Course title

Selected topics in modern optimization

Term and format

Summer 2013, June 14-28, 3 mini-blocks, each consisting of 3-hour daily lectures –approximately 15 lecture hours for each block, totalling to ~45-hour equivalent or higher load of a full-semester graduate course load– followed by a written block exam, with every block being delivered by an invited mentor; targeted enrollment of 40-50 graduate students.

Syllabus, mentors and block descriptions

Overall, the main goal of the course is to expose students to a number of cutting-edge (yet somewhat exotic) research topics in optimization, that alone are not normally featured in a standard graduate-level course work. In particular, the material is centered on the rich interplay between continuous and discrete optimization, and targets equipping the students with both theoretical and practical modeling and computational skills.

It is intended that the course receives U of Calgary Graduate School accreditation, and thus the academic credits become transferrable to other institutions via West Deans Agreement, etc.

June 14 to June 18

Antoine Deza, Professor and Canada Research Chair in Combinatorial Optimization
Department of Computing & Software, McMaster University

Combinatorial, Geometric, and Computational Aspects of Linear Optimization

Optimization has long been a source of inspiration and applications for geometers, and similarly, discrete and convex geometry has provided foundations for many efficient optimization techniques. Combinatorics forms an intrinsic part of optimization, and there is a rich interplay between geometry and combinatorics.

This module will illustrate these connections via the in-depth presentation and study of a generalization of linear optimization called colourful linear programming and the associated colourful simplicial depth question.

In statistics, there are several measures of the depth of a point p relative to a fixed set S of sample points in dimension d. One of the most intuitive is the simplicial depth of p introduced by Liu (1990), which is the number of simplices generated by points in S that contain p.

Obtaining a lower bound for the simplicial depth is a challenging problem. Caratheodory Theorem can be restated as: The simplicial depth is at least 1 if p belongs to the convex hull of S. Barany (1982) showed that the simplicial depth is a least a fraction of all possible simplices generated from S. Gromov

(2010) improved the fraction via a topological approach. Barany's result uses a colourful version of Caratheodory Theorem leading to the associated colourful simplicial depth question.

The module presents recent generalizations of the Colourful Caratheodory Theorem due to Arocha et al. and Holmsen et al. and a strengthening of these. Using a combinatorial generalization, a new lower bound for the colourful simplicial depth is explained, improving the earlier bounds of Barany and Matousek and of Stephen and Thomas. Computational approaches for small dimensions and the colourful linear programming feasibility problem introduced by Barany and Onn are discussed.

Post-module evaluation: the students will be given a set of problems from which they can choose 1 and will be marked based on a written report they have to submit within 4 weeks. The problems will cover combinatorial, geometric, and computational challenges of colourful linear programming and colourful simplicial depth so students can pick the approach they are more comfortable with. Team work is encouraged.

June 19 to June 23
UBCO group led by
Heinz Bauschke, Professor and Canada Research Chair in Convex Analysis and Optimization
Department of Mathematics and Statistics, UBC Okanagan

This course block is dedicated to general area of *convex analysis* and specifically deals with four separate aspects of modern optimization: monotone operators, variational analysis, derivative-free optimization, and computer-aided convex analysis.

Dr. Heinz Bauschke: An Invitation to Monotone Operator Theory

Monotonicity (for set-valued operators) is an elegant and powerful mathematical notion capturing both gradients of convex functions and matrices with a positive semidefinite symmetric part. Monotone operators are ubiquitous in the analysis and understanding of modern convex optimization problems. In these lectures, I aim to highlight (1) definitions and basic results for monotone operators, connections to convex analysis and duality theory; (2) applications of monotone operator theory to the Chebyshev and the Klee problem from analysis; (3) splitting methods for finding zeros of the sum. References: H.H. Bauschke and P.L. Combettes, Convex Analysis and Monotone Operator Theory in Hilbert Spaces, Springer, 1st edition, 2011. R.T. Rockafellar and R.J-B Wets, Variational Analysis, Springer, corrected 3rd printing, 2009. (Evaluation is written by homework, no group work)

Dr. Shawn Wang: Variational Analysis for Beginners

In differential calculus, Fermat's theorem states that if a differentiable function has a local minimum or a local maximum at a point, then the function derivative at that point is zero. In many practical optimization problems, the objective functions are not differentiable. What should we do if the function is not differentiable? Modern variational analysis provides basic constructs and techniques which are well-suited to this purpose. Three lectures as follows will be delivered.

(1) Subdifferentials and subderivatives for convex and nonconvex functions.

(2) Proximal averages of functions and resolvent averages of matrices.

(3) Chebyshev and Klee functions.

A standard reference is: R.T. Rockafellar and R.J-B Wets, Variational Analysis, Springer, corrected 3rd printing, 2009. Evaluation is written, no group work.

Dr. Warren Hare: Introduction to Derivative Free Optimization

Optimization, the study of minimizing or maximizing a function, arises naturally in almost every scientific research field. Applications can be found in everything from business (e.g., minimizing cost and maximizing profit), to government (e.g., minimizing hospital wait times), to engineering (e.g., maximizing structural integrity). In many modern applications, evaluating of the objective function of a problem is acquired through the output of complicated simulation software. In such situations, analytic structure to the objective function is unavailable, so gradients cannot be computed and one must resort to methods that rely on nothing more than function evaluations. The study and design of such methods is growing field of “Derivative Free Optimization” (DFO). In this lecture series we will explore some of the past, present, and future of DFO. We shall discuss the famous “Nelder-Mead” method, and explore its theory, strengths, and weaknesses. We shall examine modern “Simplex Gradient” algorithms and their convergence theory. Final, we shall see how the modern tools of nonsmooth analysis are currently being employed to generate new powerful methods in DFO.

Evaluation is written by homework, no group work.

Dr. Yves Lucet: Computer-Aided Convex Analysis

Convexity is what makes an optimization problem easy or hard: nonconvex problems are difficult to solve and one has usually to settle for a local minimum while convex optimization problems, even if there are nonsmooth, can be solved with polynomial time algorithms. Convex analysis has built numerous tools to build such efficient algorithms. Using the Computational Convex Analysis numerical library, we will compute convex operators and build an intuition on the insight they give. The lab will introduce the main operators found in convex analysis and will draw from the other 3 lectures to visualize operators in 2 and 3D. Then the lab will introduce advanced operators and several applications (computer vision, robot navigation, etc.) The focus will be on conjectures and open problems in computational convex analysis even for functions of one variable. No programming experience is required since the library contains numerous high level operators.

Evaluation: 5 questions to be answered individually with the help of a computer. First 2 hours will be tutorial like, the last 2 hours will be more free-form and will provide time to answer the questions.

June 24 to June 28
Miguel F. Anjos, Canada Research Chair in Discrete Nonlinear Optimization in Engineering
Mathematics & Industrial Engineering, Ecole Polytechnique de Montreal

Conic Relaxations for Discrete Optimization

Conic optimization involves optimizing a linear function over the intersection of an affine space and a closed convex cone. One special case of great interest is the cone of positive semidefinite matrices, for which the resulting problem is called a semidefinite optimization problem. Semidefinite and conic relaxations are known to be highly effective for approximating many hard discrete optimization problems. This course will give an introduction to conic relaxations for discrete optimization.

Assessment: students will be assessed by a combination of

(i) individual short quizzes, and

(ii) a small group project.

Textbook and reading material

A suitable textbook, research papers and reading material will be chosen by the respective mentors.

High-level list of topics include

- optimization in combinatorics, colorful linear programming,
- theoretical aspects and computational tools in convex analysis,
- efficient numerical schemes and approximation for combinatorial problems.

Prerequisites

Good level of familiarity with multivariate calculus and linear algebra, some previous exposure to optimization methods, e.g., linear programming, etc.; exposure to graduate level optimization courses is an asset but is not strictly required.