On Non-Gaussian SST Variability in the Gulf Stream and other Strong Currents

Philip Sura

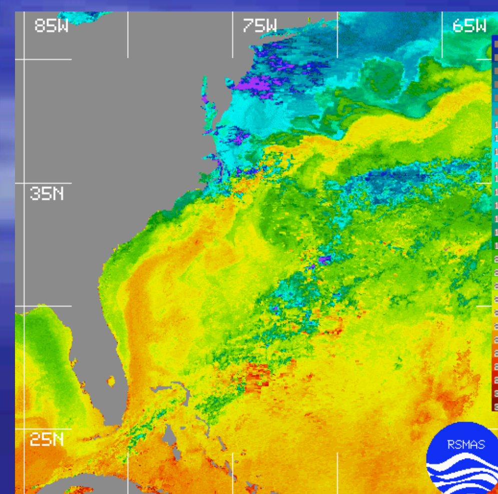
Florida State University

Department of Meteorology

Collaborator:
Prashant Sardeshmukh
NOAA/CIRES
Boulder, CO

Introduction

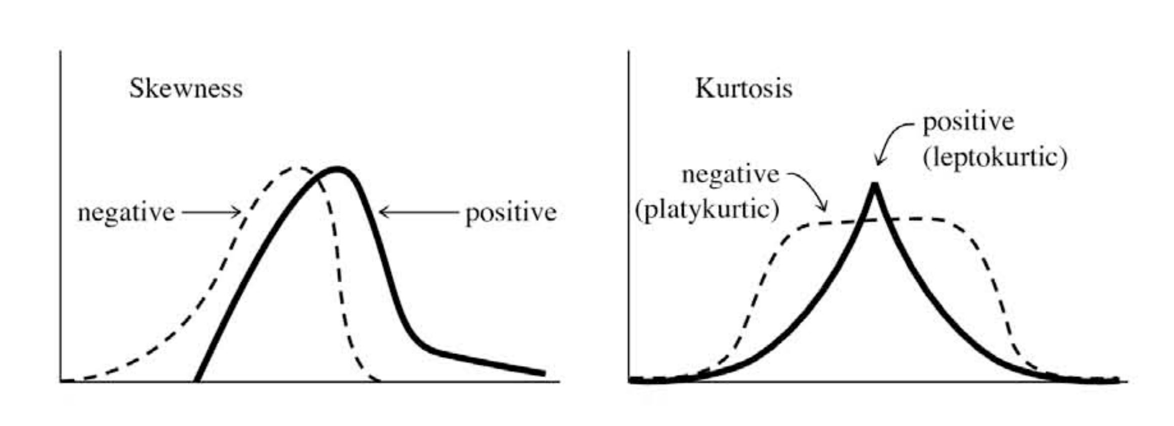
- Since the very early days of physical oceanography the Gulf Stream system plays a central role in the dynamical description of the general circulation of the ocean.
- Here we will study the physics of non-Gaussian SST variability in the Gulf Stream and other strong currents in a recently developed stochastic framework.



Measures of Non-Gaussianity: Skewness and Kurtosis

$$skew \equiv \frac{\left\langle T_o'^{\,3} \right\rangle}{\sigma^3}$$

$$kurt \equiv \frac{\left\langle T_o'^{\,4} \right\rangle}{\sigma^4} - 3$$

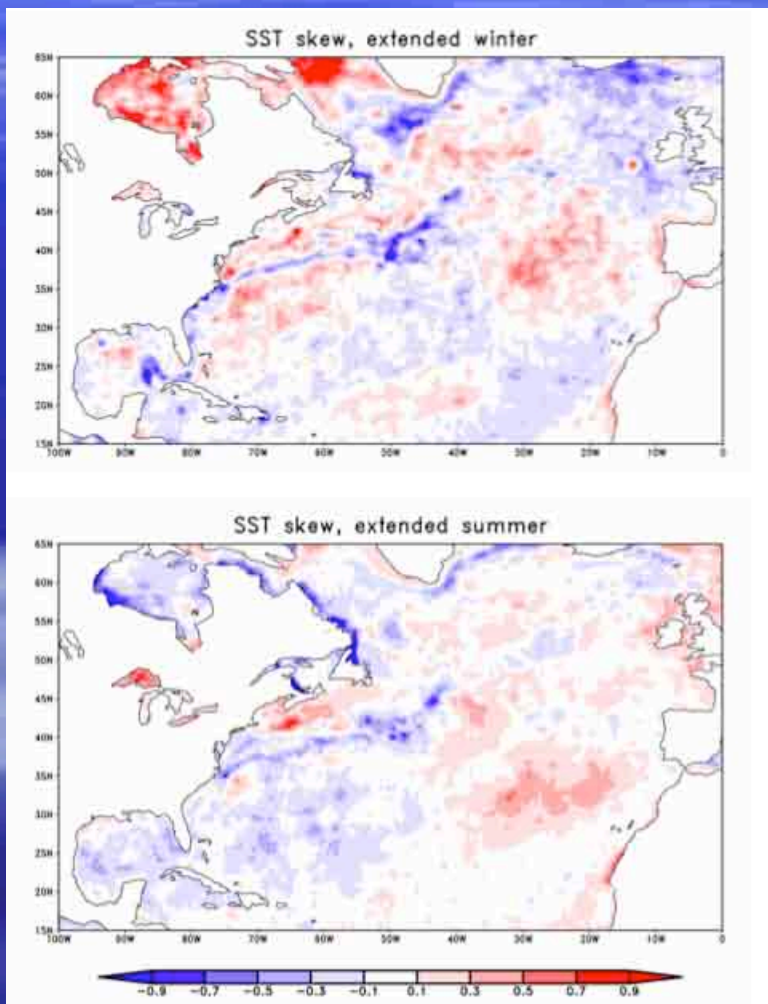


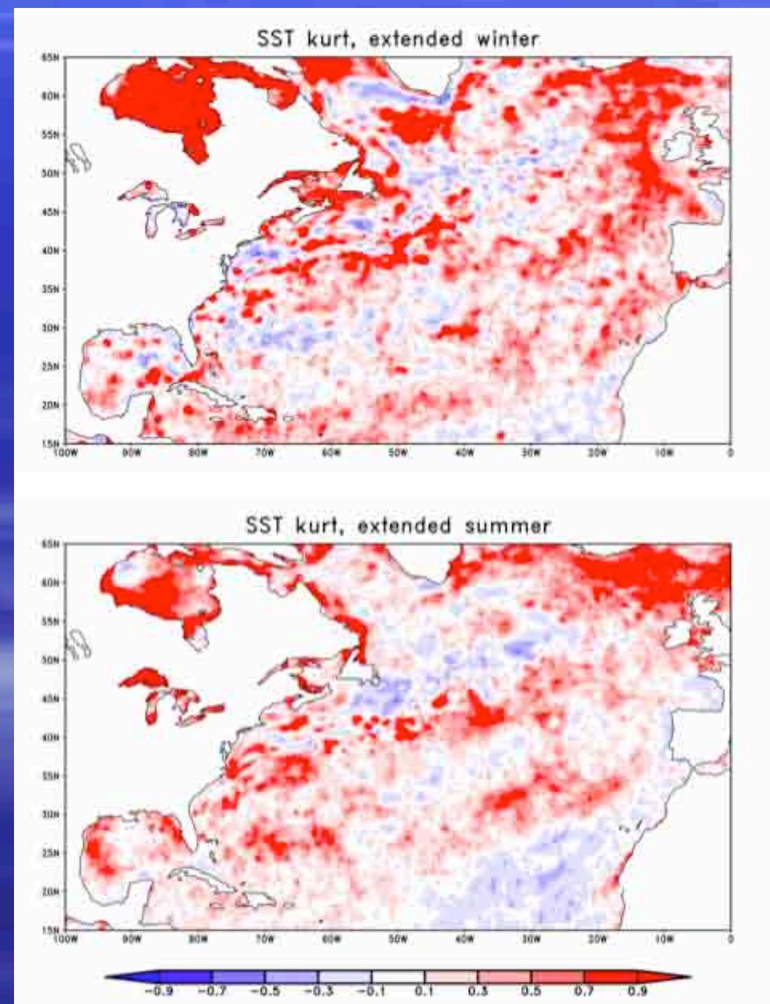
Skewness and **kurtosis** are non-dimensional measures describing the shape of a probability density function (PDF)

The shape of a PDF can provide information about the dynamics of the underlying system!

SST anomaly skewness and kurtosis in the Gulf Stream System

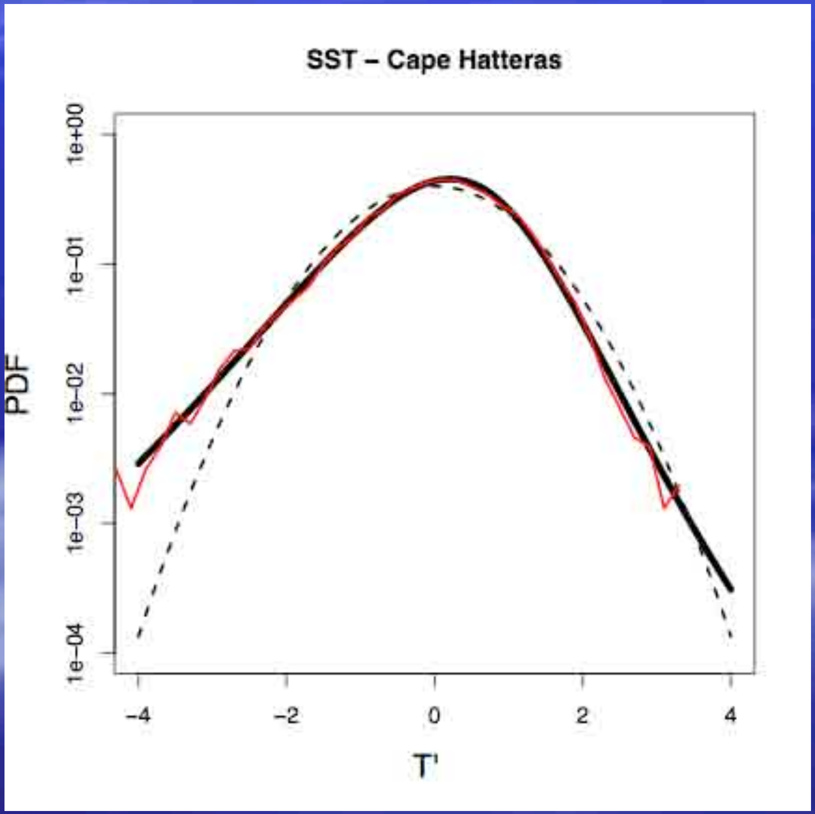
Daily AVHRR SSTs blended with in situ data, 1985-2005

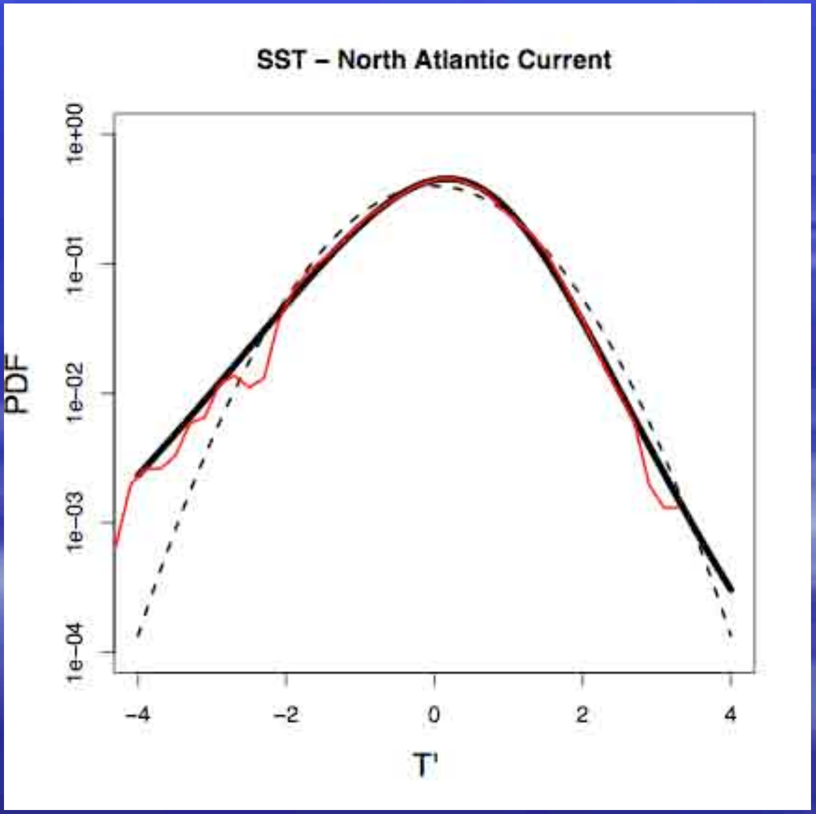




Probability Density Functions in the Gulf Stream System

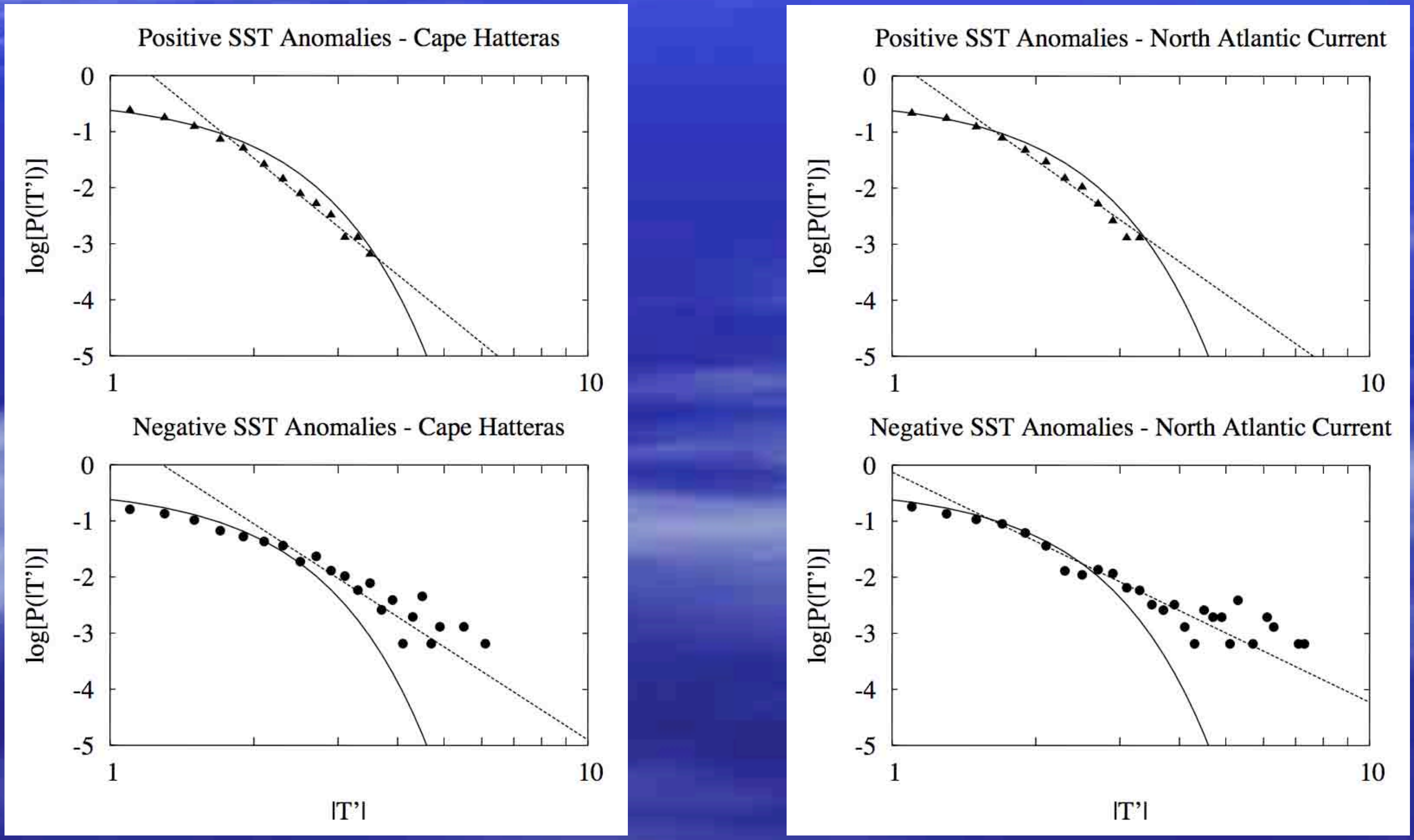
PDFs from full year daily Reynolds data





PDFs Follow a Power-Law in the Gulf Stream System

$$p(x) \propto x^{-\alpha}$$



Modeling SST Anomalies

The heat budget equation for SST T_o is:

$$\frac{\partial T_o}{\partial t} = -\mathbf{v}_o \bullet \nabla T_o + \frac{Q}{\rho C h} - \frac{w_e}{h}(T_o - T_o^b) + \kappa \nabla^2 T_o \equiv F$$

A B C D

- Advection through ocean currents (A)
- Surface heat flux (B)
- Vertical entrainment (C)
- Horizontal mixing (D)

A Taylor expansion of the total heat flux *F* yields:

$$\boxed{\frac{\partial T_o'}{\partial t} = \frac{\partial \bar{F}}{\partial T_o} T_o' + \frac{\partial F'}{\partial T_o} T_o' + F'}$$

Modeling SST Anomalies with Additive Noise

- Neglecting the anomalous heat flux derivative,
- Modeling the anomalous heat flux *F'* as white-noise yields:

$$\frac{\partial T_o'}{\partial t} = \frac{\partial \bar{F}}{\partial T_o} T_o' + \frac{\partial F'}{\partial T_o} T_o' + F' \quad \approx>$$

$$\boxed{\begin{aligned} \frac{\partial T_o'}{\partial t} &= \frac{\partial \bar{F}}{\partial T_o} T_o' + F' \\ \frac{\partial T_o'}{\partial t} &= -\lambda_{eff} T_o' + F' \end{aligned}}$$

Frankignoul and Hasselmann (1977)

Pros:

- The red-noise spectrum is consistent with observations.

Cons:

- The PDF is strictly Gaussian and not consistent with observations.

Modeling SST Anomalies with Linear Multiplicative Noise

- Including the anomalous heat flux derivative,
- Modeling the anomalous heat flux and the anomalous heat flux derivative as white noise yields:

$$\frac{\partial T_o'}{\partial t} = \frac{\partial \bar{F}}{\partial T_o} T_o' + \frac{\partial F'}{\partial T_o} T_o' + F' + R'$$

$$\frac{\partial T_o'}{\partial t} = -\lambda_{eff} T_o' - \phi F' T_o' + F' + R'$$

Pros:

- The red-noise spectrum is consistent with observations.
- Skewness and kurtosis are consistent with observation.

Equation for the Moments: Skewness and Kurtosis

The Fokker-Planck equation (FPE) for the stationary PDF *p(x)* of SST anomalies is:

$$0=\frac{d}{dx}\left[\lambda_{eff}xp\right]+\frac{1}{2}\frac{d^2}{dx^2}\left[\left(\sigma_{F'}{}^2+\sigma_{R'}{}^2+\phi^2\sigma_{F'}{}^2x^2-2\phi\sigma_{F'}{}^2x\right)p\right]$$

Moments $<x^n>$ are obtained by multiplying the FPE by x^n and integrating by parts:

$$\left[\lambda_{eff}-\frac{n-1}{2}\left(\phi\sigma_{F'}\right)^2\right]<x^n> \;=\; -(n-1)\,\phi\,\sigma_{F'}{}^2<x^{n-1}> \;+\; \frac{n-1}{2}\left(\sigma_{F'}{}^2+\sigma_{R'}{}^2\right)<x^{n-2}>$$

In particular, the second, third, and fourth moments are:

$$<x^2> \;=\; \left(\sigma_{F'}{}^2+\sigma_{R'}{}^2\right)/\left[2\lambda_{eff}-\left(\phi\sigma_{F'}\right)^2\right],$$

$$<x^3> \;=\; -2\phi\sigma_{F'}{}^2<x^2>/\left[\lambda_{eff}-\left(\phi\sigma_{F'}\right)^2\right],$$

$$<x^4> \;=\; \left[-3\phi\sigma_{F'}{}^2<x^3>+(3/2)\left(\sigma_{F'}{}^2+\sigma_{R'}{}^2\right)<x^2>\right]/\left[\lambda_{eff}-(3/2)\left(\phi\sigma_{F'}\right)^2\right].$$

Equation for the Moments: Skewness and Kurtosis

$$\boxed{kurt = A\ skew^2 + B}$$

$$A = \frac{\tfrac{3}{2}\left(-\lambda_{eff} + \left(\phi\sigma_{F'}\right)^2\right)}{\left(-\lambda_{eff} + \tfrac{3}{2}\left(\phi\sigma_{F'}\right)^2\right)}$$

$$\Rightarrow$$

$$A \geq \tfrac{3}{2}$$

$$B = \frac{3\left(-\lambda_{eff} + \tfrac{1}{2}\left(\phi\sigma_{F'}\right)^2\right)}{\left(-\lambda_{eff} + \tfrac{3}{2}\left(\phi\sigma_{F'}\right)^2\right)} - 3$$

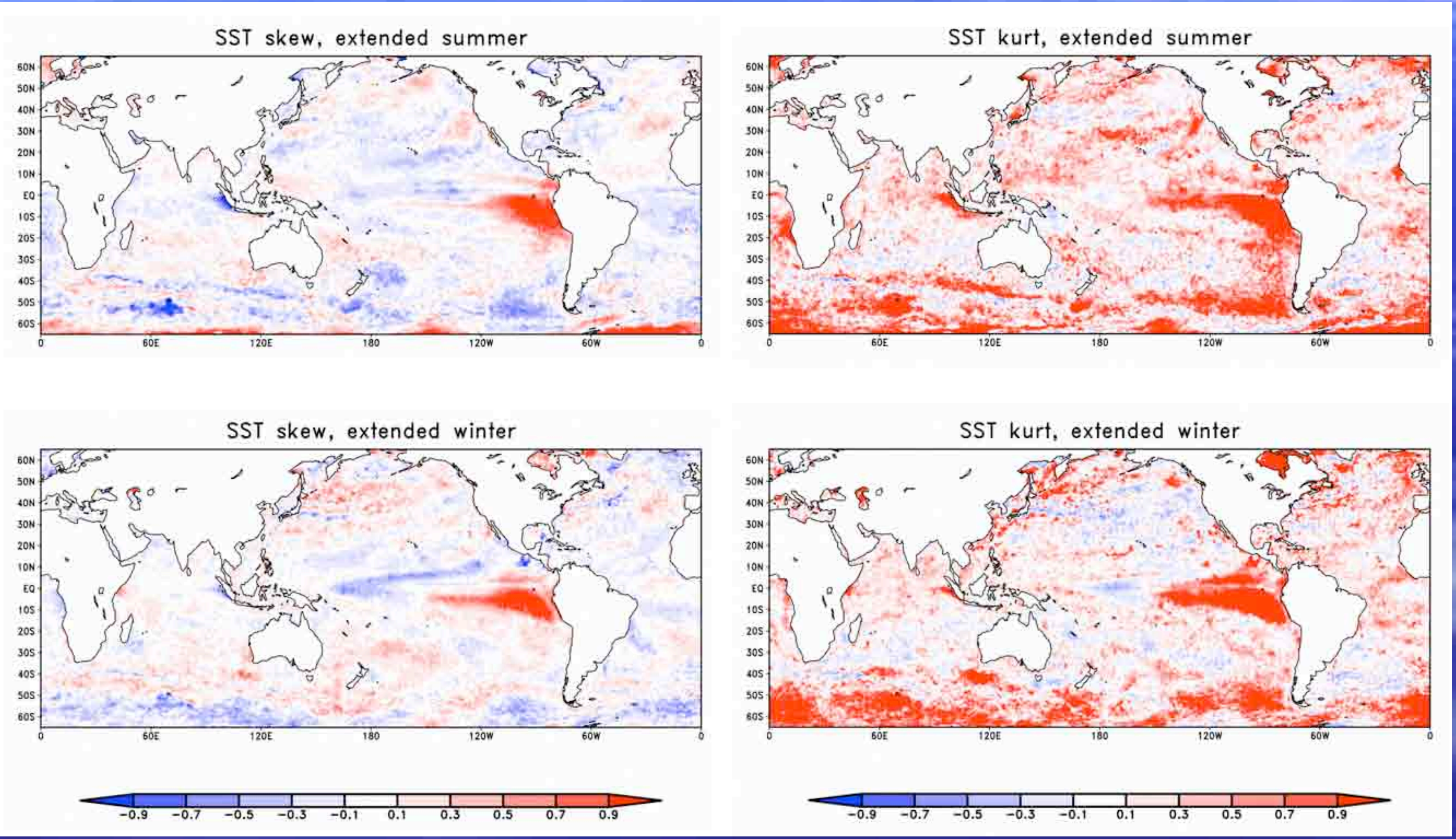
$$\Rightarrow$$

$$B \geq 0$$

$$\boxed{kurt \geq \frac{3}{2} skew^2}$$

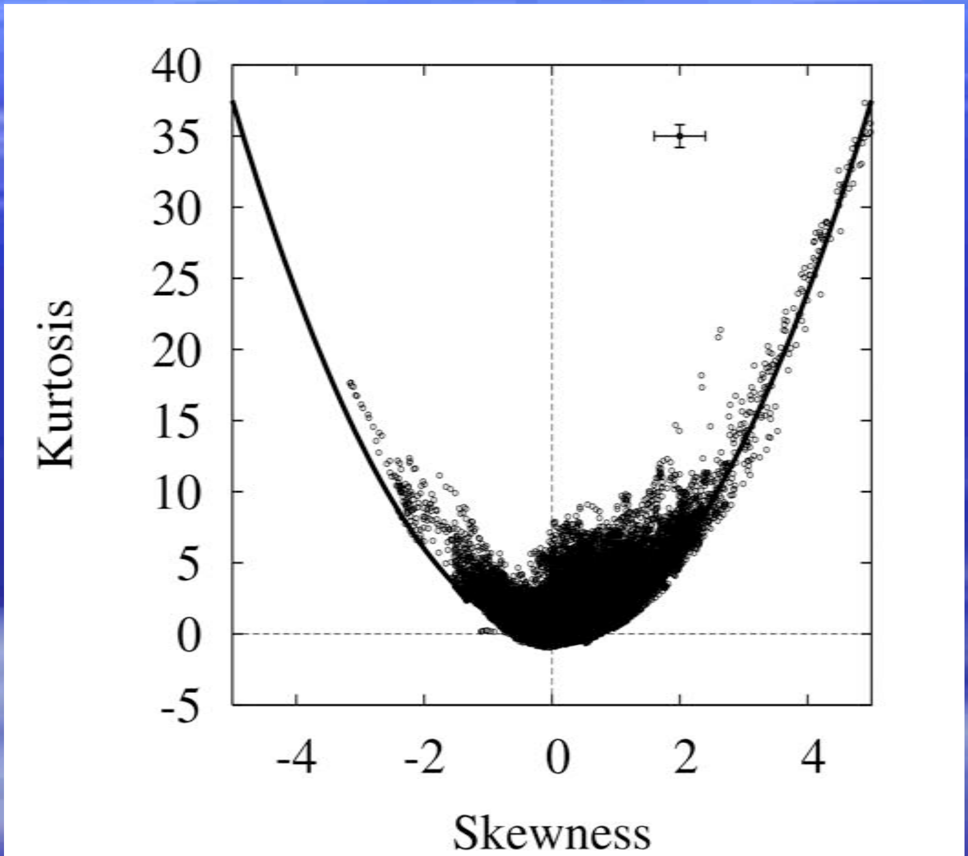
Skewness and Kurtosis - SST Anomalies

Daily AVHRR SSTs blended with in situ data, 1985-2005



Dataset from Reynolds et al. (2006)

Skewness and Kurtosis - SST Anomalies



$$kurt \geq \frac{3}{2} skew^2$$

The observed skewness-kurtosis link is consistent with our univariate linear model with linear multiplicative noise.

Power-Law - SST Anomalies

The Fokker-Planck equation (FPE) for the stationary PDF *p(x)* of SST anomalies is:

$$0=\frac{d}{dx}\left[\lambda_{eff}xp\right]+\frac{1}{2}\frac{d^2}{dx^2}\left[\left(\sigma_{F'}{}^2+\sigma_{R'}{}^2+\phi^2\sigma_{F'}{}^2x^2-2\phi\sigma_{F'}{}^2x\right)p\right]$$

For large *x* the FPE becomes:

$$0=\lambda_{eff}xp+\frac{1}{2}\frac{d}{dx}\left(\phi^2\sigma_{F'}{}^2x^2\right)p$$

$$p(x)\propto x^{-\alpha}$$

$$\alpha=2\left(\frac{\lambda_{eff}}{\phi^2\sigma_{F'}^2}+1\right)$$

The observed power-law is consistent with our univariate linear model with linear multiplicative noise.

Modeling Non-Gaussian SST Anomalies

$$\frac{\partial T_o'}{\partial t} = -\lambda_{eff} T_o' - \phi F' T_o' + F' + R'$$

This univariate linear model with linear multiplicative noise captures the observed non-Gaussianity very well.

The so far unspecified parameter ϕ determines the skewness of SST variability.

The skewness is

- positive if the additive and multiplicative noises are positively correlated ($\phi < 0$).
- negative if the additive and multiplicative noises are negatively correlated ($\phi > 0$).

Modeling Non-Gaussian SST Anomalies

The heat budget equation for SST T_o is:

$$\frac{\partial T_o}{\partial t} = -\ \mathbf{v}_o \bullet \nabla T_o \quad + \quad \frac{Q}{\rho C h} \quad - \quad \frac{w_e}{h}(T_o - T_o^b) \quad + \quad \kappa \nabla^2 T_o \quad \equiv \quad F$$

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- Advection through ocean currents (A)
- Surface heat flux (B)
- Vertical entrainment (C)
- Horizontal mixing (D)

A Taylor expansion of the total heat flux *F* yields:

$$\boxed{\frac{\partial T_o'}{\partial t} = \frac{\partial \bar{F}}{\partial T_o} T_o' + \frac{\partial F'}{\partial T_o} T_o' + F'}$$

Modeling Non-Gaussian SST Anomalies: The Surface Heat Flux

$$\frac{\partial T_o}{\partial t} = \beta(T_a - T_o)|\mathbf{U}| \equiv F_Q$$

$$\frac{\partial F_Q}{\partial T_o} = -\beta|\mathbf{U}| = -\beta(|\overline{\mathbf{U}}| + |\mathbf{U}|')$$

$$F_Q' = \beta(\overline{T_a} - \overline{T_o})|\mathbf{U}|'$$

$$(\overline{T_a} - \overline{T_o}) < 0$$

$$\Longrightarrow$$

$$\frac{\partial T_o'}{\partial t} = \frac{\partial F_Q}{\partial T_o} T_o' + F_Q'$$

$$\Leftrightarrow$$

$$\frac{\partial T_o'}{\partial t} = -\beta(|\overline{\mathbf{U}}| + |\mathbf{U}|')T_o' + \beta(\overline{T_a} - \overline{T_o})|\mathbf{U}|'$$

Parameterizing $|\mathbf{U}|'$ as white-noise we get

$$\frac{\partial T_o'}{\partial t} = -\lambda_{eff} T_o' + F' T_o' + F' + R'$$

$$\phi \equiv -1$$

The skew induced by the surface heat flux is always positive because the additive and multiplicative noises are positively correlated ($\phi < 0$).

That's because the ocean is warmer than the atmosphere.

Modeling Non-Gaussian SST Anomalies: Advection Through Ocean Currents

$$\frac{\partial T_o}{\partial t} = - \mathbf{v}_o \bullet \nabla T_o \equiv F_A$$

$$\frac{\partial F_A}{\partial T_o} = -\frac{\partial}{\partial T_o}\left(u\frac{\partial T_o}{\partial x} + v\frac{\partial T_o}{\partial y}\right) = -\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = -\nabla \mathbf{v}_o'$$

$$F_A{}' = -\bar{\mathbf{v}}_o \bullet \nabla T_o' - \mathbf{v}_o' \bullet \nabla \bar{T}_o - \mathbf{v}_o' \bullet \nabla T_o' + \overline{\mathbf{v}_o' \bullet \nabla T_o'}$$

Neglecting second order anomaly products we get

$$F_A{}' \approx -\bar{\mathbf{v}}_o \bullet \nabla T_o' - \mathbf{v}_o' \bullet \nabla \bar{T}_o = -\mathbf{v}_o' \bullet \nabla \bar{T}_o + slow$$

Neglecting slow components we get

$$F_A{}' \approx -\mathbf{v}_o' \bullet \nabla \bar{T}_o = -\nabla(\mathbf{v}_o' \bar{T}_o) + \bar{T}_o \nabla \mathbf{v}_o'$$

The oceanic velocity field is split into

- slow geostrophic
- fast ageostrophic

components.

Modeling Non-Gaussian SST Anomalies: Advection Through Ocean Currents

$$\frac{\partial T_o'}{\partial t} = \frac{\partial F_A}{\partial T_o} T_o' + F_A'$$

$$\Leftrightarrow$$

$$\frac{\partial T_o'}{\partial t} = -T_o' \nabla \mathbf{v}_o' - \nabla(\mathbf{v}_o' \overline{T}_o) + \overline{T}_o \nabla \mathbf{v}'$$

Next we parameterize the rapidly varying divergence of the ageostrophic velocity field as white-noise, and the remaining fast term as a noise residual R'. If we also include a damping term we get

$$\frac{\partial T_o'}{\partial t} = -\lambda_{eff} T_o' - F' T_o' + F' + R'$$

$$\phi \equiv +1$$

The skew induced advection through ocean currents is always negative because the additive and multiplicative noises are negatively correlated ($\phi > 0$).

Summary and Conclusions

- It is the divergence of the rapidly varying ageostrophic velocity field that most likely causes the negative skewness in strong currents.
- Through temperature advection the velocity divergence introduces negatively correlated additive and multiplicative noise terms in the evolution equation of SST anomalies.
- The required damping of SST anomalies is provided by the heat flux through the sea surface.
- A combination of SST advection and heat flux forcing could result in negative or positive skewness. It is the relative strength of each process responsible for the net skewness. In very strong currents such as the Gulf Stream the SST advection is strong enough to significantly affect the mixed-layer heat budget, resulting in the pronounced SST skewness observed along-stream strong currents.