Free Cash-Flow, Issuance Costs and Stock Volatility

by

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Introduction (1)

▷Research questions: Optimal level of cash holdings for a corporation? Implications in terms of security issuance and payout policy? When to issue new securities? Design of securities? Dynamics of prices?

guidance for a simple theoretical model

▷ Why cash holding? Use cash to finance activities and investment when other sources of funding are costly.

- Precautionary motive for holding cash is very strong Opler et al 1999, JFE, US 1971-94.
- Cost of external finance: Hennessy and Whited 2007, JOF; Lee et al 1996, JFR; (Average cost of SEO: 7.1% of the proceeds of the issuing; SEO infrequent and lumpy) Bazdresh, 2005.

▷ Why is it costly? High levels of cash induce managers to engage in wasteful activities.

- Easterbrook, 1984, Jensen, 1986
- Dittmar and Mahrt-Smith, 2007, JFE; Kalcheva and Lins, 2007, RFS

Introduction (2)

▷ Main Results

- issuance and payout policies that maximize the value of the firm.
  - firms have target cash levels (cash in excess of certain threshold is returned to shareholders) (Opler et al, 1999, DeAngelo, DeAngelo and Stulz, 2006, JFE).
  - firms optimally issue equity. Equity adjustments take place in lumpy and infrequent issues.
- asset pricing implications of financing costs and agency
  - stock prices exhibit heteroskedasticity
  - dollar volatility of stock prices increases after a negative shock on stock prices. (Black, 1976, " When things go badly for the firm, its stock price will fall, and the volatility of the stock will go up.")

▷ Contribute to complement the CTCF literature initiated by Black and Cox, 1976, Leland, 1994.

▷ Relation to the math. Fin. literature on optimal dividend and liquidity management policies: Jeanblanc and Shiryaev 1995; Sethi and Taksar, 2002; Lokka and Zervos, 2005; Cadenillas and Clark 2007.

The Model (1)

▷ Cumulative cash-flow process R_t:

$$R_0 = 0 \quad dR_t = \mu dt + \sigma dW_t.$$

▷ Frictions

- Fixed and proportional issuance costs
  $m, \quad i, \quad m + \frac{i}{p} - f$
- managerial inefficiencies

▷ Issuance policy

- dates at which new security is issued: $(\tau_n)_{n \geq 1}$
- issuance proceed: $(i_n)_{n \geq 1}$
- Total issuance proceed: $I_t = \sum_{n \geq 1} i_n 1\!\!1_{\tau_n \leq t}$
- Total fixed issuance costs: $F_t = \sum_{n \geq 1} f 1\!\!1_{\tau_n \leq t}$

▷ Cash reserves process

- $M = \{M_t;\, t \geq 0\}$

  $$M_0^- = m, \;\; dM_t = (r - \lambda) M_t dt + dR_t + \frac{1}{p} dI_t - dF_t - dL_t$$

- Bankruptcy time $\tau_B = \{t \geq 0 \,|\, M_t < 0\}$

The Model (2)

▷ Value of the firm for a given policy

$$v(m;(\tau_n)_{n\geq 1},(i_n)_{n\geq 1},L)=\mathbb{E}^m\Big[\int_0^{\tau_B} e^{-rt}\,(dL_t-dI_t)\Big],$$

▷Value function

$$V^*(m)=\sup_{(\tau_n)_{n\geq 1},(i_n)_{n\geq 1},L}\Big\{v(m;(\tau_n)_{n\geq 1},(i_n)_{n\geq 1},L)\Big\}$$

▷ Questions

- value function,
- optimal issuance and payout policies,
- optimal security,
- dynamics of security prices,
- testable asset pricing implications.

▷ First-best environment

$$V(m)=m+\mathbb{E}\left[\int_0^{\infty} e^{-rt}\,(\mu dt+\sigma dW_t)\right]=m+\frac{\mu}{r}.$$

Benchmark: $\mathbf{p = 1,\ f = 0,\ \lambda > 0}$

▷ distribute all initial cash reserve m as a special payment at date 0, hold no cash beyond that date.

▷ The pair (L, I)

$$L_t = m1_{\{t=0\}} + lt\,; \qquad I_t = (l-\mu)t - \sigma W_t$$

$$\begin{aligned} V(m) &= \mathbb{E}^m\Big[\int_0^\infty e^{-rt}\,(dL_t - dI_t)\Big] \\ & \quad m + \mathbb{E}\left[\int_0^\infty e^{-rt}\,(\mu dt + \sigma dW_t)\right] = m + \frac{\mu}{r} \end{aligned}$$

▷ Dynamics of security prices.

$S = \{S_t; t \geq 0\}$ ex-payment price of a share of the security issued by the firm
$N = \{N_t; t \geq 0\}$ number of outstanding shares

$$V(M_t) = N_t S_t$$

$$dI_t = d(N_t S_t) - N_t dS_t = -N_t dS_t = -\frac{\mu}{r}\,\frac{dS_t}{S_t}$$

$$\frac{dS_t + dD_t}{S_t} = rdt + \frac{\sigma r}{\mu}\,dW_t$$

where D_t is the cumulative payment per share process:

$$dD_t = l\frac{r}{\mu}S_t dt = \frac{l}{N_t}dt.$$

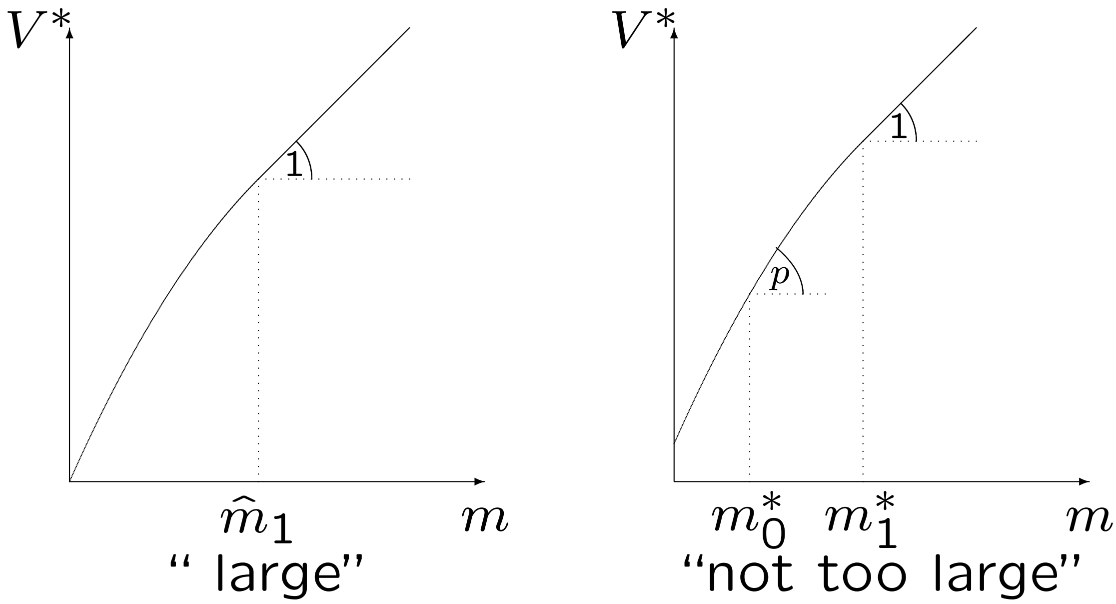
Benchmark: $\mathbf{p = 1,\ f = 0,\ \lambda > 0}$

$$\frac{dS_t}{S_t} = r\left(1 - \frac{l}{\mu}\right)dt + \frac{\sigma r}{\mu}\,dW_t$$

$$\begin{aligned} S_t &= \mathbb{E}\left[\int_t^\infty e^{-r(s-t)}\frac{lrS_s}{\mu}ds \,|\, \mathcal{F}_t\right] \\ &= \mathbb{E}\left[\int_t^\infty e^{-r(s-t)}\frac{l}{N_s}ds \,|\, \mathcal{F}_t\right]. \end{aligned}$$

$\mathbf{p > 1, \ f > 0, \ \lambda > 0}$

$$V^*(m) = \sup_{I_t, L_t} \mathbb{E}^m \Big[\int_0^{\tau_B} e^{-rt} \, (dL_t - dI_t) \Big]$$



▷ Cash reserve process M at the optimum.

- If issuance costs are “large”:
  diffusion process that is reflected back each time it hits $\widehat{m}_1$, and that is absorbed at 0.
- If issuance costs are “not too large”:
  diffusion process that is reflected back each time it hits m_1^*, and jumps to m_0^* each time it hits 0.

▷ Optimal issuance policy

- Firm value jumps from $V^*(0)$ to $V^*(m_0^*)$
- Each time M hits zero, the amount $V^*(m_0^*) - V^*(0)$ of new security is issued.

Stock price dynamics (1)

$S = \{S_t; t \geq 0\}$ ex-dividend price of a share in the firm
$N = \{N_t; t \geq 0\}$ number of shares issued by the firm

- Stock price does not jump at optimal issuance dates: $S_{\tau_n} = S_{\tau_n -}$
- $V^*(M_t) = N_t S_t$

- $dI_t = d(N_t S_t) - N_t dS_t = S_t dN_t$
- $V^*(m_0^*) - V^*(0) = S_{\tau_n}(N_{\tau_n} - N_{\tau_n -})$

Proposition. The process N modelling the number of outstanding shares is given by:

$$N_t = \begin{cases} 1 & 0 \leq t < \tau_1, \\ \left[\dfrac{V^*(m_0^*)}{V^*(0)}\right]^n & \tau_n \leq t < \tau_{n+1}. \end{cases}$$

Stock price dynamics (continuity)

AAO

$$S_t = \mathbb{E}\left[\int_t^{\infty} e^{-r(s-t)}\frac{dL_s}{N_s} \,|\, \mathcal{F}_t\right]$$

$$e^{-rt}S_t = \mathbb{E}\left[\int_0^{\infty} e^{-rs}\frac{dL_s}{N_s} \,|\, \mathcal{F}_t\right] - \int_0^t e^{-rs}\frac{dL_s}{N_s}.$$

Stock price dynamics (2)

- $V^*(M_t) = N_t S_t$
- $dS_t = d[V^*(M_t)]/N_{\tau_n} \;\; \forall t \in [\tau_n, \tau_{n+1})$.

Proposition. Between two consecutive issuance dates τ_n and τ_{n+1}, the instantaneous return on stock satisfies:

$$\frac{dS_t + dD_t}{S_t} = rdt + \sigma(N_{\tau_n} S_t) dW_t,$$

where

$$\sigma(v) \equiv \sigma \frac{V^{*\prime}\left[(V^*)^{-1}(v)\right]}{v}$$

D_t denotes the cumulative dividend per share process:

$$dD_t = \frac{dL_t^{m_1 *}}{N_{\tau_n}}.$$

Consequences:

- Changes in the volatility of stock returns are negatively correlated with stock price movements.
- Changes in the volatility of stock prices are negatively correlated with stock price movements.
- Stock price cannot take arbitrarily large values.
- A reduction in issuance costs should lead to a fall in the volatility of stock returns.

Conclusion

Introducing growth opportunities...

- Interaction between dividend policy and decision to invest in a growth opportunity
- Role of issuance costs? Does a decrease in issuance costs encourage firms to invest in more risky projects? Consequences on the dynamics of stock prices?
- non predictable growth opportunity

Conclusion

Taking into account issuance costs in corporate models allows to derive several implications on asset pricing

Issuance costs provide a natural explanation for heteroscedasticity of stock prices.

Comparative statics

Proposition

- The elasticity of the value of the firm with respect to its cash reserves,

$$\epsilon^*(m) = \frac{mV^{*\prime}(m)}{V^*(m)}; \quad m \geq 0,$$

  is an increasing function of the issuance costs p and f.

- The volatility of stock returns as a function of the firm's valuation,

$$\sigma^*(v) = \sigma \frac{V^{*\prime}((V^*)^{-1}(v))}{v}; \quad V^*(0) \leq v \leq V^*(m_1^*),$$

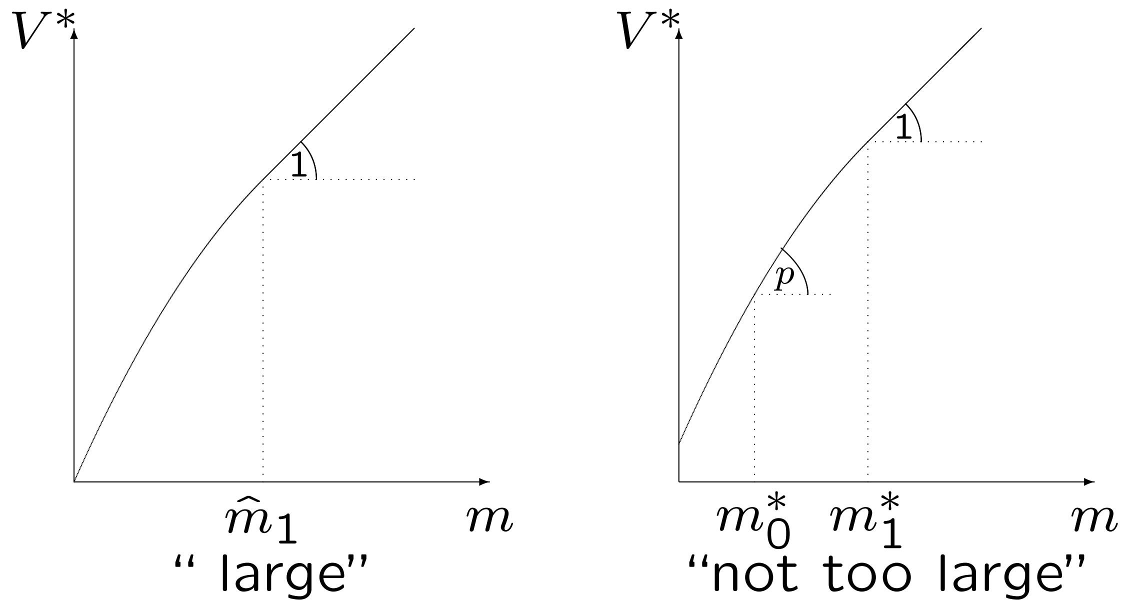
  is an increasing function of the issuance costs p and f.

$\Longrightarrow$

- A reduction in issuance costs should reduce the responsiveness of firm's valuations to changes in their cash reserves.
- A reduction in issuance costs should lead to a fall in the volatility of stock returns.

Value function

$$V^*(m) = \sup_{I_t, L_t} \mathbb{E}^m \Big[\int_0^{\tau_B} e^{-rt} \, (dL_t - dI_t) \Big]$$



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▷ Optimal issuance policy

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Value function (1)

Road map:

- Write a system of variational inequalities that the value function V^* should satisfy.
- Show that this system has a unique regular solution.
- Establish that this solution is indeed the optimal value function.

Value function (2)

▷ Heuristics

$$V^*(m) \geq V^*(m-l)+l$$
$$V^{*\prime}(m) \geq 1$$

$$V^*(m) \geq V^*(m+\frac{i}{p}-f)-i$$

$$V^*(m) \geq$$

$$\mathbb{E}^m\Big[e^{-r(t\wedge\tau_B)}\,V^*\Big(m+\int_0^{t\wedge\tau_B}[(\mu+(r-\lambda)M_s)ds+\sigma dW_s]\Big)\Big]$$

$$-rV^*(m)+\mathcal{L}V^*(m)\leq 0$$

$$\mathcal{L}u(m)=(\mu+(r-\lambda)m)u'(m)+\frac{\sigma^2}{2}\,u''(m).$$

Value function (3)

▷ Guess

- Issuance policy

$$
\begin{aligned}
V^*(0) &= \left[\max_{i\in[0,\infty)}\left\{V^*\left(\frac{i}{p}-f\right)-i\right\}\right]^+, \\
V^*(0) &= \left[\max_{m\in[-f,\infty)}\{V^*(m)-p(m+f)\}\right]^+
\end{aligned}
$$

- Dividend policy $m \geq m_1^*$,

$$V^{*\prime}(m_1^*) = 1.$$

V^* is postulated to be twice continuously differentiable over $(0,\infty)$,

$$V^{*\prime\prime}(m_1^*) = 0.$$

Value function (4)

▷ Variational system: Find (V, m_1)

$$V(m) = 0;\ m < 0, \tag{1}$$

$$V(0) = \left[\max_{m \in [-f,\infty)} \{V(m) - p(m+f)\} \right]^+, \tag{2}$$

$$-rV(m) + \mathcal{L}V(m) = 0;\ 0 < m < m_1, \tag{3}$$

$$V(m) = \frac{\mu + (r-\lambda)m_1}{r} + m - m_1; \quad m \geq m_1. \tag{4}$$

▷ Solving the system

Fix $m_1 > 0$, V_{m_1} solution to:

$$\begin{aligned} -rV_{m_1}(m) + \mathcal{L}V_{m_1}(m) = 0; \qquad 0 \leq m \leq m_1, \\ V'_{m_1}(m_1) = 1, \\ V''_{m_1}(m_1) = 0. \end{aligned}$$

V_{m_1} solution to (1)-(4) linearly extended to $[m_1, \infty)$.

Value function (5)

$$V(0) = \left[\max_{m \in [-f, \infty)} \{V(m) - p(m+f)\} \right]^{+}$$

$\exists\,!\ \hat{m}_1\ \ V_{\hat{m}_1}(0) = 0$,

(i) If $\max_{m \in [-f,\infty)} \{V_{\hat{m}_1}(m) - p(m+f)\} = 0$

$$V^* = V_{\hat{m}_1}$$

(ii) If $\max_{m \in [-f,\infty)} \{V_{\hat{m}_1}(m) - p(m+f)\} > 0$

$$\forall m_1, \quad \exists\,!\ \ m_p(m_1)\ \ s.t\ \ V'_{m_1}(m_p(m_1)) = p$$

$$V_{m_1}(0) = V_{m_1}(m_p(m_1)) - p[m_p(m_1) + f].$$

$$m_1^*, \quad m_p(m_1^*) = m_0^*, \quad V^* = V_{m_1^*}$$

$$V^*(m_0^*) - V^*(0) = p(m_0^* + f) = i^*$$