Summer School Schedule: All Sessions in ESB 2012

Course Speakers

1. **Eric Friedlander, University of Southern California, U.S.A.**
   Rational cohomology and supports for linear algebraic groups
2. **Jesper Grodal, University of Copenhagen, Denmark.**
   Group actions via homotopy theory
3. **Radha Kessar, City University, London, U.K.**
   Local representation theory
4. **Peter Symonds, University of Manchester, U.K.**
   Endo-trivial modules for infinite groups

Time/ Day	Wed June 27	Thur June 28	Fri June 29	Sat June 30
8:30am-8:50am	**Check-in (ESB Atrium) & PIMS Welcome**			
9:00am-10:15am	**Prof. Radha Kessar** Local representation theory	**Prof: Jesper Grodal** Endotrivial modules for finite groups	**Prof. Radha Kessar** Local representation theory	**Prof.Jesper Grodal** Endotrivial modules for finite groups
Coffee Break (ESB 2012 Lobby)				
10:45am-12:00pm	**Prof: Jesper Grodal** Endotrivial modules for finite groups	**Prof. Radha Kessar** Local representation theory	**Prof: Jesper Grodal** Endotrivial modules for finite groups	**Prof. Radha Kessar** Local representation theory
Lunch (See List of UBC Eateries)				
2:00pm-3:15pm	**Prof. Eric Friedlander** Rational cohomology and supports for linear algebraic groups	**Prof. Peter Symonds** Endotrivial Modules for Infinite Groups	**Prof. Eric Friedlander** Rational cohomology and supports for linear algebraic groups	**Prof. Peter Symonds** Endotrivial Modules for Infinite Groups
Coffee Break (ESB 2012 Lobby)				
3:45pm-5:00pm	**Prof. Peter Symonds** Endotrivial Modules for Infinite Groups	**Prof. Eric Friedlander** Rational cohomology and supports for linear algebraic groups	**Prof. Peter Symonds** Endotrivial Modules for Infinite Groups	**Prof. Eric Friedlander** Rational cohomology and supports for linear algebraic groups
Evening Events	**5:00pm- 6:30pm** Welcome Reception and Networking Event: **(ESB Atrium)**			

Couse readings

The summer school speakers have prepared a list of readings for the course. Please review the below courses readings an preparation material.

Eric Friedlander, University of Southern California, U.S.A

Rational cohomology and supports for linear algebraic groups

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Linear algebraic groups and rational representations

1. J. Humphreys, Linear Algebraic Groups, GTM 1975
2. J. Jantzen, Representations of Algebraic Groups, MSM 2003

Frobenius kernels

3. W. Waterhouse, Introduction to Affine Group Schemes, GTM 1979
4. F-Suslin, Cohomology of finite group schemes over a field,Invent. Math 1997

1-parameter subgroups and support varieties

5. Suslin-F-Bendel, Infinitesimal 1-parameter subgroups and cohomology & Support varieties for infinitesimal group schemes, JAMS 1997.
6. Friedlander, Support varieties for rational representations,Compos. Math 2015

Cohomological calculations

7. F-Parshall, On the cohomology of algebraic and related finite groups, Invent. Math 1982
8. Suslin-F-Bendel, ibid

Jesper Grodal, University of Copenhagen, Denmark.

Endotrivial modules for finite groups via homotopy theory

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In my 4 lectures I'll show how to use homotopy theory to determine groups of endotrivial modules for G a finite group, in particular those endotrivial modules which restrict to a trivial module direct sum a projective on a Sylow p-subgroup. We will start from from basics, and end with a solution to a conjecture of Carlson--Thevenaz and calculations for e.g., the Monster. We'll try to have plenty of concrete examples and calculations throughout, and see a number of classical concepts from algebraic topology applied to a problem in representation theory. An approximate outline is listed below:

Lecture 1: Endotrivial and Sylow-trivial modules -- an overview.
New concepts (also to be talked about during exercises?): kG-modules, categories and functors, orbit categories, cohomology of spaces and categories, limits over arbitrary indexing categories.

Lecture 2: Subgroup complexes and coefficient systems
New concepts: nerves of categories, subgroup complexes, coefficient systems, Thomason's theorem, Rickard's theorem, the Steinberg complex.

Lecture 3: Fundamental groups and fusion systems
New concepts: Fundamental groups of categories, manipulations with nerves, fusion systems, Alperin's fusion theorem.

Lecture 4: Homology decompositions and the Carlson-Thevenaz conjecture
New concepts: The isotropy spectral sequence.

Radha Kessar, City University, London, U.K.

Local representation theory

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Some familiarity with the basics of representation theory of finite groups and finite dimensional associative algebras would be helpful. Suggested background reading:

1. J.L Alperin and R. B Bell, Groups and Representations , GTM 162, Springer 1995
2. J. Thevenaz, G-algebras and modular representation theory, OUP, 1994, Chapter 1.

For a more expanded version, a good source is:

3. C. Curtis and I. Reiner, Methods of representation theory, Wiley 1982.
4. H. Nagao and Y. Tsushima, Representations of finite groups, Academic Press, 1989, Chapters 1 and 2

For a quick immersion into modular representation theory, there is:

5. (e) J. L Alperin, Local Representation Theory, CUP 1986. Chapters 1, 2 and 3

Peter Symonds, University of Manchester, U.K.

Endotrivial Modules for Infinite Groups

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Prerequisites:

- First of all, a basic knowledge of homological algebra: chain complexes, projective and injective resolutions and constructions involving them. We won't mention the derived category or spectral sequences except in passing, but a basic understanding of the former would provide some context for what we do.

- You will also want to know what a triangulated category is and why the stable category of the modular group ring of a finite group (or, more generally, of a Frobenius algebra) and various categories of chain complexes are triangulated.
  There are many texts on this, e.g.
  1. Happel, Triangulated categories and and the representation theory of finite dimensional algebras.
  2. Weibel, Introduction to homological algebra.

- Some very basic theory about the (modular) representations of groups will also be needed. Tensor product, Hom, induction (and coinduction for infinite groups). There are many books on this (and another course in this summer school).

Background:

- We will only need some basic facts about endotrivial modules for finite groups and we will go over them. But if you want to know what can be done, there is a survey article by Thevenaz in Geck, Testermann and Thevenaz, Group representation theory.

- We won't say much about the cohomology of groups, but it provided the motivation for much of this work. If you are keen, it would be good to skim enough of KS Brown, Cohomology of Groups to be able to follow Chapter X. Actually, what we will do is closer to Ikenaga, Homological dimension and Farrell Cohomology, J Algebra 1984.

- Finally, if you know anything about Gorenstein projective modules, that will provide some context. I can't think of any sources close to what we need that are not rather technical. Possibly Enochs and Jenda, Relative homological algebra.