Accident Risk, Limited Liability and Dynamic Moral Hazard

Bruno Biais, Thomas Mariotti, Jean-Charles Rochet and Stéphane Villeneuve

July 2008

I. Introduction

Motivation

We model the delegated management of a profitable but risky activity that involves infrequent "accidents" of a large magnitude

– A firm can take precautions against industrial hazards

– A hospital can take steps to prevent medical errors

– A bank can enhance the credit worthiness of its loans

Because of limited liability, one cannot make agents liable for the full cost of such risks. When prevention efforts are unobservable, there is a tension between limited liability and incentives

Modelling Strategy

In contrast with day-to-day operations, accidents in our model are rare and dramatic events

Accidents occur according to a Poisson process whose intensity depends on the level of risk prevention

Our focus on downside risk makes the size of operations a key variable for the structure of incentives

The model allows not only for downsizing, but also for size growth, subject to adjustment costs

Related Literature I : Dynamic Moral Hazard

Model with risk-averse agents (Rogerson 1985, Holmström and Milgrom 1987, Spear and Srivastava 1987, Phelan and Townsend 1991, Sannikov 2003)

Models with risk-neutral agents and limited liability (Clementi and Hopenhayn 2006, DeMarzo and Sannikov 2006, DeMarzo and Fishman 2007, Biais, Mariotti, Plantin and Rochet 2007)

Models with Brownian outcomes (Holmström and Milgrom 1987, Sannikov 2003, DeMarzo and Sannikov 2006, Biais, Mariotti, Plantin and Rochet 2007)

Related Literature II : Poisson Models

Unlike in Shapiro and Stiglitz 1984, effort makes accidents less likely but does not eliminate them altogether

Sannikov 2005 studies a Poisson model of dynamic moral hazard with upside risk in which there is no use for downsizing

Myerson 2007 considers the same model as we do, but with equally patient principal and agent. Only ε–optimal contracts exist with exogenous bounds on the agent's continuation utility

II. The Model

Agents

Time is continuous and indexed by $t \in [0, \infty)$

There are two players

- A principal with discount rate r
- An agent with discount rate $\rho > r$

Both the principal and the agent are risk-neutral

The agent has limited liability

Technology

The agent runs a project of varying size X

Size can be decreased at no cost (downsizing)

Size can increase at most at rate $\gamma < r$ (growth)

Accidents occur according to a Poisson process N

The intensity of N is λ if the agent works, $\lambda + \Delta\lambda$ if she shirks

Payoffs

The liquidation value of assets is 0

A size increase $dX \in [0, \gamma X dt]$ comes at a cost cdX

The project generates operating profits μX per unit of time

The principal bears the costs CX of accidents

The agent enjoys a private benefit BX from shirking

Some Parameter Restrictions

The expected instantaneous net output flow is positive if the agent works

$$\mu > \lambda C$$

The private benefit from shirking is lower than the social cost of increased accident risk

$$C \boldsymbol{\Delta} \lambda > B$$

Contracts and Strategies

A contract $\Gamma = (X, L)$ states downsizing/growth decisions and non-negative transfers as functions of the public history

The size process $X = X_0 + X^d + X^g$ is predictable, non-negative and of bounded variation, with initial condition $X_0 \leq X_{0-}$

The downsizing process X^d is decreasing, and the growth process X^g is absolutely continuous with density at most γX

The cumulative transfer process L is adapted, non-negative and non-decreasing, reflecting the agent's limited liability

A strategy for the agent is a predictable process Λ that takes its values in $\{\lambda, \lambda + \Delta\lambda\}$

Continuation Utilities

For the agent

$$W_t(\Gamma,\Lambda) = \mathbb{E}_t^{\Lambda}\left[\int_0^{\infty} e^{-\rho(s-t)}(dL_s + 1_{\{\Lambda_s=\lambda+\Delta\lambda\}} BX_s ds)\right]$$

For the principal

$$F_t(\Gamma,\Lambda) = \mathbb{E}_t^{\Lambda}\left[\int_0^{\infty} e^{-r(s-t)}[(\mu ds - CdN_s)X_s - cdX_s^g - dL_s]\right]$$

The Optimal Contracting Problem

An optimal long-term contract solves

$$\max_{(\Gamma,\Lambda)} \{F_0(\Gamma,\Lambda)\}$$

$$\text{s.t. } W_t(\Gamma,\Lambda) \geq W_t(\Gamma,\Lambda') \ \ \forall (t,\Lambda')$$

$$W_0(\Gamma,\Lambda) \geq W_{0^-}$$

We focus on the case where it is optimal to always induce effort

$$\Lambda_t = \lambda \ \ \forall t$$

III. Incentive Compatibility

A Martingale Representation of Utility (Sannikov 2003)

Consider the agent's total utility evaluated at date t

$$U_t = \mathbb{E}_t^{\Lambda}\Big[\int_0^{\infty} e^{-\rho s}(dL_s + 1_{\{\Lambda_s=\lambda+\Delta\lambda\}} B X_s ds)\Big]$$

$$= \int_0^{t} e^{-\rho s}(dL_s + 1_{\{\Lambda_s=\lambda+\Delta\lambda\}} B X_s ds) + e^{-\rho t} W_t$$

There exists a predictable process H such that

$$dU_t = -e^{-\rho t} H_t (dN_t - \Lambda_t dt)$$

Therefore the agent's continuation utility evolves as

$$dW_t = (\rho W_t - 1_{\{\Lambda_s=\lambda+\Delta\lambda\}} B X_t) dt - H_t (dN_t - \Lambda_t dt) - dL_t$$

Inducing Effort

Exerting effort at date t is incentive compatible if and only if

$$dW_{t \,|\, \Lambda_t = \lambda + \Delta\lambda} \geq dW_{t \,|\, \Lambda_t = \lambda} \text{ or } H_t \geq \frac{B}{\Delta\lambda} X_t \equiv b X_t$$

The agent's utility must decrease by at least bX_t after an accident

Limited Liability and Incentive Compatibility

Satisfying both constraints at date t requires

$$W_{t^-} \geq H_t \geq bX_t$$

Satisfying both constraints at date t^+ after an accident requires

$$W_t = W_{t^-} - H_t \geq H_{t^+} \geq bX_{t^+} \equiv bx_t X_t$$

Downsizing must occur after an accident at date t if $W_{t^-}/X_t < 2b$

$$x_t \leq \frac{W_{t^-} - H_t}{bX_t} \leq \frac{W_{t^-}}{bX_t} - 1 < 1$$

IV. The Optimal Contract

A Reformulation of the Optimal Contracting Problem

An optimal long-term contract that always induces effort solves

$$F(X_0, W_{0^-}) = \max_\Gamma \left\{ \mathbb{E}_0^\lambda \left[\int_0^\infty e^{-rt} \left[(\mu dt - C dN_t) X_t - c dX_t^g - dL_t \right] \right] \right\}$$

s.t. $dW_t = (\rho W_t + \lambda H_t) dt - H_t dN_t - dL_t \;\; \forall t$

$$W_{t^-} \geq H_t \geq b X_t \;\; \forall t$$

State and Control Variables

It is useful to think of some variables in size-adjusted terms

$$w_t = \frac{W_{t-}}{X_t}, \quad h_t = \frac{H_t}{X_t}, \quad x_t = \frac{X_{t+}}{X_t}$$

Incentive compatibility requires $h_t \geq b$

The downsizing policy must satisfy $x_t \leq \min\{(w_t - h_t)/b\,, 1\}$

Size growth is given by $dX_t^g = g_t X_t dt$ with $g_t \leq \gamma$

W.l.o.g. let transfers be given by $dL_t = l_t 1_{\{dN_t=0\}} dt$ with $l_t \geq 0$

The Principal's Value Function

Using the dynamics of X and W yields the HJB equation

$$\begin{aligned} rF(X_t, W_{t^-}) = (\mu - \lambda C)X_t & \\ + \max\{ -l_t + (\rho W_{t^-} + \lambda h_t X_t - l_t) F_W(X_t, W_{t^-}) & \\ + [F_X(X_t, W_{t^-}) - c] g_t X_t & \\ - \lambda [F(X_t, W_{t^-}) - F(x_t X_t, W_{t^-} - h_t X_t)] \} & \end{aligned}$$

where the maximization is with respect to admissible (h_t, x_t, g_t, l_t)

Two Simple Guesses

Because of constant returns to scale, F is homogenous

$$F(X, W_-) = Xf\left(\frac{W_-}{X}\right) = Xf(w)$$

The size-adjusted value function is concave, and linear over $[0, b]$

$$f(w) = \frac{f(b)}{b}w \ \ \forall w \in [0, b]$$

Optimality Conditions : Transfers

The FOC with respect to l_t yields

$$F_W(X_t, W_{t^-}) = f'(w_t) \geq -1 \text{ with equality if } l_t > 0$$

The optimal transfer policy is characterized by a threshold w^m

Optimality Conditions : Growth

The FOC with respect to g_t yields

$$g_t = \gamma \text{ if } F_X(X_t, W_{t^-}) = f(w_t) - w_t f'(w_t) \geq c\,;\ g_t = 0 \text{ otherwise}$$

The optimal growth policy is characterized by a threshold w^g

Optimality Conditions : Downsizing

The FOC with respect to x_t yields

$$x_t = \min\left\{\frac{w_t - h_t}{b}, 1\right\} \text{ after an accident}$$

Downsizing is used only as a last resort

Optimality Conditions : Sensitivity to Accidents

The FOC with respect to h_t yields

$$h_t = b$$

It is optimal to minimize the agent's exposure to risk

The Size-Adjusted Value Function

For $w \in [b, w^g]$ one has

$$rf(w) = \mu - \lambda C + (\rho w + \lambda b)f'(w) - \lambda[f(w) - f(w - b)]$$

For $w \in [w^g, w^m]$ one has

$$(r-\gamma)f(w) = \mu-\lambda C-\gamma c+[(\rho-\gamma)w+\lambda b]f'(w)-\lambda[f(w)-f(w-b)]$$

At the boundary $w = w^m$ one has

$$f'(w^m) = -1 \text{ and } f''(w^m) = 0$$

The Verification Argument

One first shows that there exists a size-adjusted value function that satisfies this free boundary problem. The probation region $[b, w^g]$ is empty if c is low enough, while the growth region $[w^g, w^m]$ is empty if c is high enough

One then shows that $F(X_0, W_{0-}) = X_0 f(w_0)$ is greater than the utility derived by the principal from any contract Γ that induces the agent to exert effort, and that there exists such a contract that attains this upper bound

The Optimal Contract given (X_0, W_{0^-})

The size of the firm is given by

$$X_t = X_0 \prod_{n=1}^{N_t} \min\left\{\frac{w_{\tau_n} - b}{b}, 1\right\} \exp\left(\int_0^t \gamma 1_{\{w_s \geq w^g\}}\, ds\right)$$

The transfer process is given by

$$L_t = \max\{W_{0^-} - X_0 w^m, 0\} + \int_0^t (\rho w^m + \lambda b) X_s 1_{\{W_s = w^m X_s\}}\, ds$$

The Determination of (X_0, W_{0-})

It depends on the relative bargaining power of the players. In the case where the principal is competitive, one has to solve

$$\max_{(X_0, w_0)} \{[f(w_0) + w_0] X_0\}$$

$$\text{s.t.} \quad f(w_0) X_0 \geq 0$$

$$w_0 \geq 0$$

$$X_{0-} \geq X_0$$

At the optimum $X_0 = X_{0-}$ and $f'(w_0) = -1/(1+\eta)$. When $f(w^m) \geq 0$, $\eta = 0$ and $w_0 = w^m$, otherwise $w_0 < w^m$

When Is It Optimal to Induce Effort ?

Inducing effort is optimal if

$$rf(w) \geq \mu - (\lambda + \Delta\lambda)C + (\rho w - B)f'(w)$$
$$+ \max\{[f(w) - wf'(w) - c]g\}$$

Using the HJB equation, a sufficient condition for this to hold is

$$\Delta\lambda[C + bf'(w)] \geq \lambda[f(w) - f(w-b) - bf'(w)]$$

Since $C > b$ and $f' \geq -1$, this is satisfied for all $w > b$ when $\Delta\lambda$ is high enough, and B is adjusted so as to keep b constant

V. The Dynamics of Firm Size

Low Investment Costs

Since f is not differentiable at b, one can choose c such that

$$f(b) - bf'_{+}(b) > c$$

Then it is optimal to always let firm size grow at rate γ, and

$$X_t = X_{0^-} \prod_{n=1}^{N_t} \min\left\{\frac{w_{\tau_n} - b}{b}, 1\right\} \exp(\gamma t)$$

The Law of Large Numbers (Breiman 1960)

Let μ be the invariant measure of the Markov process $\{w_{\tau_n}\}_{n\in\mathbb{N}}$

Taking logarithms yields

$$\ln(X_t) = \ln(X_{0^-}) + N_t\left[\frac{\gamma t}{N_t} + \frac{1}{N_t}\sum_{n=1}^{N_t}\ln\left(\min\left\{\frac{w_{\tau_n}-b}{b}, 1\right\}\right)\right]$$

$$\underset{\infty}{\sim} \ln(X_{0^-}) + N_t\left[\frac{\gamma}{\lambda} + \int_b^{2b}\ln\left(\frac{w-b}{b}\right)\mu(dw)\right] \quad \text{a.s.}$$

Decline versus Expansion

It is easy to construct lower and upper bounds to μ in the FOSD sense that are uniform in γ

These bounds imply that $X_t \to 0$ with probability 1 if γ is low, while $X_t \to \infty$ with probability 1 if γ is high

VI. Implementation and Testable Implications ($\gamma = 0$)

Cash Reserves

The firm holds cash reserves $M_{t^-} = W_{t^-}$ that earn interest r

Changes in this account reflect

– Operating cash-flow

– Transfers to the insurance company and the manager

– Earned interest income

One can interpret w_t as a liquidity ratio

Insurance Contract

The insurance company is liable for $(C-b)X_t$ per accident, while the firm pays the deductible b out of its cash reserves

The instantaneous premium π_t has two components

- An actuarially fair component $\pi_t^a = \lambda(C-b)X_t$
- An incentive component $\pi_t^i = -(\rho - r)W_{t^-}$

Downsizing covenant $x_t = \min\{M_t/(bX_t), 1\}$ if liquidity is too low, or if the ratio of risk exposure to inside equity is too high

Bonds and Managerial Compensation

At date 0, the firm issue a bond with coupon $\xi_t = (\mu - \lambda C)X_t$

The issuance proceeds allow to hoard cash reserves M_{0^-} and pay a commitment fee to the insurance company

The manager gets paid when the liquidity ratio $M_t/X_t = w_{t^+}$ reaches the target w^m

Cash reserves evolve as

$$dM_{t^-} = (\mu + rM_{t^-} - \pi_t - \xi_t)dt - bX_t dN_t - dL_t$$

Moral Hazard, Deductibles and Insurance Premia

Severe moral hazard implies large deductibles, and highly volatile insurance premia, decreasing significantly as long as no accident occurs, and increasing sharply after accidents

One can estimate b, λ and ρ by observing deductibles, accident rates and the evolution of cash reserves. The evolution of the incentive component of the risk premium over periods without accidents yield a further empirical restriction that allows one in principle to test the model

Pricing Credit Risk

The size adjusted price of bonds $d(w)$ satisfies the same equation as $f(w)$, with a different boundary condition $d'(w^m) = 0$. Because of downsizing, bonds incur credit risk

The credit yield spread on zero-coupon bonds is always positive, even when the maturity goes to zero. The credit yield spread at zero maturity decreases with the liquidity ratio

$$y_t(w_t) - r = \lambda \min\left\{2 - \frac{w_t}{b}, 0\right\}$$

VII. Concluding Remarks

Summary

We offer a model of large risk prevention in which, because of downside risk, size is crucial for the provision of incentives

Investment and compensation are tied to past performance, while downsizing must occur after poor performance

The immiseration result is robust to the possibility of increasing the scale of operations

Our implementation in terms of insurance and financial contracts has implications for credit risk

Applications

The model can be applied to study a large variety of situations with dynamic moral hazard

- Prevention of industrial risk
- Design of contracts for medical liability insurance
- Allocation of position limits within investment banks
- Remuneration of fund managers