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Title: Symposium on the Geometry and Topology of Manifolds

Event Type: Conference-Workshop

Location:

PIMS - UBC

Dates:

June 29 - July 9, 2015

Topic:

Geometry and Topology of Manifolds

Methodology:

There were four plenary lectures on each of the eight days of the conference. In addition there were parallel sessions, with three simultaneous talks. All lectures were a full hour in duration. Lunch was provided for participants, which facilitated interaction and the opportunity to exchange ideas.

Objectives Achieved:

The symposium brought together mathematicians with overlapping interests, but who do not often meet together. This included geometers, low- and high-dimensional topologists sharing an interest in manifolds. Many new results were explained in the lectures.

Scientific Highlights:

The Symposium featured a range of recent results in differential geometry and topology, and related topics in global analysis.

The latest developments in geometric group theory were presented at the symposium. Ruth Charney talked about her new boundary theory for CAT(0) spaces, and Mike Davis described striking estimates for the action dimension of a right-angled Artin group. Martin Bridson gave a broad overview of decision problems in group theory, and the connections between curvature, topology and algorithms.

The opening lecture was given by Gang Tian on the spectacular applications of curvature flows towards understanding the geometry and topology of manifolds. The main highlight was the recent refinements of Ricci flow techniques that have already led to Perelman's resolution of Thurston's Geometrization Conjecture.

Marco Gualtieri presented a new approach to the generalized complex structures on manifolds featuring interesting links with symplectic topology and a new method of regularization for singular ODEs on complex curves. The interplay with symplectic geometry was also featured in the lecture of

Gil Cavalcanti who showed that stable generalized complex structures have unobstructed deformations. Daniel Ruberman reported on a joint work with Tom Mrowka and Nikolai Saveliev extending the Atiyah-Patodi-Singer eta-invariant to the setting of Taubes' theory of elliptic operators on manifolds with periodic ends.

There were several talks on the latest development in studying moduli spaces of manifolds. Oscar Randal-Williams gave two lectures where he presented his work joint with Soren Galatius on the moduli space of $(n-1)$ -connected $2n$ -manifolds. The second lecture he dedicated to a geometrical application of these technology to show that a space of metrics of positive scalar curvature of any spin high dimensional manifold has at least as complicated topology as of KO -theory (this is joint work with Johannes Ebert and Boris Botvinnik).

Ib Madsen highlighted new development in studying rational homotopy type of the classifying space of the groups of homotopy automorphisms and block automorphisms of even-dimensional 'generalized surfaces'. Alexander Berglund, as a young coauthor of Ib Madsen, gave more details on this work. In addition, Soren Galatius gave two lectures explaining several homological stability results and their applications to describing topology of classifying spaces $B\text{Diff}(M)$ for even-dimensional manifolds. Along the same lines, young topologist Nathan Perlmutter presented several homological stability results for the moduli spaces of n -connected $(2n + 1)$ - dimensional manifolds.

In somewhat related theme, topological field theory, Ralph Cohen presented equivalence between two chain complex valued topological field theories: the String Topology of a manifold given manifold and the Symplectic Cohomology of its cotangent bundle.

Ben Farb gave two interesting and entertaining lectures. In the first one, he described the remarkable bridge, built by Weil, Grothendieck, Deligne and others, between topology and number theory, and in the second one, he showed new development in the topology of spaces of holomorphic maps.

Tom Farrell gave two beautiful lectures on spaces of constrained Riemannian metrics and their associated Teichmueller spaces and on bundles of negatively-curved manifolds. In particular, he presented an interesting partial result on the conjecture that negatively curved bundles with compact fibers and simply connected base space must be topologically trivial.

In his first lecture, Wolfgang Lueck highlighted new developments in studying L_2 -acyclic manifolds with torsion free fundamental group, namely, he introduced universal torsion which is defined for such manifolds and, in particular, generates new twisted L_2 -torsion and higher order Alexander polynomials. The second lecture was dedicated to the latest results on Farrell-Jones Conjecture. In a related theme, Jim Davis described his recent work (joint with Sylvain Cappell and Shmuel Weinberger) Bordism of acyclic L_2 -manifolds.

There were interesting talks of young mathematicians on the geometry and topology of 7- manifolds endowed with G_2 -structures. Diarmuid Crowley reported on new examples of 2-connected such manifolds which are homeomorphic but not diffeomorphic, Johannes Nordstrom explained how to successfully exploit a new invariant to show that the moduli space of holonomy G_2 metrics may be disconnected, Thomas Walpuski obtained a compactness result for Seiberg-Witten equations with multiple spinors in an intriguing relation to gauge theory on G_2 -manifolds.

Ailana Fraser presented new results on free boundary minimal surfaces. Andrew Dancer gave an implosion constructions related to symplectic and hyperkahler geometry of solutions spaces for Nahm's equations. Thomas Schick reported on a interesting construction (including numerical results) of a correspondence between surgery exact sequence for smooth manifolds and K -theory

groups of C^* -algebras in terms of higher index theory of the signature operator. Mihaela Pilca presented a recent result on an upper bound on the rank of Clifford structures on Riemannian manifolds, including descriptions of extreme cases of highest possible rank and rank 3. The lecture of Peng Lu featured a splitting theorem and a rigidity theorem for complete gradient Ricci solitons. Jason Lotay reported on the recent progress on a challenging 'filling in' problem of existence and deformations of hyperkaehler 4-manifolds given the appropriate structure on the boundary 3-manifold. Michael Davis gave an interesting lecture on the 'action dimension' of a discrete group. Octav Cornea presented his work on Lagrangian cobordism and Fukaya categories.

In the first lecture, Ciprian Manolescu presented his work on the triangulation conjecture: he showed that it is false in the dimensions 4 and higher. In the second talk, on involutive Heegaard Floer homology, he presented new invariants of homology cobordism, explicitly computable for surgeries on L-space knots and thin knots. Ryan Budney gave a talk on a related topic, on triangulations of 4-manifolds, he outlined some new developments, algorithmic challenges and some simple attacks on long-standing open problems.

Steve Boyer discussed the intriguing conjecture that for a rational homology 3-sphere, being a Heegaard-Floer L-space is equivalent to its fundamental group to be NOT left-orderable. These conditions are also conjectured to be equivalent to the nonexistence of a co-orientable taut codimension-one foliation on the manifold. Techniques for proving these conjectures for Seifert-fibred and graph-manifolds were discussed by Adam Clay.

Organizers:

Boris Bottvinnik, Mathematics, University of Oregon
Ian Hambleton, Mathematics, McMaster University
Alexei Kovalev, Cambridge University
Dale Rolfsen, University of British Columbia

Speakers:

Plenary Speakers:

Martin Bridson: (Oxford University) Lecture 1: Decision problems, curvature and topology. I shall discuss a range of problems in which groups mediate between topological/geometric constructions and algorithmic problems elsewhere in mathematics, with impact in both directions. I shall begin with a discussion of sphere recognition in different dimensions. I'll explain why there is no algorithm that can determine if a compact homology sphere of dimension 5 or more has a non-trivial finite-sheeted covering. I'll sketch how ideas coming from the study of CAT(0) cube complexes were used by Henry Wilton and me to settle isomorphism problems for profinite groups, and to settle a conjecture in combinatorics concerning the extension problem for sets of partial permutations. Lecture 2: Profinite rigidity and low-dimensional orbifolds Abstract: This lecture shares with Lecture 1 the theme of recognition problems, but the focus now is on low-dimensional manifolds and finite quotients of groups. I shall begin by discussing the history of the following problem: to what extent is a residually-finite group determined by its profinite completion. Fundamental groups of orbifolds of dimension at most 3 are determined to a greater extent than arbitrary groups. After surveying what is known in this context, I'll present recent work with Reid and Wilton showing that the fundamental groups of punctured torus bundles can be distinguished from each other and from other 3-manifold groups by means of their profinite completions.

Ruth Charney: (Brandeis University) Morse Boundaries. Boundaries play an important role in the study of hyperbolic spaces and hyperbolic groups. Analogous boundaries exist for CAT(0) spaces but they are not quasi-isometry invariant and hence do not give a well-defined boundary for a CAT(0) group. In joint work with H. Sultan, we define a 'contracting boundary' for CAT(0) spaces by restricting to rays with hyperbolic-like behavior and we prove that the contracting boundary is a quasi-isometry invariant. More recently, M. Cordes has shown that these ideas can be generalized to any proper geodesic metric space by restricting to Morse geodesics.

Octav Cornea: (Universite de Montreal) Lagrangian cobordism and Fukaya categories. This talk is based on joint work with Paul Biran (ETH). The derived Fukaya category has emerged out of work of Gromov, Floer, Donaldson, Kontsevich, Fukaya, Seidel and many others starting in the 1980's. It is a triangulated category associated to a symplectic manifold that, in favourable cases, allows one to use iterated exact triangles to recover properties of a given Lagrangian submanifold from knowledge of a class of simpler objects. Constructions of exact triangles associated to specific geometric settings are essential from this perspective. The first important construction is due to Seidel (2003) and is associated to a Dehn twist. Another class of examples has emerged more recently in the work of Biran and myself (2013) and is a reflection of the relation given by Lagrangian cobordism, a natural notion introduced by Arnold at the end of the 1970's. After giving the relevant background, the purpose of this talk is to explain the relation between cobordism and the derived Fukaya category and show how Seidel's exact triangle fits into the picture.

Michael Davis: (Ohio State University) The action dimension of RAAGs. The 'action dimension' of a discrete group G is the smallest dimension of a contractible manifold which admits a proper action of G . Associated to any flag complex L there is a right-angled Artin group, A_L . We compute the action dimension of A_L for many L . Our calculations come close to confirming the conjecture that if the L^2 -Betti number of A_L in degree l is nonzero, then the action dimension of A_L is greater than or equal to $2l$. This is a report on joint work with Grigori Avramidi, Boris Okun and Kevin Schreve.

Tom Farrell: (Tsinghua University) Lecture 1: Bundles with extra geometric structure. The structure of smooth fiber bundles whose concrete fibers are each equipped with a Riemannian metric whose sectional curvatures are constrained to lie in a fixed interval S of real numbers (called S -bundles) will be explored in this talk. Some interesting cases being $S = (-\infty, 0)$, $(-\infty, 0]$, $(-4, -1)$, $[0, +\infty)$, $(0, +\infty)$ and $(1, 4)$. In examining negatively curved bundles -- i.e. the case where $S = (-\infty, 0)$ -- important use of Anosov flow bundles is made -- i.e. bundles whose concrete fibers are each equipped with an Anosov flow. This leads to an interesting partial result on the conjecture that negatively curved bundles with compact fibers and simply connected base space must be topologically trivial. Lecture 2: Space of constrained Riemannian metrics and their associated Teichmueller spaces. This talk will focus on trying to make transparent some of the key ideas underlying the results mentioned in my first talk. For example how it is seen that the Teichmueller space of negatively curved metrics on many high dimensional closed smooth manifolds is not contractible. Also recent results about quarter pinched positively curved bundles and the space of non-negatively curved Riemannian metrics on certain open manifolds will be discussed. These lectures represent work of many people including Pedro Ontaneda, Andrey Gogolev, Igor Belegradek, Vitali Kapovitch, Dan Knopf and Zhou Gang.

Benson Farb: (University of Chicago) Lecture 1: Point counting for topologists. In this talk I will try to describe the remarkable bridge, built by Weil, Grothendieck, Deligne and others, between topology and number theory. I will concentrate on explaining how this bridge can be used to make topological (resp. arithmetic) predictions, and prove them, using arithmetic (resp. topology). A crucial intermediary in this story is the Grothendieck ring of varieties, or "baby motives". Lecture 2: Topology of spaces of holomorphic maps, revisited. In 1979, Segal computed the stable homology of the space of degree d holomorphic maps from CP^1 to CP^n , inspiring a flurry of activity in the

1980's. In this talk I will describe a new (and still-developing) point-of-view on these theorems (and generalizations). Applications include answers to questions such as: how many degree d rational curves are there in projective n -space $P^n(F_q)$? One theme is the transmission of information between three viewpoints: topology, algebraic geometry, and arithmetic. This is joint work with Jesse Wolfson.

Soren Galatius: (Stanford University) Lecture 1: Homological stability and non-stability for moduli spaces of closed manifolds. Two closed manifold M and M' of dimension $2n$ are said to be stably diffeomorphic if they become diffeomorphic after replacing each with its connected sum with a finite number of copies of $S^n \times S^n$. In general there doesn't seem to be any good maps between the classifying spaces $B\mathrm{Diff}(M)$ and $B\mathrm{Diff}(M')$, but nevertheless it turns out that they have isomorphic rational cohomology in a range of degrees, provided the numbers $(-1)^n \chi(M)$ and $(-1)^n \chi(M')$ are both sufficiently large, and $n > 2$ and the manifolds are simply connected. The analogous statement in integral cohomology is false, but with p -local coefficients it is true provided in addition that the p -adic valuations of $\chi(M)$ and $\chi(M')$ agree. This is joint work with Oscar Randal-Williams. Lecture 2: Tautological rings for high-dimensional manifolds. To each fiber bundle $f: E \rightarrow B$ whose fibers are closed oriented manifolds of dimension d and each polynomial $p \in H^*(BSO(d))$ there is an associated "tautological class" $\kappa_p \in H^*(B)$ defined by fiberwise integration. The set of polynomials in these classes which vanish for all bundles whose fibers are oriented diffeomorphic to M forms an ideal $I_M \subset \mathbb{Q}[\kappa_p]$ and the quotient ring $R_M = \mathbb{Q}[\kappa_p]/I_M$ is the 'tautological ring' of M . In this talk I will discuss some recent results about the structure and particularly Krull dimension of this ring for various M . This is joint work with Ilya Grigoriev and Oscar Randal-Williams.

Marco Gualtieri: (University of Toronto) Lecture 1: Generalized complex vs logarithmic symplectic geometry. I will review the notion of a generalized complex structure and describe recent developments in our understanding of this type of geometry, which interpolates between usual complex and symplectic manifolds. I will emphasize a new approach which reformulates the structure in terms of a usual symplectic form, but on a modification of the tangent bundle, a construction which may have independent interest in symplectic topology. Lecture 2: The Stokes Groupoid. The solution to a singular ordinary differential equation is not well-defined on the original curve; I will explain that it is only well-defined on the corresponding Stokes groupoid, a complex surface equipped with groupoid structure. I will explain how this point of view leads to a new functorial regularization procedure for divergent perturbation series solutions as well as a canonical geometric solution of the isomonodromy system.

Wolfgang Lueck: (Universitaet Bonn) Lecture 1: Universal torsion, L^2 -invariants, polytopes and the Thurston norm. We introduce universal torsion which is defined for L^2 -acyclic manifolds with torsionfree fundamental group and takes values in certain K_1 -groups of a skew field associated to the integral group ring. It encompasses well-known invariants such as the Alexander polynomial and L^2 -torsion. We discuss also twisted L^2 -torsion and higher order Alexander polynomials which can also be derived from the universal invariant and assign certain polytopes to the universal torsion. This gives especially in dimension 3 interesting invariants which recover for instance the Thurston norm. Lecture 2: Introduction to the Farrell-Jones Conjecture. The Farrell-Jones Conjecture identifies the algebraic K - and L -groups for group rings with certain equivariant homology groups. We will give some details of its formulation, its status and indicate some ideas of proofs for certain classes of groups. We will try to convince the audience about its significance by considering special cases and presenting the surprising large range of its applications to prominent problems in topology, geometry, and group theory.

Ib Henning Madsen: (Copenhagen University) Automorphisms of manifolds and graph homology. The lecture will describe the rational cohomology of the classifying space of the groups of homotopy

automorphisms and block automorphisms of 2d-dimensional "generalized surfaces". The results will be given in terms of Lie algebra cohomology, and graph homology. The lecture represents joint work with Alexander Berglund.

Ciprian Manolescu: (University of California, LA) Lecture 1: The triangulation conjecture. The triangulation conjecture stated that any n -dimensional topological manifold is homeomorphic to a simplicial complex. It is true in dimensions at most 3, but false in dimension 4 by the work of Casson and Freedman. In this talk I will explain the proof that the conjecture is also false in higher dimensions. This result is based on previous work of Galewski-Stern and Matsumoto, who reduced the problem to a question in low dimensions (the existence of elements of order 2 and Rokhlin invariant one in the 3-dimensional homology cobordism group). The low-dimensional question can be answered in the negative using a variant of Floer homology, $\text{Pin}(2)$ -equivariant Seiberg-Witten Floer homology. Lecture 2: Involutive Heegaard Floer homology. Using the conjugation symmetry on Heegaard Floer complexes, we define a three-manifold invariant called involutive Heegaard Floer homology, which is meant to correspond to \mathbb{Z}_2 -equivariant Seiberg-Witten Floer homology. Further, we obtain two new invariants of homology cobordism, explicitly computable for surgeries on L-space knots and thin knots. This is joint work with Kristen Hendricks.

John Pardon: (Stanford University) Existence of Lefschetz fibrations on Stein/Weinstein domains. I will describe joint work with E. Giroux in which we show that every Weinstein domain admits a Lefschetz fibration over the disk (that is, a singular fibration with Weinstein fibers and Morse singularities). We also prove an analogous result for Stein domains in the complex analytic setting. The main tool used to prove these results is Donaldson's quantitative transversality.

Oscar Randal-Williams: (Cambridge University) Lecture 1: Infinite loop spaces and positive scalar curvature. It is well known that there are topological obstructions to a manifold M admitting a Riemannian metric of everywhere positive scalar curvature (psc): if M is Spin and admits a psc metric, the Lichnerowicz-Weitzenboeck formula implies that the Dirac operator of M is invertible, so the vanishing of the \hat{A} genus is a necessary topological condition for such a manifold to admit a psc metric. If M is simply-connected as well as Spin, then deep work of Gromov-Lawson, Schoen-Yau, and Stolz implies that the vanishing of (a small refinement of) the \hat{A} genus is a sufficient condition for admitting a psc metric. For non-simply-connected manifolds, sufficient conditions for a manifold to admit a psc metric are not yet understood, and are a topic of much current research. I will discuss a related but somewhat different problem: if M does admit a psc metric, what is the topology of the space $\mathcal{R}^+(M)$ of all psc metrics on it? Recent work of V. Chernysh and M. Walsh shows that this problem is unchanged when modifying M by certain surgeries, and I will explain how this can be used along with work of Galatius and myself to show that the algebraic topology of $\mathcal{R}^+(M)$ for M of dimension at least 6 is "as complicated as can possibly be detected by index-theory". This is joint work with Boris Botvinnik and Johannes Ebert. Lecture 2: Moduli spaces of high-dimensional manifolds. I will explain recent and ongoing work with Soren Galatius, in which we study moduli spaces of smooth $2n$ -dimensional ($2n > 4$) manifolds (or, what is equivalent, the classifying spaces of diffeomorphism groups of such manifolds). I will describe a homotopy-theoretic approximation to these spaces which allows us to compute their cohomology in many cases. This gives a high-dimensional extension of well-known results in dimension 0 (Nakaoka's stability theorem and the Barratt-Priddy-Quillen theorem) and dimension 2 (Harer's stability theorem and the Madsen-Weiss theorem).

Daniel Ruberman: (Brandeis University) End-periodic index theory. We extend the Atiyah, Patodi, and Singer index theorem from the context of manifolds with cylindrical ends to manifolds with periodic ends. This theorem provides a natural complement to Taubes' Fredholm theory for general end-periodic operators. Our index theorem is expressed in terms of a new periodic eta-invariant that equals the Atiyah-Patodi-Singer eta-invariant in the cylindrical setting. We apply this periodic eta-invariant to the study of moduli spaces of Riemannian metrics of positive scalar curvature. This

is joint work with Tom Mrowka and Nikolai Saveliev.

Gang Tian: (Princeton University) Curvature flows and geometric applications. In last two decades, curvature flows have provided effective tools for studying geometry and topology of manifolds. A famous example is Perelman's solution of the Poincaré conjecture by using Hamilton's Ricci flow. In this talk, I will start with Ricci flow and discuss some of its applications, then I will discuss some other curvature flows and show how they can be used to studying geometry and topology of underlying manifolds.

Invited Speakers:

R. Inanc Baykur: (University of Massachusetts) Multisections of Lefschetz fibrations and topology of symplectic 4-manifolds. We initiate an extensive study of positive factorizations in framed mapping class groups, which allows us to effectively build symplectic 4-manifolds with essential information on various surfaces in them. In this talk, we will demonstrate how these techniques can be used to reformulate and address several interesting problems related to the topology of symplectic 4-manifolds and Lefschetz pencils. Different parts of this work are joint with K. Hayano and N. Monden.

Maxime Bergeron: (University of British Columbia) The topology of representation varieties. Let H be a finitely generated group and let G be a complex reductive linear algebraic group (e.g. a special linear group). The representation space $\text{Hom}(H, G)$, carved out of a finite product of copies of G by the relations of H , has many interesting topological features. From the point of view of algebraic topology, these features are easier to understand for the compact subspace $\text{Hom}(H, K)$ of $\text{Hom}(H, G)$ where K is a maximal compact subgroup of G (e.g. a special unitary group). Unfortunately, the topological spaces $\text{Hom}(H, G)$ and $\text{Hom}(H, K)$ usually have very little to do with each other; for instance, some of the components of $\text{Hom}(H, G)$ may not even intersect $\text{Hom}(H, K)$. Accordingly, I will discuss exceptional classes of groups H for which $\text{Hom}(H, G)$ and $\text{Hom}(H, K)$ happen to be homotopy equivalent, thereby allowing one to obtain otherwise inaccessible topological invariants.

Alexander Berglund: (Stockholm University) Stable cohomology of automorphisms of high dimensional manifolds. There is classical programme for understanding diffeomorphisms of high dimensional manifolds whereby one studies, in turn, the monoid of homotopy automorphisms, the block diffeomorphism group, and finally the diffeomorphism group. The difference in each step is measured by, respectively, the surgery exact sequence and, in a range, Waldhausen's algebraic K -theory of spaces. In recent joint work with Ib Madsen, we calculated the stable rational cohomology of the block diffeomorphism group of the $2d$ - dimensional "generalized genus g surface", i.e., the g -fold connected sum of $S^d \times S^d$ minus a disk ($2d > 4$). Here, stable means g should be large compared to the cohomological degree. Our result is expressed in terms of a certain decorated graph complex. Curiously, the complex we obtain is closely related to the "hairy graph complex" that was introduced recently by Conant-Kassabov-Vogtmann in the study of the homology of automorphism groups of free groups.

Steven Boyer: (Université du Québec à Montréal) Foliations, left-orders, and L -spaces. Much work has been devoted in recent years to examining relationships between the existence of a co-oriented taut foliation on a closed, connected, prime 3-manifold W , the left-orderability of the fundamental group of W , and the property that W not be a Heegaard-Floer L -space. Classic work shows that each of these conditions holds when W has a positive first Betti number and it has been conjectured that they coincide when the first Betti number of W is zero. In this talk I will discuss the known connections between these conditions and survey the current status of the conjectures.

Ryan Budney: (University of Victoria). Triangulations of 4-manifolds. I will outline some

developments in the study of PL-triangulations of 4-manifolds, a developing census, algorithmic challenges and some simple attacks on long-standing open problems.

Gil Cavalcanti: (Universiteit Utrecht) Stable generalized complex structures. Stable generalized complex structures are a special class of generalized complex manifolds which are not too far from being symplectic. We show that the stable condition can be rephrased by saying that the structure is equivalent to a symplectic structure on a Lie algebroid. This equivalence allows us to show that deformations of these structures are unobstructed and we obtain a local normal form for the set of points where the structure fails to be symplectic. Some topological restrictions to the existence of such structures follow from the normal form.

Weimin Chen: (University of Massachusetts Amherst) Toward an equivariant version of Gromov-Taubes invariant. Gromov-Taubes invariant of a symplectic four-manifold is defined by counting EMBEDDED pseudo-holomorphic curves (maybe disconnected) whose homology class is Poincare dual to a given cohomology class. The seminar work of Taubes asserts that the Gromov-Taubes invariant equals the gauge-theoretic Seiberg-Witten invariant of the underlying smooth four-manifold. Taubes' work has profoundly influenced the research in four-manifold topology (and beyond). Some of the most important consequences include a symplectic characterization and classification of rational and ruled surfaces (following the pioneering works of Gromov and McDuff), the equivalence of symplectic minimality and smooth minimality, and a new, differential and symplectic topology interpretation of the Kodaira dimension of complex surfaces. More recently, Taubes' work has been extended to the level of Floer homology, which, in particular, resulted in a resolution of the Weinstein conjecture in dimension three and an isomorphism between the various Floer homologies of three-manifolds. It is a natural problem to extend Taubes' work to certain singular spaces. Two important cases which may yield interesting geometric or topological applications are normal projective surfaces and symplectic finite group actions (in which case the corresponding singular space is the quotient space of the group action). In this talk, we will discuss the problem of constructing an equivariant version of Gromov-Taubes invariant -- what can be done and what are the obstacles, as well as some of the geometric and topological applications obtained so far.

Adam Clay (University of Manitoba) Foliations of graph manifolds Boyer, Gordon and Watson have conjectured that the fundamental group of a closed, orientable, irreducible 3-manifold M is left-orderable if and only if M is not an L-space. The latter was also conjectured by Juhasz to be equivalent to the existence of a co-orientable, taut foliation of M . When M is a Seifert fibred manifold these conjectures all hold, and there are well-understood constructions which show us exactly why. In this talk I will discuss these conjectures in the context of graph manifolds, with particular focus on the foliations needed in order to extend existing constructions from the case of Seifert fibred manifolds to the case of graph manifolds. This is joint work with Steve Boyer.

Ralph Cohen: (Stanford University) Comparing Topological Field Theories: the string topology of a manifold and the symplectic cohomology of its cotangent bundle. I will describe joint work with Sheel Ganatra, in which we prove an equivalence between two chain complex valued topological field theories: the String Topology of a manifold M , and the Symplectic Cohomology of its cotangent bundle, T^*M . I will also discuss how the notion of Koszul duality appears in the study of TFT's.

Diarmuid Crowley: (University of Aberdeen) Exotic G_2 -manifolds. I shall present examples of smooth 2-connected 7-manifolds M_0 and M_1 which admit G_2 holonomy metrics and which are homeomorphic but not diffeomorphic. These are the first examples of exotic pairs which admit Ricci flat special holonomy metrics. The key invariant is a generalisation of the classical Eells-Kuiper invariant for spin 7-manifolds. The generalised Eells-Kuiper invariant also appears in complete classifications for 2-connected 7-manifolds and 2-connected 7-manifolds with G_2 structure. This work is joint with Johannes Nordstrom.

Andrew Dancer: (University of Oxford) Hyperkahler implosion and Nahm's equations. We describe implosion constructions in symplectic and hyperkahler geometry. We show how the latter case may be approached via Nahm moduli spaces, and also describe quasi-Hamiltonian analogues of implosion.

Jim Davis: (Indiana University) Bordism of L^2 -acyclic manifolds. A manifold is acyclic if all of its betti numbers vanish. (It is also called anharmonic since there are no nontrivial L^2 -harmonic forms.) For a manifold with fundamental group Z^n , a manifold is L^2 -acyclic if, and only if, it is acyclic with $Q(t_1, \dots, t_n)$ -local coefficients. We are interested in $\Omega_n(BG)$, oriented bordism of L^2 -acyclic manifolds with respect to a regular G -cover. Theorem 1: There is a long exact sequence
... $\Omega_n(BZ^n) \rightarrow \Omega_k(BZ^n) \rightarrow L_k(Q(t_1, \dots, t_n)) \rightarrow \dots$
and $L_k(Q(t_1, \dots, t_n))$ vanishes if k is odd. This is proven by modifying the surgery program with a few tricks. Interesting connections with Witt groups of Hermitian forms will be discussed, as well as generalizations to virtually abelian groups. This is a joint work with Sylvain Cappell and Shmuel Weinberger.

Nathan Dunfield: (University of Illinois) Random knots: their properties and algorithmic challenges. I will discuss various models of random knots in the 3-sphere, surveying what is known about them theoretically and what is conjectured about them experimentally. In particular, I will discuss experiments that probe the practical/average case complexity of questions like computing the genus of a knot. I will then fit this into a broader picture of open questions about the computational complexity of various problems in 3-dimensional topology.

Fuquan Fang: (Capital Normal University) Nonnegative curvature and Tits buildings. In this talk I will explain a surprise link between non-negatively curved manifolds with polar actions and Tits buildings. A new geometric characterization of normal homogeneous spaces (of certain types) can be described in terms of polar symmetric, where no transitive action is assumed.

Ailana Fraser: (University of British Columbia) Minimal surface in the ball. I will discuss questions and results on existence, uniqueness, and compactness of free boundary minimal surfaces in the ball of fixed topological type.

Bernhard Hanke: (University of Augsburg) Inessential Brown-Peterson homology and bordism of elementary abelian groups. We revisit the bordism theory of free oriented G -manifolds, where G is an elementary abelian p -group. Complementing previous approaches we pay special attention to elements coming from proper subgroups of G . Our results can be applied to the Gromov-Lawson-Rosenberg conjecture concerning the existence of positive scalar curvature metrics on non-simply connected closed manifolds.

Matthew Hedden: (Michigan State University) Khovanov-Floer theories. Khovanov homology is an easily defined homological invariant of links in the 3-sphere, which generalizes the Jones polynomial. An abundance of much less easily defined homological invariants of links have been defined using symplectic or gauge theoretic Floer homology theories. Quite surprisingly, these invariants are often related to Khovanov homology through spectral sequences. It is natural to wonder why Khovanov homology is connected to so many theories, and what types of structures persist through the spectral sequences. In this talk I'll define an abstract algebraic notion of a "Khovanov-Floer" theory, and sketch a proof that such theories behave naturally with respect to link cobordisms. I'll then show that all the known spectral sequences from Khovanov homology satisfy our definition, implying that link cobordisms induce invariant homomorphism between spectral sequences. I'll assume no knowledge of Khovanov homology, Floer homology, or spectral sequences. This is joint work with John Baldwin and Andrew Lobb.

Ian Leary: (University of Southampton) Concerning KS Brown's question. Using finite extensions of right-angled Coxeter groups we construct groups G for which the minimal dimension of a classifying space for proper actions is strictly greater than the virtual cohomological dimension. In contrast to previous examples these groups do admit a cocompact model for this classifying space. This is joint work with Nansen Petrosyan.

Peng Lu: (University of Oregon) A rigidity theorem for codimension one shrinking gradient Ricci Solutions in Euclidean space. I will present a joint work with Pengfei Guan and Yiyang Xu. In the work we prove a splitting theorem for complete gradient Ricci soliton with nonnegative curvature and establish a rigidity theorem for codimension one complete shrinking gradient Ricci soliton in \mathbb{R}^{n+1} with nonnegative Ricci curvature.

Steven Lu: (Universite du Quebec a Montreal) Orbifold uniformization and a DUY-Simpson's correspondence for singular varieties. The solution to the Calabi conjecture by S.T. Yau (1978) implies directly the following uniformization results: An n -dimensional compact Kahler manifold with positive (respectively trivial) canonical class and vanishing of the integral of $(n c_1^2 - 2(n+1) c_2)w^{n-2}$, where c_1 and c_2 are the Chern classes and w the Kahler class, is uniformized by the hyperbolic ball (respectively by the complex Euclidean space, with the standard Euclidean metric). These generalize directly to the case of compact Kahler orbifolds (i.e. Kahler varieties with at worst quotient singularities) if one replaces the respective Chern classes by their orbifold counterparts. In 1994, a remarkable paper of Shepherd-Barron and Wilson shows that complex projective threefolds with at worst canonical singularities with "numerically trivial" first and second orbifold-Chern classes are uniformized by Abelian three-folds, giving the first instance of uniformization in higher dimensions for non-quotient singularities. Very little has taken place since until the recent article of Greb-Kebekus-Peternell that dealt with varieties that are nonsingular in codimension two but which avoids dealing directly with orbifolds (such as orbifold Chern classes ...). In this talk, I will give the outline of at least a couple of ways in a joint work with Behrouz Taji that generalizes the result of Shepherd-Barron and Wilson to arbitrary dimension and to the case of klt singularities (which are singularities that are more general than quotient singularities but natural in the setting of birational geometry). This also represents a full orbifold generalization of the said result of Greb-Kebekus-Peternell. The first uses the polystability of the tangent sheaves of such varieties and the second the semistability thereof. The first requires a generalization to the orbifold setting of the classical theorem of Narasimhan-Seshadri and Donaldson-Uhlenbeck-Yau while the second that of Simpson's correspondence between semi-stable sheaves and flat vector bundles.

Jason Lotay: (University College London). Hyperkahler 4-manifolds with boundary. Hyperkahler geometry, which arises in the study of special holonomy and Ricci-flat metrics, is also important for theoretical physics and moduli space problems in gauge theory. In dimension 4, hyperkahler geometry takes on a special character, and a natural question arises: given a compact 3-dimensional manifold N which can be a hypersurface in a hyperkahler 4-manifold, when can it actually be "filled in" to a compact hyperkahler 4-manifold with N as its boundary? In particular, starting from a compact hyperkahler 4-manifold with boundary, which deformations of the boundary structure can be extended to a hyperkahler deformation of the interior? I will discuss recent progress on this problem, which is joint work with Joel Fine and Michael Singer.

Johannes Nordstrom: (University of Bath) Disconnecting the G_2 moduli space. Little is currently known about the global properties of the G_2 moduli space of a closed 7-manifold, i.e. the space of Riemannian metrics with holonomy G_2 modulo diffeomorphisms. A holonomy G_2 metric has an associated G_2 -structure, and I will define a $\mathbb{Z}/48$ valued homotopy invariant of a G_2 -structure in terms of the signature and Euler characteristic of a $\text{Spin}(7)$ -coboundary. I will describe examples of manifolds with holonomy G_2 metrics where the invariant is amenable to computation in terms of eta invariants, and which are candidates for having a disconnected moduli space. This is joint work

in progress with Diarmuid Crowley and Sebastian Goette.

Jongil Park (Seoul National University, Korea) On symplectic fillings of quotient surface singularities. One of active research areas in 4-manifold theory is to classify symplectic fillings of certain 3-manifolds equipped with a natural contact structure. Among them, people have long studied symplectic fillings of the link of a normal complex surface singularity. Note that the link of a normal complex surface singularity carries a canonical contact structure which is also known as the Milnor fillable contact structure. For example, P. Lisca classified symplectic fillings of cyclic quotient singularities whose corresponding link is lens space, and A. Nemethi and P. Popescu-Pampu identified the correspondence between the symplectic fillings in Lisca's classification and the Milnor fibers for cyclic quotient singularities. Furthermore, M. Bhupal and K. Ono tried to extend these results, so that they classified all possible symplectic fillings of quotient surface singularities. In this talk, I'd like to investigate the correspondence between the symplectic fillings in Bhupal-Ono's classification and the Milnor fibers of quotient surface singularities. This is a joint work with Heesang Park, Dongsoo Shin, and Giancarlo Urzfiua.

Nathan Perlmutter (University of Oregon) Homological stability for diffeomorphism groups of odd dimensional manifolds. I will present a new homological stability result for the diffeomorphism groups of manifolds of dimension $2n + 1 \geq 9$, with respect to forming the connected sum with copies of an arbitrary $(n - 1)$ -connected, $(2n + 1)$ -dimensional manifold that is stably parallelizable. This work can be viewed as an odd dimensional analogue of a recent result of Galatius and Randal-Williams regarding the homological stability of the diffeomorphism groups of manifolds of dimension $2n \geq 6$, with respect to forming connected sums with $S^n \times S^n$.

Mihaela Pilca (University of Regensburg) Homogeneous Clifford structures on Riemannian manifolds. Clifford structures on Riemannian manifolds generalize almost Hermitian and almost quaternion-Hermitian structures. These recently introduced structures are in a certain sense dual to spin structures. In this talk I will present some recent results on Clifford structures, in particular on their classification in the homogeneous setting. Namely, we will show that there exists an upper bound for their rank on compact manifolds of non-vanishing Euler characteristic. Furthermore, we will give the complete description of the limiting cases of highest possible rank, which involves four of the exceptional Lie groups. In the other extreme case of rank 3 Clifford structures, i.e. homogeneous almost quaternion-Hermitian manifolds, we show that the manifold is either a Wolf space, the product of two spheres S^2 or the complex quadric $SO(7)/U(3)$. The talk is based on joint work with Andrei Moroianu and Uwe Semmelmann.

Tali Pinsky (University of British Columbia) Templates for geodesics on surfaces and the volumes of their complements. A closed geodesic on a hyperbolic surface has a natural lift to the unit tangent bundle of the surface, and is a knot therein. At the same time, it is a periodic orbit of the geodesic flow on the surface. In the talk I'll show how templates, which are embedded branched surfaces carrying the set of periodic orbits of a flow, can be computed for the geodesic flows on certain surfaces called Hecke triangles. I'll then show how Ghys' template for the modular surface can be used to obtain results about the growth of the volumes of complements of closed geodesics.

Piotr Przytycki: (McGill University and Polish Academy of Sciences) Arcs intersecting at most once. I will show that on a punctured oriented surface with negative Euler characteristic χ , the maximal cardinality of a set of essential simple arcs that are pairwise non-homotopic and intersecting at most once is $2|\chi|(|\chi| + 1)$. This gives a cubic estimate in $|\chi|$ for a set of $2|\chi|$ curves pairwise intersecting at most once, which to a great extent answers a question of Farb and Leininger.

Holger Reich: (Free University of Berlin) Algebraic K-theory of group algebras and the cyclotomic trace. The talk will report on joint work with Wolfgang Lueck (Bonn), John Rognes (Oslo) and Marco Varisco (Albany). The Whitehead group $\mathcal{W}h(G)$ and its higher analogues defined using algebraic

K-theory play an important role in geometric topology. There are vanishing conjectures in the case where KG is torsionfree. For groups containing torsion the Farrell-Jones conjectures give a conjectural description in terms of group homology. After an introduction to this circle of ideas, I will report on the following new result, which for example detects a large direct summand inside the rationalized Whitehead group of a group like Thompson's group T . The Farrell-Jones assembly map for connective algebraic K-theory is rationally injective, under mild homological finiteness conditions on the group and assuming that a weak version of the Leopoldt-Schneider conjecture holds for cyclotomic fields. This generalizes a result of Boekstedt, Hsiang and Madsen, and leads to a concrete description of a large direct summand of $K_n(ZG) \otimes_{\mathbb{Z}} \mathbb{Q}$ in terms of group homology. Since the number theoretic assumption holds in low dimensions, this also computes a large direct summand of $Wh(G) \otimes_{\mathbb{Z}} \mathbb{Q}$. In many cases the number theoretic assumptions always hold, so we obtain rational injectivity results about assembly maps, in particular for Whitehead groups, under homological finiteness assumptions on the group only. The proof uses the cyclotomic trace to topological cyclic homology, Boekstedt-Hsiang-Madsen's functor C , and new general injectivity results about the assembly maps for THH and C .

Nikolai Saveliev: (University of Miami) On the deleted squares of lens spaces. The congruence space $F_2(M)$ of ordered pairs of distinct points in a manifold M , also known as the deleted square of M , is not a homotopy invariant of M : Longoni and Salvatore produced examples of homotopy equivalent lens spaces M and N of dimension three for which $F_2(M)$ and $F_2(N)$ are not homotopy equivalent. We study the natural question whether two arbitrary 3-dimensional lens spaces M and N must be homeomorphic in order for $F_2(M)$ and $F_2(N)$ to be homotopy equivalent. Among our tools are the Cheeger-Simons differential characters of deleted squares and the Massey products of their universal covers. This is a joint work with Kyle Evans-Lee.

Thomas Schick: (Georg-August Universitaet Goettingen) Signature and higher index theory. Higson and Roe have used homological algebra over C^* -algebras to map the surgery exact sequence for smooth manifolds to an exact sequence of K-theory groups of C^* -algebras (the latter containing as particular case the Baum-Connes assembly map). Jointly with Paolo Piazza, we have developed an appropriate secondary large scale index theory to directly construct all the maps involved in terms of higher index theory of the signature operator. This allows in particular to extend the result to the topological category. We present this result. To obtain numerical results we show how one can systematically map further to cyclic homology groups to obtain numerical invariants.

Andras Stipsicz: (Alfred Renyi Institute of Mathematics) Concordance homomorphisms from knot Floer homology. Knot Floer homology is a rather effective tool to study concordance properties of knots. By deforming the differential defining the homology, a 1-parameter family of concordance invariants can be defined. One of these invariants, indeed, can be used to derive a bound on the unoriented genus of the knot. This is joint work with Peter Ozsvath and Zoltan Szabo.

Ozgun Unlu: (Bilkent University) Free Group Actions on Manifolds. In this talk we will discuss some methods for constructing free group actions on manifolds. Then we will talk about applications of these methods when these manifolds are products of spheres. Lastly, using the known group theoretic restrictions on finite groups that can act freely on these manifolds, we will give the characterization of the finite groups which can act freely on certain manifolds.

Thomas Walpuski: (MIT) A compactness theorem for the Seiberg-Witten equation with multiple spinors. Unlike the usual Seiberg-Witten equation, its version with multiple spinors does not enjoy a priori estimates which lead to compactness. In fact, compactness can fail; however, in a rather controlled way: after suitable rescaling sequences of solutions of the Seiberg-Witten equation converge to Fueter sections of a bundle of moduli spaces of ASD instantons. I will give a brief sketch of our proof (based on ideas developed by Taubes for $PSL(2, \mathbb{C})$ -connections). Moreover, I

will explain our point of view of the Seiberg- Witten equation within the context of Seiberg-Witten equations with ADHM targets, which will illuminate the connection with gauge theory on G2-manifolds. This is joint work with Andriy Haydys.

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