no connection—just a principal bundle. The collection of G-bundles is a *groupoid*, not a set. Discuss how to count: need to sum over π_0 of the groupoid and for each object divide by the cardinality of the group π_1 of automorphisms. An action is an invariant function on the groupoid, such as the counting function. Construct another by weighting by a characteristic class in $H^n(BG;\mathbb{R}/\mathbb{Z})$. (You'll need to exponentiate it.) Compute the sum on a connected compact n-manifold in terms of homomorphisms from the fundamental group to the group G. For $n=2$ we'll see a formula for this in a subsequent lecture.

Date: May 18, 2014.

Next, consider what happens on a compact n-manifold with boundary. There is no fundamental class on which to evaluate the characteristic class. So you'll have to pick a cocycle representing that class, and then work out that the integral lives in a complex line which just depends on the boundary.

Finally, make a similar story using differential geometry and the Chern-Simons functional of a connection on a principal bundle. Begin with a circle bundle ($G = \mathbb{T}$ the circle group of unit norm complex numbers) with connection in $n = 1$ dimensions, where the action is holonomy and for an interval it is parallel transport. Then look into a similar story in $n = 3$ dimensions for the Chern-Simons function for $G = SU_2$. In both cases the principal bundle admits sections.

References

A beautiful account of the principle of least action is in the Feynman lecture. More formal aspects are covered in the first part of Deligne-Freed, which also has a brief review of connections. The appendix of Freed-Quinn has an account of integration of singular cocycles. The classical Chern-Simons functional is the subject of the paper by those authors; for manifolds with boundary, see Freed.

R. Feynman, Lectures on Physics, Volume II, Lecture 19, `http://www.feynmanlectures.caltech.edu/`II_1`9.html`

Pierre Deligne and Daniel S. Freed, *Classical field theory*, Quantum fields and strings: a course for mathematicians, Vol. 1, 2 (Princeton, NJ, 1996/1997), Amer. Math. Soc., Providence, RI, 1999, pp. 137-225.

Daniel S. Freed and Frank Quinn, *Chern-Simons theory with finite gauge group*, Comm. Math. Phys. 156 (1993), no. 3, 435-472, `arXiv:hep-th/911100`.

Shiing Shen Chern and James Simons, *Characteristic forms and geometric invariants*, Ann. of Math. (2) 99 (1974), 48-69.

Daniel S. Freed, *Classical Chern-Simons theory. I*, Adv. Math. 113 (1995), no. 2, 237-303, `arXiv:hep-th/9206021`.

1.2. Lecture 2: 2-dimensional field theories

Lecture 2.1 will introduce the definition of a topological field theory. This lecture, which comes after, gives the standard argument that a (1,2)-dimensional theory on oriented manifolds gives a Frobenius algebra if the target is the category of complex vector spaces. You can also consider the target to be the category of abelian groups and work out that you get a Frobenius ring. The converse is a nice theorem which is proved using Morse theory. As this is the simplest nontrivial theorem which illustrates the techniques underlying some of the subject, give some details of the proof.

Examples: Give a structure theorem for Frobenius algebras over $\mathbb{C}$. Compute in that case the invariant attached to a closed oriented surface—this is the Verlinde formula. A finite group has a Verlinde algebra attached to it, as does a finite group G with a cohomology class in $H^2(BG; \mathbb{R}/\mathbb{Z})$. More explicitly, a class is represented by a central extension $\widetilde{G}$ of G by $\mathbb{R}/\mathbb{Z} \cong \mathbb{T}$, and a basis of the Frobenius ring is provided by representations of $\widetilde{G}$ for which the center $\mathbb{T}$ acts as scalar multiplication. Give an example, say a nontrivial extension of $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$. A non-semisimple example is the cohomology ring of a compact oriented manifold whose cohomology is in even degrees. Examples include complex Grassmannians, say complex projective spaces. The *quantum cohomology ring* is a deformation which encodes Gromov-Witten invariants. That can be mentioned, and perhaps the example of complex projective space can be explained.

The field theories considered so far give invariants for a single manifold at a time. There is an important generalization which gives invariants of families of manifolds. In 2 dimensions this sometimes goes under the name *cohomological field theory*. If there is time, give an introduction to this structure. One possibility is to explain—by extension of the ideas in the first paragraph—that one obtains a BV (Batalin-Vilkovisky) algebra.

References

Many expositions of topological field theory show how to extract a Frobenius algebra from a 2-dimensional field theory. The proof of the converse is in an appendix to the Moore-Segal paper; an exposition of the proof is in the Freed lecture notes. The examples can be worked out without references, though for quantum cohomology see the Givental reference. For families of field theories see the papers of Teleman and Getzler.

G. W. Moore and G. B. Segal, *D-branes and K-theory in 2D topological field theory*, Dirichlet branes and mirror symmetry (Paul S. Aspinwall, Tom Bridgeland, Alastair Craw, Michael R. Douglas, Mark Gross, Anton Kapustin, Gregory W. Moore, Graeme Segal, Balazs Szendroi, and P. M. H. Wilson, eds.), Clay Mathematics Monographs, vol. 4, American Mathematical Society, Providence, RI, 2009, pp. x+681. arXiv:hep-th/0609042.

D. S. Freed, *Bordism: Old and New*, Lecture 23, `http://www.ma.utexas.edu/users/dafr/M392C/index.html`.

A. Givental, *A tutorial on quantum cohomology*, `http://math.berkeley.edu/~giventh/papers/lqc.pdf`.

Constantin Teleman, *The structure of 2D semi-simple field theories*, Invent. Math. 188 (2012), no. 3, 525-588, `http://math.berkeley.edu/~teleman/paperlist.html`.

E. Getzler, *Batalin-Vilkovisky algebras and two-dimensional topological field theories*, Comm. Math. Phys. 159 (1994), no. 2, 265-285.

1.3. Lecture 3: Finite gauge theories

We pick up the idea of a path integral, which was introduced in Lecture 1.1, especially for finite gauge theory. Recall the finite sum path integral from that lecture, the weighting necessary since the collection of G-bundles (G a finite group) is a groupoid, and the answer on a closed n-manifold if the characteristic class in $H^n(BG;\mathbb{R}/\mathbb{Z})$ vanishes.

Now extend the finite path integral to not only sum the action in the top dimension n, but also in dimension $n-1$ where one sums lines to get a vector space. More formally, the action is a line bundle over a groupoid with finite π_0 and finite π_1, and we take the *limit* in the sense of categories and functors. This can be interpreted as the space of invariant sections. Show it is naturally also the *colimit*, which exhibits it as a finite sum, so a finite path integral. This is the starting point for a discussion of *ambidexterity*, but there is no time to explore that in general.

Work out the finite path integral for $n = 2$ to construct the Frobenius algebras described in Lecture 1.1, including the one for a nonzero cocycle. The crucial computation is for a pair of pants, which gives the multiplication, and for a disk, which gives the trace. You should also say something about the $n = 3$ case (which does not involve Frobenius algebras), but the crucial computation is in the next paragraph.

By now the idea of an extended topological field theory will have been introduced. Extend the action and finite path integral to dimension $n-2$. Carry this out in finite gauge theory for $n=2$ and $n=3$. The latter case is a categorified computation of the $n=2$ computations for the pair of pants and disk in the last paragraph. The answer is a category which is very special: a modular tensor category. If the cocycle is zero, then it is the category of G-equivariant vector bundles over G where G acts on itself by conjugation. There is a monoidal product defined by pushforward of vector bundles under multiplication $G \times G \to G$, i.e., by convolution of vector bundles. There is also a braiding. When the cocycle is nonzero the corresponding category consists of twisted G-equivariant bundles over G. These bundles represent twisted equivariant K-theory classes. You may not have time for the twisted case, but at least introduce the modular tensor category for the untwisted case as that is important in the general Chern-Simons theories of Lecture 1.4.

References

Finite gauge theories are introduced in Dijkgraaf-Witten. The paper by Freed introduces the finite path integral and computes the $n=3$ example. Be warned that the underlying category theory is treated very informally (and was not well-developed at the time). A more modern treatment is in the Freed-Hopkins-Lurie-Teleman paper, especially in §3 and §8, and in the lectures by Lurie. Similar field theories can be constructed from finite homotopy types, as in the paper of Turaev.

Robbert Dijkgraaf and Edward Witten, *Topological gauge theories and group cohomology*, Comm. Math. Phys. 129 (1990), no. 2, 393-429.

Daniel S. Freed, *Higher algebraic structures and quantization*, Comm. Math. Phys. 159 (1994), no. 2, 343-398, `arXiv:hep-th/9212115`.

Daniel S. Freed, Michael J. Hopkins, Jacob Lurie, and Constantin Teleman, *Topological quantum field theories from compact Lie groups*, A celebration of the mathematical legacy of Raoul Bott, CRM Proc. Lecture Notes, vol. 50, Amer. Math. Soc., Providence, RI, 2010, pp. 367-403. `arXiv:0905.0731`.

Gijs Heuts and Jacob Lurie, *Ambidexterity*, Topology and Field Theories, ed. Stephan Stolz, American Mathematical Society, 2014, pp. 79–110.

Vladimir Turaev, Homotopy quantum field theory, EMS Tracts in Mathematics, vol. 10, European Mathematical Society (EMS), Zürich, 2010.

1.4. Lecture 4: Quantum Chern-Simons theory

We turn from finite groups to continuous Lie groups. The bottom line is that (conjecturally) for every compact Lie group G and class in $H^4(BG;\mathbb{Z})$ there exists a (1,2,3)-dimensional topological field theory. It assigns a modular tensor category to S^1 and quantum Chern-Simons invariants to closed 3-manifolds. Jones' link invariants are also included. A theorem of Reshetikhin-Turaev gives a rigorous construction of a theory starting from a modular tensor category. A more modern treatment would base the construction on the cobordism hypothesis (Lecture 2.3). An important feature is that these topological field theories are for oriented manifolds with a p_1-structure; the need for the latter is called the "framing anomaly". As far as I know, the story sketched here has not been carried out in this generality.

So in this lecture you will just bite off a small piece, with a focus on the physics origins and the implications of the path integral. For this Witten's paper is the main reference. First recall the Chern-Simons functional from Lecture 1.1. Restrict to $G = SU(2)$ and choose a level $k \in \mathbb{Z}^{>0}$.

($H^4(BSU(2);\mathbb{Z}) \cong \mathbb{Z}$.) Write the path integral over the space of connections on a compact oriented 3-manifold X, emphasizing that the measure is not something we can construct mathematically. In case $X = \mathbb{R} \times Y$ for Y a compact oriented surface, explain informally how "canonical quantization" leads to a finite dimensional complex vector space. Then for a compact oriented 3-manifold with boundary set up the formal path integral to get elements of the vector space of the boundary, so formally indicate how the physics constructions lead to a (2,3)-topological theory F.

You can explain formally how to obtain invariants of knots. Cut out an open tubular neighborhood of a knot $K \subset X$, X a closed oriented 3-manifold, to obtain a manifold with boundary ∂X a torus. If the knot is framed, there is a distinguished isotopy class of oriented diffeomorphisms of the standard torus T with ∂X. So we get invariants in the vector space $F(T)$, and we can fix a basis of $F(T)$ to obtain numerical invariants. Relate them to the Jones' invariants, which will be discussed in Lecture 3.2.

Allow sufficient time (at least half the lecture) to say something nontrivial about the path integral, which is more than a cartoon. What you can do is discuss the asymptotics in the limit $k \to \infty$, where k is the level. You'll need to learn a bit about some important geometric and topological invariants—Reidemeister torsion, spectral flow, η-invariants,. . .—but you needn't explain them in detail in the lecture. Indicate how a Riemannian metric is introduced to define the integral and how the dependence on the metric is traded for a p_1-structure.

References

The rigorous mathematical construction of quantum Chern-Simons theory starting from a modular tensor category is in Turaev's book. It uses special features of 3-dimensional topology; an approach based on Morse theory is in Walker. But this lecture only mentions these mathematical ideas in passing. The main focus is on the physics constructions, for which the main reference is Witten's paper. The free theory ($G = U(1)$) is discussed in Schwarz and is important for the asymptotics. (The precise formula for $G = SU(2)$ is corrected very slightly in Freed-Gompf, which also contains numerical computations which demonstrate that features of the path integral work as advertised.) The p_1-structure as a type of framing is described in Atiyah.

V. G. Turaev, Quantum invariants of knots and 3-manifolds, de Gruyter Studies in Mathematics, vol. 18, Walter de Gruyter & Co., Berlin, 1994.

K. Walker, *On Wittens 3-manifold invariants*. `http://canyon23.net/math/tc.pdf`.

Edward Witten, *Quantum field theory and the Jones polynomial*, Comm. Math. Phys. 121 (1989), no. 3, 351-399.

A. S. Schwarz, *The partition function of degenerate quadratic functional and Ray-Singer invariants*, Lett. Math. Phys. 2 (1977/78), no. 3, 247-252.

Daniel S. Freed and Robert E. Gompf, *Computer calculation of Wittens 3-manifold invariant*, Comm. Math. Phys. 141 (1991), no. 1, 79-117.

Michael Atiyah, *On framings of 3-manifolds*, Topology 29 (1990), no. 1, 1-7.

1.5. Lecture 5: The Verlinde Ring and K-theory

Begin with the dimensional reduction f to 2-dimensions of a quantum Chern-Simons theory F (defined by a compact Lie group G and a level k). So for any 1- or 2-manifold M we have

$$f(M) = F(S^1 \times M). \tag{2}$$

The theory takes values in complex vector spaces and complex linear maps; it is defined on oriented manifolds. (The dimensional reduction is independent of the p_1-structures which are part of the 3-dimensional theory.) From Lecture 1.2 we know that this dimensional reduction is given by a commutative Frobenius algebra over $\mathbb{C}$. This algebra is called the *Verlinde algebra*. It refines to a ring over $\mathbb{Z}$. Describe it for $G = SU(2)$: it is a quotient of the representation ring. You can also describe it for a finite group G from the picture at the end of Lecture 1.3: if the level in $H^3(BG; \mathbb{R}/\mathbb{Z})$ vanishes, then it is the equivariant K-theory group $K_G(G)$ with convolution product (which is pushforward under multiplication $G \times G \to G$). For nonzero level it is a twisted version.

In the second part of the lecture you can summarize a construction for general compact Lie groups G which uses a topological version of the path integral ideas described in Lecture 1.4. A similar idea is used in Lecture 3.1. The "integral" is done in topology as a wrong-way map, or pushforward, and for that one needs orientations, which are the topological analog of the measure needed to execute the path integral as an honest integral. The obstruction to *consistent* orientations is itself a topological field theory—a very special *invertible* theory—and this obstruction vanishes. You should sketch the push-pull construction of the (1,2)-theory, which takes values in the category of abelian groups, and also explain how the orientations work.

References

The reference for the second part of the lecture is FHT. For invertible field theories the main mathematical theorem is in GMTW, but you will not have time to cover that. Verlinde's original paper describes the $SU(2)$ case. There are many other references for that; Chapter 22 of the Husemöller et. al. book is a recent one with more details.

Daniel S. Freed, Michael J. Hopkins, and Constantin Teleman, *Consistent orientation of moduli spaces*, The many facets of geometry, Oxford Univ. Press, Oxford, 2010, pp. 395-419. arXiv:0711.1909.

Soren Galatius, Ulrike Tillmann, Ib Madsen, and Michael Weiss, *The homotopy type of the cobordism category*, Acta Math. 202 (2009), no. 2, 195-239, arXiv:math/0605249.

Erik Verlinde, *Fusion rules and modular transformations in 2D conformal field theory*, Nuclear Phys. B 300 (1988), no. 3, 360-376.

D. Husemöller, M. Joachim, B. Jurčo, and M. Schottenloher, Basic bundle theory and K-cohomology invariants, Lecture Notes in Physics, vol. 726, Springer, Berlin, 2008. With contributions by Siegfried Echterhoff, Stefan Fredenhagen and Bernhard Krötz.

2. Lecture Series II: General topological field theories and the cobordism hypothesis

This lecture series explores the general setup of extended topological field theories, including a lecture on duality and one on the cobordism hypothesis, and then lectures on extended examples, including string topology and factorization homology.

2.1. Lecture 1: Bordism groups and categories

Goals:

- Motivate and define the bordism category, and at least indicate some of its variants.
- Give Atiyah-Segal's definition of an nTFT.
- Give some simple examples of TFT's in low dimensions.

Fix a dimension n. Review the definition of the the Euler characteristic, and of the signature, of a compact $(n-1)$-manifold. Motivate the consideration of these numbers as *invariants* of compact $(n-1)$-manifolds. Define what a cobordism between two compact $(n-1)$-manifolds is, and show that it defines an equivalence relation on compact $(n-1)$-manifolds that is coarser than diffeomorphism – give some attention to the collar data required to glue cobordisms. Explain that the above constructed numbers are invariant with respect to cobordism. Explain that disjoint union makes the resulting set of equivalence classes into a commutative monoid $\Omega^{\mathsf{un}}_{n-1}$; and in fact this commutative monoid is a group – emphasize the geometric procedure for constructing inverses. Point out that Euler characteristic and signature can be viewed as homomorphisms from cobordism groups. For $\mathsf{B} \xrightarrow{\Theta} \mathsf{BO}(n)$ a fibration, give/indicate the Θ-version of the cobordism group Ω^{Θ}_{n-1}. (Maybe state that Ω^{Θ}_{n-1} is the $(n-1)^{st}$ homotopy group of a spectrum (Thom-Pontrjagin theory).)

Motivate a Topological Field Theory as an invariant of compact $(n-1)$-manifolds that is not, per se, invariant with respect to cobordism, but functorial with respect to cobordism; and state Atiyah's definition of an nTFT. This will amount to defining the cobordism category: an object is a compact $(n-1)$-manifold and the set of morphisms from M to M' is

$$\{\overline{W}^n, \partial\overline{W} \underset{\alpha}{\cong} M \sqcup M'\}_{/\mathsf{Diff}(\overline{W},\partial\overline{W})} \ .$$

– cobordisms between M and M', up to isomorphism. Composition is given by gluing along common boundary – give some attention to the collar data required to compose morphisms. Disjoint union gives a symmetric monoidal structure on this category. Observe that every object in this bordism category has a dual – point out that this amounts to the same geometric procedure for noticing that the monoid of cobordism classes was a group. Give/indicate the Θ-version of this cobordism symmetric monoidal category, which might be denoted as Cob^{Θ}_n.

Define an nTFT as a symmetric monoidal functor $\mathsf{Cob}^{(\Theta)}_n \to \mathcal{C}$. Here are two easy examples that could be mentioned.

(1) Consider the category of vector spaces over $\mathbb{C}$ and linear maps among them, with symmetric monoidal structure given by tensor product (over $\mathbb{C}$). Explain how a smooth vector bundle $E \to X$ equipped with a *flat* connection gives a functor $\mathsf{Cob}^X_1 \to \mathsf{Vect}$ given on objects as $x \mapsto V_x$ and on morphisms as $([-1,1] \xrightarrow{\gamma} X) \mapsto V_{\gamma(-1)} \xrightarrow{P_\gamma} V_{\gamma(1)}$ where P is the parallel

transport operator associated to the flat connection, and on $(S^1 \xrightarrow{\gamma} X)$ as the monodromy of γ.

(2) Consider the category whose objects are rings, and whose morphisms from R to S are $R-S$-bimodules, with composition given by mutual tensor product; and with symmetric monoidal structure given by tensor product (over $\mathbb{Z}$) of rings. Explain how a ring R determines a 1TFT valued in this category by the assignments $* \mapsto R$ and $S^1 \mapsto \mathsf{Trace}(R)$ – the largest quotient of R on which left and right multiplication coincide.

Point out that there is a naturally defined symmetric monoidal Top-enriched category whose (morphism-wise) set of components gives this cobordism category just constructed. Namely, the *space* of morphisms from M to M' is (a model for)

$$\coprod_{[\overline{W}^n, \partial\overline{W} \underset{\alpha}{\cong} M \sqcup M']} \mathsf{BDiff}(\overline{W}; \partial\overline{W}) \ .$$

Give/indicate the Θ-version of the symmetric monoidal Top-enriched cobordism category. Explicate the $n = 1$ case, and the oriented $n = 2$ case as it relates to moduli spaces of Riemann surfaces.

Point out that π_0 on morphisms gives a functor from this enriched cobordism category to the unenriched one. Indicate that, without the flatness assumption on the connection, example (1) gives a *continuous* 1TFT (for this, use that Vect is enriched over itself, and there is the map of enrichments $\mathsf{Top} \to \mathsf{Vect}$ given by $C^*(-;\mathbb{C})$); and that it descends to the unenriched cobordism category if and only if the connection is flat. Likewise, indicate that example (2) lifts to a *continuous* 1TFT by improving the value on S^1 to be HH_*R, the Hochschild homology of R, with rotations acting by Connes B operator. (More interesting examples of nTFT's is the subject of this workshop.)

One might regard the above symmetric monoidal Top-enriched category as a symmetric monoidal $(\infty, 1)$-category. *Indicate* the symmetric monoidal (∞, n)-category of bordisms, by drawing pictures of manifolds with corners. Draw pictures to indicate that, not only does every object have a dual, but every k-morphism has a left and a right adjoint, provided $0 < k < n$. Indicate a Θ-version of this symmetric monoidal (∞, n)-category.

References

Background for this talk will be familiarity with basic definitions in category theory, and with classical results in differential topology. A good reference for most of this material is Freed's course notes. A good reference for cobordism groups, equipped with Θ-structures is Stong's book. Atiyah-Segal's axioms for a TQFT is a standard one for the cobordism category. A careful account of the (symmetric monoidal) topological category Cob_n (as well as Cob_n^{Θ}) can be found in Galatius-Madsen-Tillmann-Weiss. For the (∞, n)-version, $\mathsf{Bord}_n^{(\Theta)}$, one could look at Lurie.

D. S. Freed, *Bordism: Old and New*, `http://www.ma.utexas.edu/users/dafr/M392C/index.html`.

Stong, Robert E. Notes on cobordism theory. Mathematical notes Princeton University Press, Princeton, N.J.; University of Tokyo Press, Tokyo 1968

M. Atiyah, *Topological quantum field theories*, Inst. Hautes Études Sci. Publ. Math. (1988), no. 68, 175-186 (1989).

Soren Galatius, Ulrike Tillmann, Ib Madsen, and Michael Weiss, *The homotopy type of the cobordism category*, Acta Math. 202 (2009), no. 2, 195-239, arXiv:math/0605249.

Jacob Lurie, *On the classification of topological field theories*, Current developments in mathematics, 2008, Int. Press, Somerville, MA, 2009, pp. 129-280. arXiv:0905.0465.

2.2. Lecture 2: Duality

This lecture starts with the abstract notion of dualizability, and then gives several concrete examples illustrating the notion. Begin with the familiar definition of adjoint functors between categories in terms of hom sets and show this is equivalent to the definition in terms of unit and counit. Introduce the classical theory of dualizability in a symmetric monoidal category, and give examples of dualizable objects in the categories of spectra and chain complexes. State some of the standard properties of dualizable objects.

Extend to a notion in any bicategory, and illustrate with the example of a monoidal category (one-object bicategory). Give a relevant concrete example of this, such as the fact that the dualizable objects in vector spaces are the finite dimensional vector spaces. Explain the relationship between the bicategorical perspective and the classical theory.

Next explain some basic facts about duals/adjoints. Prove that if duals exist, then they are unique. It would be good to formulate this as an equivalence between certain categories. Illustrate that in a symmetric monoidal category, the left dual is the same as the right dual. Conclude the classical fact that for finite dimensional vector space there is a canonical isomorphism $V \cong V^{**}$.

Now introduce the notion of *fully-dualizable object* in an n-category. Here, due to time constraints, we have to be a little vague on the precise definition of n-category. The point is to emphasize the fact that out of an n-category you can extract ordinary bicategories by throwing out certain non-invertible morphisms and the passing to quotients. Then fully-dualizable objects are defined in terms of the dualizability properties of these associated bicategories.

The main example of a category which is fully-dualizable is the bordism category introduced in Lecture 2.1. At this point it is probably enough to describe the oriented bordism n-category and show it is fully-dualizable. Another interesting example is to consider the dualizability properties of the 2-category of algebras, bimodules, and maps.

References

The basic properties about adjoints are describe in the chapter on adjunctions in Mac Lane's book. A good reference for the classical theory of dualizability is Chapter III, section 1 of LMS. Most of the rest is discussed in Schommer-Pries' notes on dualizability. The dualizability in the category of algebras, bimodules, and maps is discussed in some detail in Schommer-Pries *The Classification of ...* in one of the appendices.

S. Mac Lane *Categories for the working mathematician* Graduate Texts in Mathematics, 5. Springer-Verlag, New York, 1998.

G. Lewis, J.P. May, and M. Steinberger *Equivariant stable homotopy theory*, Springer Lecture Notes in Mathematics 1213, 1980. `http://www.math.uchicago.edu/$\sim$may/BOOKS/equi.pdf`

C. Schommer-Pries *Dualizability in Low-Dimensional Higher Category Theory*, Topology and Field Theories, Contemp. Math. , vol. 613, Amer. Math. Soc., Providence, RI, 2014. ArXiv:math/1308.3574

C. Schommer-Pries *The Classification of Two-Dimensional Extended Topological Field Theories*. Ph. D. Dissertation, UC Berkeley. ArXiv:math/1112.1000

2.3. Lecture 3: Extended field theories and the cobordism hypothesis

By now several things will have been introduced in other lectures. The bordism category will have appeared, as well as the definition of a TQFT as a functor. Also in the previous lecture of this series the notion of fully-dualizable will have been introduced. We are all ready for the cobordism hypothesis.

Start be recalling the notion of fully-dualizable symmetric monoidal n-category. Then recall the bordism n-category. This might not of been done very precisely yet, so try to give some details here. It might be a good idea to focus on dimension two. You will definitely want to introduce several variants of the bordism category corresponding to different tangential structures: oriented, spin, tangentially framed, G-principal bundles, maps to X, etc. Show that the bordism category is fully-dualizable.

Now you can state the cobordism hypothesis. The first version states that the tangentially framed bordism n-category is free on a fully-dualizable object. Make sure to explain this in words and to also write a more mathematically precise statement. Describe how the n-groupoid of fully-dualizable objects gets a (homotopy) $O(n)$-action, and then state the second version of the cobordism hypothesis (for tangential structures). This is in terms of homotopy fixed point spaces.

The next goal will be to try to go into some detail in dimension 2. Look specifically at the 2D framed bordism 2-category. This is a nice example where the left duals and right duals of morphisms are not equal. Consider TQFT's with values in the 2-category of algebras, bimodules, and maps. The fully-dualizable objects in this 2-category will have been introduced in previous lectures. Note that there are very few framed surfaces, and so we want an oriented theory.

This means we want to trivialize the $SO(2)$-action. The best way to think of a homotopy $SO(2)$-action is as a map $\mathbb{CP}^\infty = BSO(2) \to BAut(C)$ where C is the n-groupoid/space of fully-dualizable objects. We can trivialize the action cell by cell. There is only one interesting one in these dimensions, the 2-cell of $BSO(2)$ which gives rise to a natural automorphism of the identity functor of C. This is called the Serre automorphism.

Give the formula for the Serre automorphism in terms of basic duality data. Then look at it in the 2-framed bordism category. It is the interval as a bordism from the point to the point, but where the framing has been twisted once around. Next compute it in the case of algebra bimodules and maps. Show that for an algebra A, the Serre is given by the bimodule ${}_A\hat{A}_A$, where $\hat{A} = Hom(A, k)$ is the linear dual. A trivialization of the Serre, $\lambda : {}_A\hat{A}_A \cong {}_AA_A$, is the same as a symmetric Frobenius algebra. Maybe finish by describing the value of this field theory on several bordisms, and tie it back into Lecture 1.2.

References

The main reference for this lecture is Lurie. The cobordism hypothesis is stated in §2.4; the Serre automorphism is discussed in §4.2. There is more on these topics, including the Serre automorphism, in Schommer-Pries and in Teleman. Davidovich gives a detailed treatment of dualizability and $SO(2)$-invariance for algebras.

Jacob Lurie, *On the classification of topological field theories*, Current developments in mathematics, 2008, Int. Press, Somerville, MA, 2009, pp. 129-280. arXiv:0905.0465.

C. Schommer-Pries *Dualizability in Low-Dimensional Higher Category Theory*, Topology and Field Theories, Contemp. Math. , vol. 613, Amer. Math. Soc., Providence, RI, 2014. ArXiv:math/1308.3574

Constantin Teleman, *Five lectures on topological field theory*, `http://math.berkeley.edu/~teleman/math/barclect.pdf`

Orit Davidovich, *State sums in two-dimensional fully extended topological field theories*, `http://repositories.lib.utexas.edu/bitstream/handle/2152/ETD-UT-2011-05-3139/DAVIDOVICH-DISSERTATION.pdf`

2.4. Lecture 4: String topology

This lecture begins with an introduction to string topology operations via the definition of Chas and Sullivan's commutative

$$\mathbb{H}_*(LM) \otimes \mathbb{H}_*(LM) \xrightarrow{\bullet} \mathbb{H}_*(LM)$$

loop product for the free loop space LM of a closed, orientable manifold M. Explain the product geometrically, as it was originally presented in Chas-Sullivan *String topology*, as well as via the homotopy theoretic realization as given in Cohen-Jones *A homotopy theoretic realization of string topology*.

Explain the algebraic structure the loop product is an aspect of; i.e., a *Batalin-Vilkovisky* algebra structure on $\mathbb{H}_*(LM)$, reflecting the action of the framed little disks operad. Briefly describe the relationship to Hochschild cohomology and homology of the cochains of M or chains of ΩM, respectively.

A major component of the lecture should be devoted to an explicit description of the constructions of more generalized *string topology operations* $H_*(LM)^{\otimes k} \to H_*(LM)^{\otimes \ell}$ of Cohen and Godin in *A polarized view of string topology*. With the Cohen-Jones version of the loop product in mind, Cohen-Godin produce these from *Sullivan chord diagrams* and show that

- two Sullivan chord diagrams connected by a path give rise to the same string topology operation, and
- the space of Sullivan chord diagrams with a fixed genus and with a fixed number of inputs and a fixed number of outputs is path connected.

Therefore, the collection of string topology operations arising from Sullivan chord diagrams forms a *positive boundary* TQFT.

Explain how string topology fits into the framework of the cobordism hypothesis, perhaps expanding on remark 4.2.16 in Lurie's notes. Perhaps also mention Costello's work on open-closed field theories in relation to this discussion.

The lecture should then include at least a short discussion of Godin's and Kupers' later results which generalize the Cohen-Godin TQFT using diagrams related to the *moduli space of Riemann surfaces*, giving rise to a *homological conformal field theory*. (The lecturer may wish to include details of these generalizations if interested and if time permits.)

A nice outline appears in Blumberg, Cohen, and Teleman's *Open-closed field theories, string topology, and Hochschild homology*.

References

Moira Chas and Dennis Sullivan, *String Topology*, arXiv:math/9911159

Ralph Cohen and J.D.S. Jones, *A homotopy theoretic realization of string topology*, Math. Annalen, vol 324. 773-798 (2002), arXiv:math/0107187

Veronique Godin and Ralph Cohen, *A polarized view of string topology* Topology, Geometry, and Quantum Field theory, Lond. Math. Soc. lecture notes vol. 308 (2004), 127-154, arXiv:math/0303003

Veronique Godin, *Higher string topology operations*, arXiv:0711.4859

Sander Kupers, *Constructing higher string operations for manifolds using radial slit configurations*, `http://math.stanford.edu/~kupers/radialslitoperationsnew.pdf`

Andrew Blumberg, Ralph Cohen, and Constantin Teleman, *Open-closed field theories, string topology, and Hochschild homology*, arXiv:0906.5198

2.5. Lecture 5: Factorization homology and E_n-algebras

This lecture will introduce E_n algebras and factorization homology (also known as topological chiral homology). Factorization homology is a lot like a homology theory: it takes in a space M (in this case, a framed n-manifold) and an algebraic object A acting as "coefficients" (in this case, an E_n algebra in a symmetric monoidal category $\mathcal{C}$), and it outputs an object $\int_M A$ in $\mathcal{C}$. From the perspective of this workshop, factorization homology is useful because it gives a construction of a large class of extended TFTs.

The talk should start by explaining how associative algebras can be viewed as "1-dimensional algebra," i.e., living on the real line. Following the notation in Francis' paper, let $Mfld_1^{fr}$ denote the category whose objects are one-dimensional manifolds (without boundary) and with finitely many connected components and whose morphisms are framed embeddings. Let $\mathcal{C}^{\otimes}$ denote a symmetric monoidal category. (Feel free to let the category be vector spaces with tensor product, or topological spaces with Cartesian product.) Given an associative algebra A in $\mathcal{C}$, we can construct a symmetric monoidal functor $F_A : Mfld_1^{fr} \to \mathcal{C}$ that sends an interval with n components to $A^{\otimes n}$. The embedding of two disjoint intervals into a single interval is sent by F_A to the multiplication map $m : A \otimes A \to A$. (You've got to keep track here of "order of the intervals" by using the framing.) The associativity will appear as the different ways of "factoring" the inclusion of the three intervals into a single interval. Allowing manifolds with boundary, you can incorporate bimodules.

A natural question is what you should assign to the circle. You can explain that you should guess F_A assigns $A/[A, A]$ because there are two ways of including two disjoint intervals in a circle into a big interval. (One way amounts to the multiplication ab and the other to ba.) You can explain that $F_A(S^1) \cong A \otimes_{A \otimes A^{op}} A$ is a tensor analog of the excision axiom for a homology theory. You should also mention that this procedure is interpretable as a 1-dimensional framed TFT with values in the Morita category (whose objects are associative algebras and whose morphisms are bimodules, up to bimodule isomorphism).

The next question to address is what the generalization to framed n-manifolds should be. It would be natural here to give the definition of an E_n algebra. The model of little n-cubes sitting inside a big n-cube is a nice generalization of the 1-dimensional pictures. (You should draw lots of pictures.) The motivating example of an E_n algebra (in topological spaces) is the n-fold based loop space $\Omega^n X$ of a based space X. You should mention May's recognition theorem, that a grouplike space Y equipped with an E_n algebra structure is homotopy equivalent to $\Omega^n X$ for some space X.

Once you have the notion of an E_n algebra, you can define factorization homology $\int_M A$ for M a framed n-manifold and A an E_n algebra. You can now introduce a "homology theory for framed n-manifolds," following Francis, and explain the theorem that there there is an equivalence between such homology theories and E_n algebras. (It's good to emphasize the analogy with the Eilenberg-Steenrod theorem.)

To conclude, you can explain, following Lurie's exposition, how this procedure also gives you a framed n-dimensional TFT. The tricky part here is to sketch the target (∞, n)-category, which is an E_n version of the Morita category. The idea is easy to communicate with pictures (focus on $n = 2$). The nonabelian Poincare duality theorem (of Salvatore and Lurie) gives a different and useful description of the values of factorization homology in special cases.

References

Francis, "Factorization homology of topological manifolds" `http://arxiv.org/abs/1206.5522`
Ginot, "Notes on factorization algebras, factorization homology and applications", `http://arxiv.org/abs/1307.5213`
Lurie, "On the classification of topological field theories", section 4.1 "Topological Chiral Homology" `http://www.math.harvard.edu/~lurie/papers`
Lurie, *Higher Algebra*, Chapter 5 "Little Cubes and Factorizable Sheaves", `http://www.math.harvard.edu/~lurie/papers`
Scheimbauer, "The higher category of E_n algebras and factorization homology as a fully extended TFT," `http://www.math.ethz.ch/~clausche/LesDiablerets.pdf`

3. Lecture Series III: Heegaard-Floer homology and related topics

3.1. Lecture 1: Morse field theory

Goals:

- Construct a 1TFT (with singularities (think, "graphs")) from the datum of a compact smooth manifold X; but acknowledge that this is more a slogan than an articulate construction. Show that this 1TFT organizes familiar operations on the cohomology of X, such as cup product and the Euler class.
- Express that this is to be taken as a toy model for other, higher dimensional, TFT's, such as Gromov-Witten theory, and other Floer theories – do this by emphasizing the structural aspects of this toy example.

Fix a field k. Fix a closed smooth n-manifold X. We will construct familiar operations on the cochains $C^*(X) := C^*(X; k)$:

Consider a finite directed graph Γ, and construct the moduli space $\mathcal{M}_\Gamma$ of graphs equivalent to Γ – it has the homotopy type of $\mathsf{BAut}(\Gamma)$. Construct the space $\widetilde{\mathcal{M}}_\Gamma(X)$ for which a point is roughly:

- a distinct Morse function f_e for each edge e of Γ,
- a continuous map $|\Gamma| \to X$;

such that each $\gamma_{|e}$ is a Morse flow line with respect to f_e, and each $\gamma(v)$ is a critical point for each of the Morse functions labeling v's emanating edges – define this as efficiently as you can manage, perhaps being light on certain details. Modding out by reparametrizations of each flow line gives the *moduli space* $\mathcal{M}_\Gamma(X)$. Evaluation at the incoming and outgoing leaves gives a span of abelian groups

$$C_*(\mathcal{M}_\Gamma) \otimes C_*(X)^{\otimes p} \xleftarrow{(\pi_p)_*} C_*(\mathcal{M}_\Gamma(X)) \xrightarrow{(\pi_q)_*} C_*(X)^{\otimes q} \ . \tag{1}$$

Fiber integration along π_p gives

$$(\pi_p)^! \circ (\pi_q)_* \colon H_*(\mathcal{M}_\Gamma) \otimes H_*(X)^{\otimes p} \to H_*(X)^{\otimes q} \ .$$

This fiber integration can be achieved in a few ways – for instance using Pontrjagin collapse maps – so you can choose a way that you prefer. Concatenating graphs along common leaves makes these maps into a functor from the cobordism category of directed graphs, into abelian groups, whose value on $*$ is $H_*(X)$. Explain all this, concisely, and explain how familiar operations on $H_*(X)$ are constructed through this.

References

This is a lot of material to pack into one lecture, and it would be impossible to do this thoroughly. So you should emphasize the structural aspects of the constructions of these maps (1). Be impressionistic as necessary, without getting hung-up on technical details. In doing this, err toward giving precise definitions and statements, over explaining proofs/arguments.

The moduli spaces $\mathcal{M}_\Gamma(X)_{(x_1,\ldots,x_p;y_1,\ldots,y_q)}$ admit compactifications as compact manifolds with corners. This is an interesting, and important, aspect – make a choice on whether or not to omit or include this, and if so, explain how the above operations can be viewed in terms of integration over this compact manifold with corners.

Cohen-Norbury: `http://arxiv.org/pdf/math/0509681v1.pdf`

3.2. Lecture 2: The Jones polynomial

Goals:

- Define the Jones polynomial. Gives some computations of it, and express that it is a good knot invariant.
- Define Khovanov homology of a knot, and explain in what sense it recovers the Jones polynomial.

Explain what a knot projection is. Explain what the Reidemeister moves are, and that any two projections of a knot are connected through Reidemeister moves; in this way knot invariants can be constructed from knot projections.

Define the Jones polynomial, as it assigns to each knot projection a polynomial. The fastest way to do this is using the Kauffman bracket; a more involved way to do this is through Temperley-Lieb algebra representations of braid groups, using that every knot is a trace of a braid (a result of Alexander) – choose which presentation to give. Indicate/state that this polynomial is an invariant of the isotopy class of the knot (not just the projection). Indicate, or at least state, some computations of the Jones polynomial, and give examples of knots that it distinguishes.

Motivate (however desired, for instance for gained functoriality) "categorification", which is to say, identifying the Jones Polynomial as the Euler characteristic of a Knot homology theory. Define Khovanov homology – this will amount to a tour through some homological algebra. Explain/indicate why it too is a knot invariant (not just an invariant of knot projections). Explain/indicate why Khovanov homology indeed categorifies the Jones Polynomial.

References

This is a lot of material to pack into one lecture, and it would be impossible to do this thoroughly. So you should emphasize the conceptual aspects of the construction of Khovanov homology; there will be more in the next lecture. Be impressionistic as necessary, without getting hung-up on technical details during the lecture, though you should learn as many as possible while preparing the lecture. In the lecture, err toward giving precise definitions and statements, over explaining proofs/arguments.

There are a number of expositional choices that can be made. Do this as you wish, keeping in mind the goals (and the time constraint, of course). When making choices on what to not include, err on giving precise definitions, but not on verifying various statements.

The two Bar-Natan references are easy to go through, especially for definitions, but beware that they are missing important conceptual aspects. For these aspects, see the Jones (for the Jones polynomial) and Khovanov (for the Khovanov homology) references. The Khovanov reference is very thorough, and you are encouraged to use parts of this, but do your best to extract the structural aspects.

Bar-Natan on Khovanov homology computations (a good survey),
`http://arxiv.org/pdf/math/0201043.pdf`
Bar-Natan on Khovanov homology and surface-tangles, `http://arxiv.org/pdf/math/0410495v2.pdf`
Jones, `http://math.berkeley.edu/~vfr/jones.pdf`
Khovanov, `http://arxiv.org/pdf/math/9908171v2.pdf`

3.3. Lecture 3: Khovanov homology

Goals:

- Explain that Khovanov homology is a knot invariant (not just an invariant of a knot diagram). Explain that it can be phrased as a sort of 2TFT.
- Indicate that it is related to Chern-Simons theory.

Define the Cobordism category of surfaces in $\mathbb{R}^4$. Explain what is meant by stating that Khovanov homology is a functor from this category. Explain/indicate why Khovanov homology is such a functor. In short, explain the statements of Theorems 4/5 and 6 of the Bar-Natan account "... surface-tangles", and indicate their proofs.

If you have time, and can understand it, explain Witten's statement (not with proof) on the relationship between Khovanov homology and Chern-Simons theory.

References

This is a lot of material to pack into one lecture, and it would be impossible to do this thoroughly. So you should emphasize the formal/structural aspects of the "2TFT". Be impressionistic as necessary, without getting hung-up on technical details. In doing this, err toward giving precise definitions and statements, over explaining proofs/arguments. Note that Khovanov homology is introduced in the previous lecture.

You could follow Bar-Natan's account on "... surface-tangles". Try to frame the lecture to give precise statements of the main results, and merely indicate how the arguments go that prove these results.

Bar-Natan on Khovanov homology computations (a good survey),
`http://arxiv.org/pdf/math/0201043.pdf`
Bar-Natan on Khovanov homology and surface-tangles, `http://arxiv.org/pdf/math/0410495v2.pdf`
Jones, `http://math.berkeley.edu/~vfr/jones.pdf`
Khovanov, `http://arxiv.org/pdf/math/9908171v2.pdf`
Witten, "QFT and the Jones polynomial",
`http://projecteuclid.org/download/pdf_1/euclid.cmp/1104178138`
Witten, "Khovanov homology and gauge theory", `http://arxiv.org/pdf/1108.3103v2.pdf`

3.4. Lecture 4: Heegaard-Floer homology

Goals:

- Define Heegaard-Floer homology, and its variations. This will amount to a tour through a lot of alternative definitions.
- Give/state some computations, and express that it is a good invariant of 3-manifolds.

Explain that every oriented closed 3-manifold admits a Heegaard splitting. Use this to give a procedure for constructing 3-manifolds from the data of an oriented closed surface (of genus g) and a pair of g isotopy classes of disjoint essential circles in the surface, which generate the surface's homology.

Given such data, consider the g-fold symmetric product of the surface. Explain/indicate why it is equipped with a canonical symplectic structure, with respect to which each member of the pair of g isotopy classes determines a Lagrangian, and these intersect transversely. Construct a chain complex whose underlying graded module is freely generated by the intersection points of transverse representatives of circles in the surface; and whose differential is given by counting moduli spaces of holomorphic strips. If you wish/have time, describe Lipschitz's combinatorial formula for such counting. Define the Heegaard Floer homology of an oriented closed 3-manifold as the homology of this chain complex – this amounts to defining all of its variants ($\widehat{HF}$, HF^+, etc.). Indicate that this group is independent of choices. Gives/state some explicit calculations.

References

This is a lot of material to pack into one lecture, and it would be impossible to do this thoroughly. So you should emphasize the fundamental steps in the construction of HF (and its variants). Be impressionistic as necessary, without getting hung-up on technical details. In doing this, err toward giving precise definitions and statements, over explaining proofs/arguments. It might be helpful to break arguments/constructions/definitions into steps.

A good reference for most of this is Ozsvath-Szabo's account, especially for explicitness. You will be wise to trim the technicalities surrounding moduli spaces of homomorphic disks; specifically about the well-definedness of these moduli spaces on the choices of isotopy classes of simple closed curves. Likewise for Betti number assumptions. Juhasz gives another account that is somewhat more conceptual/structural, but more involved. Juhasz gives a nice historical framing of HF.

Juhasz, `http://arxiv.org/pdf/1310.3418v3.pdf`
Ozsvath-Szabo, `https://web.math.princeton.edu/~szabo/clay.pdf`

3.5. Lecture 5: Gluing for Heegaard-Floer homology

Goals:

- Give some applications of Heegaard-Floer homology.
- Define the map $HF(Y) \to HF(Y')$ associated to a cobordism between Y and Y', reminiscent of a 4TFT.
- Define HF of a compact 3-manifold with boundary. Define HF of a surface. Explain how HF glues, and point out that this gluing expression is reminiscent of a 3TFT valued in the Morita category; and so as a partially extended 4TFT.

- Indicate how HF is related to other TFT's, such as Donaldson theory, and perhaps to Khovanov homology.

Elaborate on a few applications of Heegaard homology.

Explain how to get 4-manifold invariants from HF. Indicate how they are related to Seiberg-Witten invariants, which detect smooth structures on 4-manifolds.

Indicate the definition of Heegaard-Floer homology for oriented 3-manifolds with boundary ('bordered' HF). Define Heegaard-Floer homology on surfaces, the output which is an A_∞-algebra. Explain how $HF(\partial\overline{M})$ acts on $HF(\overline{M})$. Explain the statement

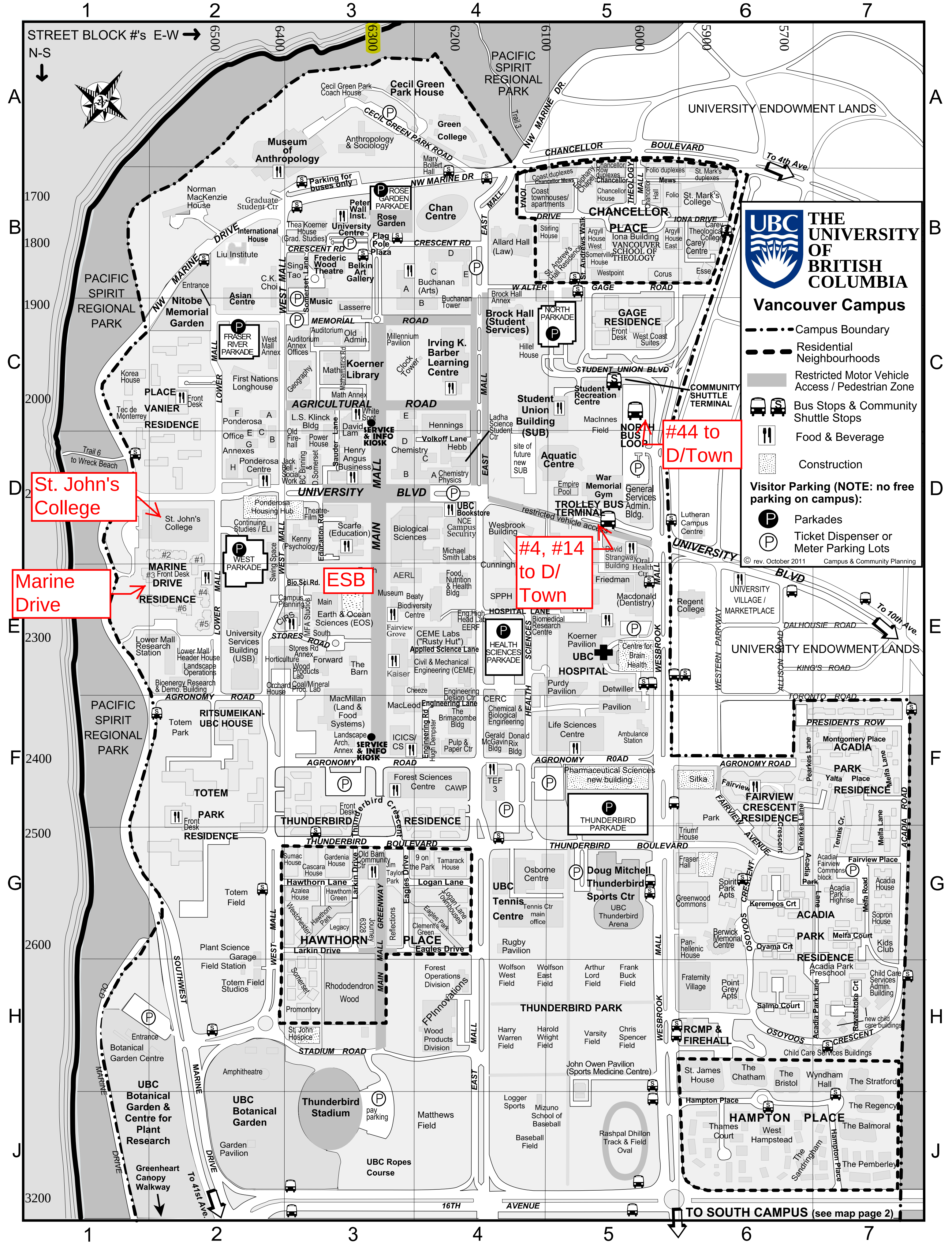
$$HF(\overline{M}_1) \bigotimes_{HF(\partial)} HF(\overline{M}_2) \simeq HF(\overline{M}_1 \coprod_{\partial} \overline{M}_2).$$

References

This is a lot of material to pack into one lecture, and it would be impossible to do this thoroughly. So you should emphasize the construction of the A_∞-algebra associated to a surface. Be impressionistic as necessary, without getting hung-up on technical details. In doing this, err toward giving precise definitions and statements, over explaining proofs/arguments. It might be helpful to break arguments/constructions/definitions into steps.

The applications and the 4TFT aspects can be found in Juhasz's survey. The 3TFT aspects can be extracted from the Lipshitz works; for this be impressionistic, emphasizing the formal structures rather than the immense details of the construction and proof of the tensor-product rule.

Lipshitz-Ozsvath-Thurston, `http://arxiv.org/pdf/0810.0687.pdf`
Juhasz, `http://arxiv.org/pdf/1310.3418v3.pdf`
Ozsvath-Szabo, `https://web.math.princeton.edu/~szabo/clay.pdf`


STREET BLOCK #'s E-W
N-S
THE UNIVERSITY OF BRITISH COLUMBIA
Vancouver Campus
Campus Boundary
Residential Neighbourhoods
Restricted Motor Vehicle Access / Pedestrian Zone
Bus Stops & Community Shuttle Stops
Food & Beverage
Construction
Visitor Parking (NOTE: no free parking on campus):
Parkades
Ticket Dispenser or Meter Parking Lots
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Campus & Community Planning
St. John's College
Marine Drive
ESB
#44 to D/Town
#4, #14 to D/ Town
PACIFIC SPIRIT REGIONAL PARK
UNIVERSITY ENDOWMENT LANDS
Museum of Anthropology
Cecil Green Park House
Green College
CHANCELLOR BOULEVARD
To 4th Ave.
NW MARINE DR
CHANCELLOR PLACE
VANCOUVER SCHOOL OF THEOLOGY
Chan Centre
Rose Garden
Koerner Library
Irving K. Barber Learning Centre
Nitobe Memorial Garden
Asian Centre
PLACE VANIER RESIDENCE
GAGE RESIDENCE
Brock Hall (Student Services)
Student Union Building (SUB)
Aquatic Centre
COMMUNITY SHUTTLE TERMINAL
NORTH BUS LOOP
TROLLEY BUS TERMINAL
UNIVERSITY BLVD
MARINE DRIVE RESIDENCE
Biological Sciences
Earth & Ocean Sciences (EOS)
UBC HOSPITAL
Regent College
UNIVERSITY VILLAGE / MARKETPLACE
To 10th Ave.
UNIVERSITY ENDOWMENT LANDS
TOTEM PARK RESIDENCE
THUNDERBIRD RESIDENCE
THUNDERBIRD PARKADE
FAIRVIEW CRESCENT RESIDENCE
ACADIA PARK RESIDENCE
HAWTHORN PLACE
Doug Mitchell Thunderbird Sports Ctr
UBC Tennis Centre
THUNDERBIRD PARK
RCMP & FIREHALL
HAMPTON PLACE
UBC Botanical Garden & Centre for Plant Research
UBC Botanical Garden
Thunderbird Stadium
Matthews Field
UBC Ropes Course
16TH AVENUE
To 41st Ave.
TO SOUTH CAMPUS (see map page 2)

Map Directory

Site or Building Name & Address	Grid
Abdul Ladha Science Student Ctr, 2055 East Mall	D4
Acadia/Fairview Commonsblock, 2707 Tennis Cres	G7
Acadia House, 2700-2720 Acadia Rd	G7
Acadia Park Residence	F/H-6/7
Acadia Park Highrise, 2725 Melfa Rd	G7
Acadia Park Preschool, 2750 Acadia Park Lane	H7
Allard Hall [Faculty of Law], 1822 East Mall	B4
Anthropology & Sociology Bldg, 6303 NW Marine Dr	A3
Aquatic Centre, 6121 University Blvd	D5
Aquatic Ecosystems Research Lab (AERL), 2202 Main Mall	E3
Asian Centre, 1871 West Mall	B2
Auditorium (a.k.a. "Old Auditorium"), 6344 Memorial Rd	C3
Auditorium Annex Offices, 1924 West Mall	C3
Barn (daycare), 2323 Main Mall	E3
B.C. Binning Studios (formerly Hut M-17), 6373 University Blvd	D3
Beaty Biodiversity Centre & Museum, 2212 Main Mall	E3/4
Belkin (Morris & Helen) Art Gallery, 1825 Main Mall	B3
Berwick Memorial Centre, 2765 Osoyoos Cres	G6
Bioenergy Research & Demonstration Bldg., 2337 Lower Mall	E2
Biological Sciences Bldg [Science Faculty office], 6270 University Blvd	D3
Biomedical Research Ctr, 2222 Health Sciences Mall	E4
Biotechnology Laboratory, 2125 East Mall	D4
Bollert (Mary) Hall, 6253 NW Marine Dr	A4
Bookstore, 6200 University Blvd	D4
Botanical Garden Centre/Gatehouse, 6804 SW Marine Dr	H1
Botanical Garden Pavilion (enter at Gatehouse, 6804 SW Marine Dr)	J2
Botan. Gard. Greenhses/ Workshops, 6088 S. Campus Rd	South Campus
Brimacombe Building, 2355 East Mall	F4
BROCK HALL: Student Services & Welcome Centre, 1874 East Mall	**C4**
Brock Hall Annex, 1874 East Mall	C4
Buchanan Building (Blocks A, B, C, D, & E) [Arts], 1866 Main Mall	B3/4
Buchanan Tower, 1873 East Mall	C4
C.K. Choi Building for the Institute of Asian Research, 1855 West Mall	B2
Campus & Community Planning, 2210 West Mall	E3
Campus Security, 2133 East Mall	D4
Carey Centre, 5920 Iona Drive	B6
Carey Theological College, 1815 Wesbrook Mall	B6
CAWP (Centre for Advanced Wood Processing), 2424 Main Mall	F4
Cecil Green Park Coach House, 6323 Cecil Green Park Rd	A3
Cecil Green Park House, 6251 Cecil Green Park Rd	A3
CEME — *see Civil & Mechanical Engineering Building*	
Centre for Comparative Medicine, 4145 Wesbrook Mall	South Campus
Centre for Interactive Research on Sustainability (CIRS), 2260 West Mall	E3
CERC (Clean Energy Research Ctr), 2360 East Mall	F4
Chan Centre for the Performing Arts, 6265 Crescent Rd	B4
Chancellor Place neighbourhood	B5
Chemical & Biological Engineering Bldg, 2360 East Mall	F4
Chemistry A Block - Chemistry Physics Building, 6221 University Blvd	D4
Chemistry B,C,D & E Blocks, 2036 Main Mall	D3
Child Care Services Administration Bldg, 2881 Acadia Rd	H7
Child Care Services Bldgs, Osoyoos Cresc and Revelstoke Crt	H7
CIRS — *see Centre for Interactive Research on Sustainability...*	
Civil & Mechanical Engineering Bldg (CEME), 6250 Applied Science Lane	E4
Civil & Mechanical Eng. Labs ("Rusty Hut"), 2275 East Mall	E4
Coal & Mineral Processing Lab, 2332 West Mall	E3
Continuing Studies Bldg [English Language Institute], 2121 West Mall	D2
Copp (D.H.) Building, 2146 Health Sciences Mall	D5
Cunningham (George) Building [Pharmaceutical Sc.], 2146 East Mall	E4
David Lam Learning Centre, 6326 Agricultural Rd	C3
David Lam Management Research Ctr, 2033 Main Mall	C3
Donald Rix Building, 2389 Health Sciences Mall	F4
Doug Mitchell Thunderbird Sports Centre, 6066 Thunderbird Blvd	G5
Dorothy Somerset Studios (formerly Hut M-18), 6361 University Blvd	D3
Earth Sciences Building (ESB) under construction, 2207 Main Mall	E3
Earth & Ocean Sciences (EOS) - Main and South, 6339 Stores Rd	E3
Earthquake Engineering Research Facility (EERF), 2235 East Mall	E4
Engineering High Head Room Lab, 2225 East Mall	E4
English Language Institute (E.L.I.) — *see Continuing Studies Building*	
Environmental Services Facility, 6025 Nurseries Rd	South Campus
Fairview Crescent Residence, 2600-2804 Fairview Cres	F6
Fire Department, 2992 Wesbrook Mall	H6
First Nations Longhouse, 1985 West Mall	C2
Flag Pole Plaza (Main Mall & Crescent Rd)	B3
Food, Nutrition and Health Bldg, 2205 East Mall	E4
Forest Sciences Centre [Faculty of Forestry], 2424 Main Mall	F4
Forward (Frank) Building, 6350 Stores Rd	E3
FPInnovations (Forest Operations & Wood Products), 2601/2665 E. Mall	H4
FPInnovations (Pulp & Paper Division), 3800 Wesbrook Mall	South Campus
Fraser Hall (public rental housing), 2550 Wesbrook Mall	G6
Fraternity Village, 2880 Wesbrook Mall	H6
Frederic Wood Theatre, 6354 Crescent Rd	B3
Friedman Bldg, 2177 Wesbrook Mall	E5
Gage Residence, 5959 Student Union Blvd	C5
General Services Administration Bldg (GSAB), 2075 Wesbrook Mall	D5
Geography Building, 1984 West Mall	C3
Gerald McGavin Building, 2386 East Mall	F4
Graduate Student Centre — *see Thea Koerner House*	
Green College, 6201 Cecil Green Park Rd	A4
Greenheart Canopy Walkway, Botanical Garden, 6804 SW Marine Dr	H1
Greenwood Commons (public rental housing), 2660 Wesbrook Mall	G6
Hampton Place neighbourhood	H/J-6/7
Hawthorn Place neighbourhood	G/H3
Hebb Building, 2045 East Mall	D4
Hennings Building, 6224 Agricultural Rd	C4
Henry Angus Building [Sauder School of Business], 2053 Main Mall	D3
Hillel House - The Diamond Foundation Centre for Jewish Campus Life, 6145 Student Union Blvd	C4
Horticulture Building/Greenhouse, 6394 Stores Rd	E2/3
Hugh Dempster Pavilion, 6245 Agronomy Rd	F4
ICICS/CS (Institute for Computing, Information & Cognitive Systems/Computer Science), 2366 Main Mall	F4
Instructional Resources Centre (IRC), 2194 Health Sciences Mall	E5
International House, 1783 West Mall	B2
In-Vessel Composting Facility, 6035 Nurseries Road	South Campus
Irving K. Barber Learning Centre, 1961 East Mall	C4
Jack Bell Building for the School of Social Work, 2080 West Mall	D3
John Owen Pavilion & Allan McGavin Sports Medicine Centre, 3055 Wesbrook Mall	H5
Kaiser (Fred) Building [Faculty of Applied Science], 2332 Main Mall	E3
Kenny (Douglas T) Building, 2136 West Mall	D3
Kids Club, 2855 Acadia Rd	G7
Klinck (Leonard S.) Bldg, 6356 Agricultural Rd	C3
Koerner (Walter C.) Library, 1958 Main Mall	C3
Landscape Architecture Annex, 2371 Main Mall	F3
Lasserre (Frederic) Building, 6333 Memorial Rd	C3
Law, Faculty of — *see Allard Hall*	
Leon and Thea Koerner University Centre, 6331 Crescent Rd	B3
Life Sciences Centre, 2350 Health Sciences Mall	F5
Liu Institute for Global Issues, 6476 NW Marine Dr	B2
Lower Mall Header House, 2269 Lower Mall	E2
Lower Mall Research Station, 2259 Lower Mall	E2
Macdonald (J.B.) Building [Dentistry], 2199 Wesbrook Mall	E5
MacLeod (Hector) Building, 2356 Main Mall	F3
MacMillan (H.R.) Bldg [Faculty of Land & Food Systems], 2357 Main Mall	F3
Marine Drive Residence (Front Desk in Bldg #3), 2205 Lower Mall	E2
Material Recovery Facility, 6055 Nurseries Rd	South Campus
Mathematics Annex, 1986 Mathematics Rd	C3
Mathematics Building, 1984 Mathematics Rd	C3
Medical Sciences Block C, 2176 Health Sc. Mall	E4
M.F.A. Studios (formerly B.C. Binning MFA Studios), 6363 Stores Rd	E3
Michael Smith Laboratories, 2185 East Mall	D4
Museum of Anthropology (MOA), 6393 NW Marine Dr	A2/3
Music Building, 6361 Memorial Rd	B/C3
Networks of Ctrs of Excellence (NCE), 2125 East Mall	D4
Nitobe Memorial Garden, 1895 Lower Mall	B/C2
Nobel Biocare Oral Heath Centre (David Strangway Bldg), 2151 Wesbrook Mall	E5
Norman MacKenzie House, 6565 NW Marine Dr	B2
NRC Institute for Fuel Cell Innovation, 4250 Wesbrook Mall	South Campus
Old Administration Building, 6328 Memorial Rd	C3
Old Auditorium — *see Auditorium*	
Old Barn Community Centre, 6308 Thunderbird Blvd	G3
Old Firehall, 2038 West Mall	D3
Orchard House, 2336 West Mall	E2
Osborne (Robert F.) Centre/Gym, 6108 Thunderbird Blvd	G4
Panhellenic House, 2770 Wesbrook Mall	G6
Peter Wall Institute for Advanced Studies, 6331 Crescent Rd	B3
Place Vanier Residence, 1935 Lower Mall	C/D2
Plant Ops Nursery/Greenhouses, 6029 Nurseries Rd	South Campus
Plant Science Field Station & Garage, 2613 West Mall	H2
Point Grey Apartments, 2875 Osoyoos Cresc	H6
Police (RCMP) & Fire Department, 2990/2992 Wesbrook Mall	H6
Ponderosa Centre, 2071 West Mall	D2
Ponderosa Office Annexes: A, B, & C, 2011-2029 West Mall	C/D2
Ponderosa Office Annexes: E to H, 2008-2074 Lower Mall	C/D2
Power House, 2040 West Mall	D3
Pulp and Paper Centre, 2385 East Mall	F4
Ritsumeikan-UBC House, 6460 Agronomy Rd	F2
Rose Garden	B3
Roy Barnett Recital Hall - in Music Building	
Rugby Pavilion, 2584 East Mall	G4
Scarfe (Neville) Building [Education], 2125 Main Mall	D3
School of Population & Public Health (SPPH), 2206 East Mall	E4
Simon K.Y. Lee HKU-UBC House — Bldg #1, Marine Drive Residence	E2
Sing Tao Building, 6388 Crescent Rd	B3
Sopron House, 2730 Acadia Rd	G7
South Campus Warehouse, 6116 Nurseries Rd	South Campus
Spirit Park Apartments, 2705-2725 Osoyoos Cresc	G8
St. Andrew's Hall/Residence, 6040 Iona Dr	B5
St. John's College, 2111 Lower Mall	D2
St. Mark's College, 5935 Iona Dr.	B6
Staging Research Centre, 6045 Nurseries Rd	South Campus
Stores Road Annex, 6368 Stores Rd	E3
Student Recreation Ctr, 6000 Student Union Blvd	C5
Student Union Bldg (SUB), 6138 Student Union Blvd	C4
TEF3 (Technology Enterprise Facility 3), 6190 Agronomy Rd	F4
Thea Koerner House [Faculty of Graduate Studies], 6371 Crescent Rd	B3
Theatre-Film Production Bldg, 6358 University Blvd	D3
Thunderbird Residence, 6335 Thunderbird Cresc	F3/4
Thunderbird Stadium, 6288 Stadium Rd	J3
Thunderbird Winter Sports Ctr — *see Doug Mitchell Thunderbird Sports...*	
Totem Field Studios, 2613 West Mall	H2
Totem Park Residence, 2525 West Mall	F/G2
TRIUMF, 4004 Wesbrook Mall	South Campus
Triumf House (TRIUMF Visitor's Residence), 5835 Thunderbird Blvd	G6
UBC Bookstore, 6200 University Blvd	D4
UBC Farm, 6182 Wesbrook Mall	South Campus
UBC Hospital, 2211 Wesbrook Mall	E5
UBC Tennis Centre, 6160 Thunderbird Blvd	G4
UBC Thunderbird Arena (in Doug Mitchell Centre), 2555 Wesbrook Mall	G5
University Centre (Leon & Thea Koerner), 6331 Crescent Rd	B3
University Neighbourhoods Association, 5923 Berton Ave	South Campus
University Services Building (USB), 2329 West Mall	E2
Vancouver School of Theology, 6000 Iona Drive	B5
Walter H. Gage Residence, 5959 Student Union Blvd	C5
War Memorial Gymnasium, 6081 University Blvd	D5
Wayne & William White Engineering Design Ctr, 2345 East Mall	E4
Wesbrook Bldg, 6174 University Blvd	D4
Wesbrook Place neighbourhood	South Campus
Wesbrook Village shopping centre	South Campus
West Mall Annex, 1933 West Mall	C2
West Mall Swing Space Bldg, 2175 West Mall	D2
Wood Products Laboratory, 2324 West Mall	E3
Woodward IRC, 2194 Health Sciences Mall	E4/5
Woodward Library, 2198 Health Sciences Mall	E4/5

SOUTH CAMPUS MAP

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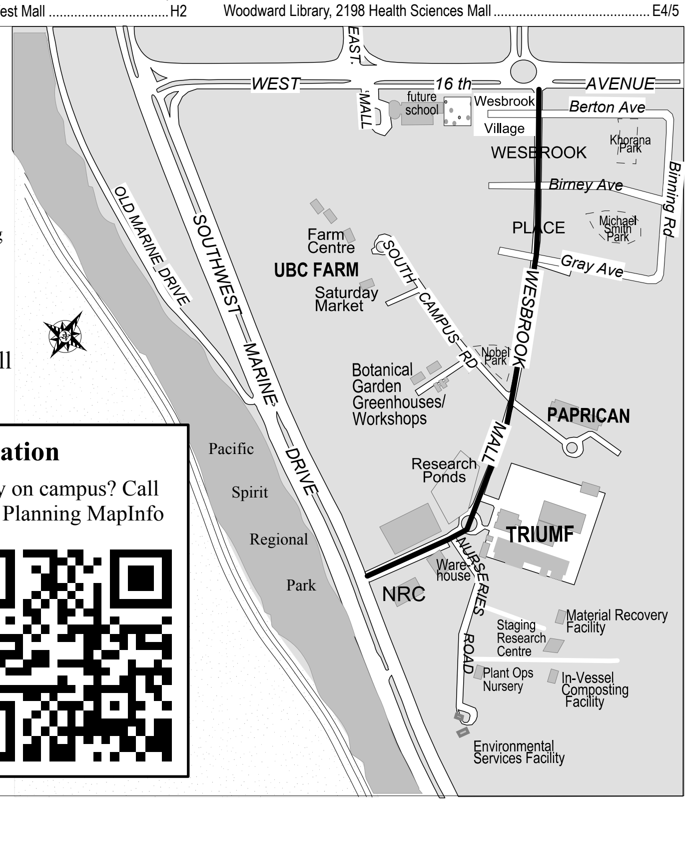
Campus & Community Planning
www.planning.ubc.ca

Note:
Local traffic only along Wesbrook Mall on South Campus



Map Information

Need help finding your way on campus? Call the Campus & Community Planning MapInfo Line at 604-827-5040, M-F, 8:30-4:30

Or use the online searchable colour map at **www.maps.ubc.ca**

Recreational Activities
at the University of British Columbia

Aquatic Centre
6121 University Boulevard
(604) 822- 4522 www.aquatics.ubc.ca
The UBC Aquatic Centre features a 50-metre indoor pool, seasonal 55-yard outdoor pool, whirlpool, fitness/weight room, sauna/steam rooms, seasonal patio area and diving boards from one to ten meters.

Summer Hours: Please call for swim times, lessons, etc.

Beaty Biodiversity Museum
2212 Main Mall
(604) 827- 4955 www.beatymuseum.ubc.ca
A new public museum dedicated to enhancing the public's understanding and appreciation of biodiversity. It is home to over 20,000 fossils from all over the world, including the largest blue whale exhibit in Canada

Summer Hours: Wed- Sun: 11:00am-5:00pm

Belkin Art Gallery
1825 Main Mall (beside Fredric Wood Theatre)
(604) 822- 2759 www.belkin.ubc.ca
The Morris and Helen Belkin Art Gallery's mandate is to research, exhibit, collect, publish, educate and develop programs in the field of contemporary art and in contemporary approaches to the practice of art history and criticism.

Summer Hours: Tues-Fri: 10:00am-5:00pm
Sat-Sun: 12:00pm-5:00pm

Botanical Garden
6804 Marine Drive
(604) 822- 9666 www.ubcbotanicalgarden.org
Established in 1916, the UBC Botanical Garden has an outstanding collection of temperate plants displayed according to their geographic areas. Exhibits of regional plants include the Native Garden and Alpine Garden.

Summer Hours: Daily 9:00am-5:00pm

Tennis Courts
2525 West Mall & 6010 Thunderbird Boulevard
(604) 822- 2505
All guests staying at the University of British Columbia are welcome to use the tennis courts located at Place Vanier and Totem Park Residences. There are additional courts at the UBC Coast Club located at 6160 Thunderbird Blvd. Please call for information on reservations, fees and special packages.

Please Note: Rates and Schedules are subject to change.

Museum of Anthropology
6393 NW Marine Drive
(604) 822- 5087 www.moa.ubc.ca

The Museum of Anthropology is one of North America's premier museums. School programs focusing on the Northwest Coast First Nations are available. All programs encourage discussion, observation and hands-on experience with touchable objects to learn about people and cultures. School programs must be arranged in advance.

Summer Hours: Daily 10:00am-5:00pm
Tues: 10:00an-9:00pm

Nitobe Memorial Garden
1903 Lower Mall
(604) 822- 9666 www.nitobe.org

Considered to be the best traditional, authentic Japenese Tea and Stroll garden in North America and among the top five Japanese gardens ouside Japan, the Nitobe Garden includes a rare authentic Tea Garden with a ceremonial Tea House. The exquisite work of art was created out of two=and-a-half acres (one hectare) of pristien forest by landscape architects and gardeners recommended by the government of Japan.

Summer Hours: Daily 10:00am-5:00pm

Pacific Spirit Regional Park
Park Office 4915 West 16th Avenue
(604) 224- 5739

The Pacific Spirit Regional Park encompasses 763 hectares of forest and foreshore surrounding UBC, and boasts 35 kilometres of walking trails. Experience a variety of landscapes, from estuary marshes, rock and cobble beaches, wooded ravines, ancient bog and upland forests. Regional Park Interpreters offer customized group programs on themes ranging between edible plants, birds, and bog ecology.

Student Recreation Centre
6000 Student Union Boulevard
(604) 822- 6000 www.rec.ubc.ca or www.birdcoop.ubc.ca

The SRC is one of Canada's premier University fitness facilities. It includes 1,800 square-feet of gym space, a full service fitness and weight room, a 2,300 square-foot dance studio, and a 1,600 square-foot traditional martial arts dojo.

Summer Hours: Mon-Thurs: 6:30am-9:00pm Saturday: 10:00am-6:00pm
Friday: 630am-7:00pm 12:00pm- 6:00pm

University Golf Course
5185 University Boulevard
(604) 224- 1818 www.universitygolf.com

Designed to satisfy players of every level, the course features low-mowed rough and few hazards of water to carry over. Still, it does present challenges even for the experienced golfer. Greens on Par 3's are well protected by sand and require stealth accuracy. Move back to the championship tees and put a little more distance between you and the pins. 18 holes, Par 72.

Summer Hours: First tee time: 6am Last tee time: 8pm

Please Note: Rates and Schedules are subject to change.

Campus Dining

at the University of British Columbia

From world-class catering to casual dining, coffee shops and internationally-inspired food outlets, UBC offers a delicious assortment of food services solutions. Here is an overview of food service providers certain to deliver a satisfying campus dining experience.

UBC Food Services

www.food.ubc.ca

Serving only locally-roasted fair trade organic shade-grown coffee at all UBC Food Services non-franchise locations

Wescadia Catering

Conference and special event catering

www.catering.ubc.ca

Sage Bistro at University Centre

Casual fine dining available for breakfast, lunch and special events

www.sage.ubc.ca

The Point Grill at Marine Drive Residence

New upscale casual dining restaurant open for brunch, lunch, and dinner. Open M-F

Triple O's at David Lam Research Centre

Casual dining in a family-friendly environment. Open daily

Residence Dining

Totem Park and Place Vanier Cafeterias

For information about group meal plans, please call 604-822-6204 or email rene.atkinson@ubc.ca

Pacific Spirit Place Cafeteria at the SUB

Student Union Building, 6138 Student Union Blvd

Pacific Spirit Place is open weekdays for breakfast and lunch. For information about group meal plans, please call 604-822-9310 or email fred.cheng@ubc.ca

Bakeshop	A&W
Pasta Bar	Koya Japan
Salad Bar	Manchu Wok
Pizza Pizza	Subway

Proudly Brewing Starbucks Coffee

Starbucks Coffee at Student Union Building
The Barn at Main Mall
Starbucks Coffee at Fred Kaiser
Steamies Café at the Bookstore
Pond Café at Ponderosa Centre

More Great Locations…

Niche Café at Beaty Biodiversity Museum
Caffé Perugia at Life Sciences Centre
Café MOA at Museum of Anthropology
Ike's Café at Irving K. Barber Learning Centre
Tim Horton's at Forest Sciences Centre

For guests, visitors, or groups visiting the UBC Campus, the UBC Food Services gift card is the easiest way for you and your group to dine at any of our locations.

Food Outlets

at the Student Union Building (SUB)

The SUB features a variety of food outlets all under one roof and conveniently located at the heart of campus. Get a delicious bagel or muffin to go, grab a slice of pizza at Pie R Squared, pick up some freshly made sushi or sit and enjoy a juicy beef burger at Pit Pub. The SUB has something for everyone!

Concourse and Sub-Level

Blue Chip Cookies

Proudly serving organic, fair trade coffees, cappuccinos and lattés. All our cookies and fabulous baked goods are made in-house and baked fresh daily.

Bernoulli's Bagels

Montreal-style bagels, sandwiches, and bagel melts using high-quality ingredients and freshly squeezed vegetable or citrus juice!

The Delly

Fresh sandwiches made to order. A wide selection of salads, wraps, curries, soups and pasta made daily.

The Honour Roll

Maki rolls, nigiri, sushi, donburi rice bowls and bento boxes are made fresh throughout the day. Ask about party platters and catering.

The Pit Burger Bar

Charbroiled hamburger specials, veggie burgers, hot wings, beer-battered fish & chips and more!

The Pit Pub

Satellite big-screen sports, six high-definition TV's, great drink prices, and a great atmosphere!

The Moon Noodle House

Great wonton soup, daily specials, fresh steamed veggies, combos and hot & sour soup.

The Patio BBQ

On the south side of the SUB, Monday to Friday (weather permitting) offering grilled 1/4 pound burgers, veggie burgers, smokies and drinks.

The Pendulum Restaurant

Delicious grilled sandwiches and panninis, and lots of vegetarian and vegan dishes!

Pie R Squared

Great house-made pizza slices, great prices, cold drinks. Now offering soft-serve ice cream and doughnuts.

www.catering.ubc.ca

NEED CATERING? For catered events or meals on the go, Wescadia Catering offers a multitude of menu ideas to meet a range of dietary needs. We pride ourselves on our knowledgeable, friendly staff, professional service and quality ingredients.

University Boulevard

Restaurants and Food Outlets

University Boulevard boasts a vibrant neighbourhood feel, and features dozens of places to enjoy a sit-down meal, people-watch over coffee, or grab a quick bite on the run. Visitors will feel right at home choosing from internationally-recognized franchises and unique offerings from local entrepreneurs.

The Boulevard Coffee Roasting Co.

at David Strang, 5870 University Blvd.
theboulevard.ca

Mahony & Sons Public House

at David Strang, 5990 University Blvd.
www.mahonyandsons.com

The Well Café

at Regent College, 5800 University Blvd.

University Village

5700 Block, University Blvd.

- Blenz Coffee Shop
- Booster Juice Juice & Snack Bar
- Mio Japan Japanese Fast Food
- McDonald's Breakfast – Late-Night Fast Food
- Pearl Fever Tea House & Snack Bar
- Pita Pit Lunch – Late-Night Take-Out & Delivery
- One More Sushi Japanese Dining
- Only U Café Deli & Diner
- Starbuck's Coffee Shop
- University Pizza Take-Out & Delivery
- Vera's Burger Shack Diner
- Village Restaurant Chinese Dining

International Food Fair

University Marketplace, Lower Level

- A-1 Vietnamese Food Pho & Noodle House
- Curry Point East Indian
- Donair Town Persian, Mediterranean, Catering
- Leona Mediterranean Food Lebanese
- Malaysian Cuisine Malaysian, Thai
- Osaka Sushi Japanese
- Timpo Mongolian BBQ Stir-Fry
- Yi Kou Xiang Chinese

ALSO RECOMMENDED… ***Westward Ho! PublicHouse & Grill Room** at the University Golf Club*
www.universitygolf.com/dine