2012 SÉMINAIRE DE MATHÉMATIQUES SUPÉRIEURES SCIENTIFIC REPORT.

OVERVIEW

The 2012 Séminaire de Mathématiques Supérieures introduced nearly eighty young researchers from eastern and western Canada, the USA, Australia, Belgium, Brazil, the Czech Republic, England, France, Germany, Hungary, India, the Netherlands, and Sweden, to some of the most exciting subjects of active research in the area of probabilistic combinatorics. The subjects addressed at the summer school can be roughly arranged into two overlapping themes: (i) properties of discrete Markov chains; and (ii) new techniques for understanding structural properties of deterministic and random graphs.

The majority of the eleven invited speakers stayed for at least one full week of the SMS, and four of the speakers (Hatami, McDiarmid, Scott, Winkler) stayed for both weeks. This gave the students plenty of opportunity to interact with the speakers outside of the lecture hall, which contributed substantially to the scientific quality of the meeting. (For example, one speaker, Prasad Tetali, ended up giving a supplementary “mini-course” to a subset of the students who were interested in hearing more detail about some of the research Tetali touched on in his lectures.)

THEME 1: DISCRETE MARKOV CHAINS

The flagship lectures on this theme were by Peter Winkler, who gave a sequence of five ninety-minute talks on random walks on graphs. This (by now) classical subject has lots of beautiful theorems and scores of applications in mathematics and computer science. Nonetheless, new and remarkable results keep coming in. Winkler started by reviewing the classical results in the area, including the connection between random walks and electrical networks and its extensions. He then moved to some exciting new research, including recent results and open problems on covering the vertices and edges of a graph, the use of potential functions to prove universal bounds for cover times, and cat-and-mouse (or cop-and-robber) games on graphs.

James Lee’ s talks dovetailed beautifully with Winkler’s, while consisting of more classically probabilistic content. Lee presented his recent *tour de force* with Ding and Peres, relating the cover time of reversible Markov chains to the extremes of an associated Gaussian process. This research has now appeared in the Annals of Mathematics. Lee provided a brief background on Gaussian processes and beautifully presented Talagrand’s majoring measures theorem. He then explained how he, Ding, and Peres used the majorizing measures theory to exhibit a close connection between the cover time of a graph and the expected square of its Gaussian free field.

The main tool allowing results for the Gaussian free field to be transferred to the setting of Markov chains is the the Dynkin isomorphism theory for Markov processes. While this connection is extremely useful and has already resulting in solutions to some open questions on cover times, it is also rather mysterious even in extremely simple examples. Lee discussed some natural starting points for possible research into the deeper structure behind the Dynkin theory.

Prasad Tetali's talks, on geometric and functional analysis on graphs, were conceptually linked to those of James Lee via the connection between isoperimetric inequalities and extremes of Gaussian processes. Tetali began with a review of some classical isoperimetric and functional inequalities in discrete spaces, with applications to concentration of measure and convergence to equilibrium of finite Markov chains. He then presented recent results on generalizations of Cheeger-type inequalities and refinements of Brunn-Minkowski inequalities, which suggest new directions for interesting research in geometric and functional analysis on graphs.

Perla Sousi presented her recent result, joint with Yuval Peres (and independently proved by Roberto Oliveira) on the equivalence of a broad family of notions of mixing time. Most notable among these is the fact that for reversible Markov chains, the mixing time is equivalent to the hitting time of large sets. This easily-stated fact provides a robust equivalent of the mixing time which can be used both to simplify many existing proofs and to derive new results. Sousi highlighted one such result, related to a geometric characterization of the mixing time for random walks on trees.

Yuval Peres's lectures were on the subject of random walks on *infinite graphs*, which despite being more classical is still rife with open problems and areas where our understanding is incomplete. At the outset, Peres motivated his lectures with the following question: which of the following random walks on $\mathbb{Z}^2$ are transient and which are recurrent?

1. In $\mathbb{Z}^2$, at times $t \in [4^k, 2 \cdot 4^k)$ we go up or down with equal probability. At times $t \in [2 \cdot 4^k, 4^{k+1})$, we go left or right with equal probability.

2. In $\mathbb{Z}^2$, if the current node has been visited before, then move left or right with equal probability; otherwise go up or down with equal probability.

3. In $\mathbb{Z}^2$, if $|x| \geq |y|$ then we go up or down each with probability 0.3, and left or right each with probability 0.2. This is reversed if $|y| > |x|$.

4. In $\mathbb{Z}^3$, fix two mean-zero measures μ_1, μ_2 that are truly 3D (that is, doesn't assign probability 1 to any hyperplane) with bounded support. If X_t has been visited before, then $X_{t+1} - X_t \sim \mu_2$, else $X_{t+1} - X_t \sim \mu_1$.

(It turns out that 1, 3, and 4 are transient, and it is an open problem to determine transience or recurrence for number 2.) Peres then presented a wide range of questions and results on transience, recurrence, and speed of random walks on various models of infinite graphs, with a particular focus on highlighting basic gaps in our conceptual understanding and current techniques.

Finally, Eric Vigoda's talks formed a bridge between the first and second themes, presenting results related to Markov Chain Monte Carlo algorithms for generating random colourings of graphs of bounded degree. Vigoda explained the basic coupling technique, and its refinement – known as path coupling – due to Bubley and Dyer. He then explained the well-known result of Mark Jerrum on rapid mixing of the Glauber dynamics for colouring when the number of colours exceeds twice the maximum degree Δ. Vigoda followed this up with various improvements, beginning with his own famous result showing rapid mixing for the Glauber dynamics with $11\Delta/6$ colours, via the analysis of a more complicated chain that flips 2-color components. He also showed how a multi-step coupling can be used to get improved results assuming lower bounds on the girth and on the maximum degree, Δ. Finally, he explained a beautiful use of spectral graph theory to obtain improved results for planar graphs or graphs embeddable on a fixed surface.

THEME 2: COLOURINGS, CLIQUES, AND CONNECTIVITY

Many questions in combinatorics concern the relationship between the local and global structure of a graph or set system. For instance, what can we say about the subgraphs of a graph with large chromatic number? What about graphs without large cliques or independent sets? How uniformly is it possible to distribute edges in a graph? In 7.5 hours of lectures, Alex Scott presented a wide range of results and conjectures of this flavour, touching on the Erdos-Hajnal Conjecture, the Gyarfas-Sumner Conjecture, discrepancy for graphs and hypergraphs, and recently developed VC-dimension techniques.

Nikhil Bansal gave another extended mini-course of 7.5 hours, which brought the participants to the edge of existing knowledge in discrepancy theory. Discrepancy theory deals with the following type of question. Given a set-system, find a red-blue coloring of the elements such that each set is colored as evenly as possible. Perhaps surprisingly, this notion has a wide variety of applications both in computer science and mathematics, and several techniques (many of them non-constructive) have been developed to understand the discrepancy of various set-systems.

Recently, there have been several new developments in discrepancy based on connections to semidefinite programming. This connection is useful is various ways. It gives efficient polynomial time algorithms for several problems for which only non-constructive results were previously known. It also leads to several new structural results, such as tightness of the so-called determinant lower bound, and bounds on the discrepancy of union of set systems. Bansal presented these results in detail and touched on several related concepts such as correlated Brownian motions, the non-constructive entropy method, Gaussian rounding, and SDP duality.

Penny Haxell's lectures had the intriguing title "a topology-free topological method." Over the last dozen years or so, certain topological methods have been developed and used to prove a family of results related to the following general problem. Let G be a graph whose vertex set is partitioned into nonempty sets $V_1, ..., V_r$. What conditions will guarantee that G contains an independent set $\{v_1, ..., v_r\}$ such that $v_i \in V_i$ for each i? This family of results includes theorems on matchings in hypergraphs, list colouring, strong colouring, and Aharoni's proof of Ryser's longstanding conjecture on packing and covering in tripartite hypergraphs. The topological arguments used are based on the notion of topological connectivity of simplicial complexes. Haxell has recently developed a method for establishing this entire theory using only elementary combinatorial arguments, and this approach was the subject of her SMS lectures.

Colin McDiarmid presented some recent breakthroughs on a classic question in probabilistic graph theory: what is the typical behaviour of the chromatic number $\chi(G)$ of a graph G? If R_n denotes some sort of random graph on n vertices, can we determine a function $f(n)$ such that $\chi(R_n)/f(n) \to 1$ in probability as $n \to \infty$? If so, what is $f(n)$? Can we bound the typical spread of the values $\chi(R_n)$? Is $\chi(R_n)$ usually close to $\omega(R_n)$, the maximum size of a complete subgraph?

McDiarmid presented a variety of his recent results; his lectures focussed primarily on the classical Erds-Rnyi or Bernouilli random graph $G(n, p)$ (both in the dense case when p is a constant and in the sparse case when np is constant), and on random geometric graphs. He also touched on other graph invariants such as edge chromatic number (chromatic index), list chromatic number, total chromatic number, achromatic number, improper chromatic number, and span. Perhaps most notably, he presented a recent technique that yields improved estimates for $\chi(G(n, p))$ in the dense case; and a surprising 'phase change' that occurs when colouring random geometric graphs.

The subject of influences is key to the understanding of phase transitions and sharp thresholds for various properties of discrete systems, including colouring of graphs, satisfiability of random formulas, and connectivity of random networks. Hatami presented the basic notion of the influence of a variable on a Boolean function, then sketched the proof of the Friedgut's theorem which says that if $f : \{0, 1\}^n \to \{0, 1\}$ has small total influence then it essentially depends on few coordinates. This theorem does not hold when the uniform distribution on $\{0, 1\}^n$ is replaced with the p-biased distribution for a small value of p. He discussed the relevance of this case to the study of the threshold phenomenon, and then sketch the proof of his own recent result, which characterizes the structure of Boolean functions with small total influences on general product probability spaces. The latter result has garnered substantial attention and has recently appeared in the Annals of Mathematics.

Organization and administration

We received 135 applications from which we selected to fund 58 participants other than speakers. Approximately 20 other participants attended without our support, five of whom were funded by their home institutions and/or research supervisors, and the remainder of whom were mostly local graduate students and postdocs. Of the funded participants, eight are based at Fields institute member universities (University of Toronto x3, University of Waterloo x3, Carleton University x2), and six are based at PIMS member or affiliated universities (University of Washington x2, UBC, SFU, University of Victoria, University of Portland). We also had eighteen participants based at CRM member universities (McGill x16, Université de Montréal x2).

About 90% of the funding for participants went to graduate students, of which we tried to select those who were already advanced in their studies and working in areas closely related to the topic of the school. The remaining funding was directed primarily to three recent PhDs (who received their doctorates in 2009, 2010 and 2012). There were a few exceptions such as the graduate students selected by MSRI based on other criteria, and a very advanced undergraduate student from Carleton who was about to enter graduate school. In the selection process, we gave priority to the applicants for whom the school could have a significant impact on their research activities and development. In this regard, a letter from the advisor explaining the relevance of the school for the students program of studies was often a decisive factor. About 27% of the total number of participants, with or without funding, were female.

With the exception of the two CMS scholarships, and the 19 graduate students funded through MSRIs contribution (covering both local and travel expenses), the majority received a somewhat basic local support: 2 weeks in the student residences of the Université de Montréal and a contribution for travel expenses. The accommodation costs were somewhat reduced, compared with previous years, as we were able to negotiate a lower rate for the rental of student residences. To make up for the lack of support for local expenses, we provided a breakfast every day of the school, as well as coffee breaks. The CRMs administrative assistant was essential in the planning and the organization of the latter.

The CMS scholarships were awarded to two exceptionally qualified students enrolled in Canadian PhD programs. Both recipients actively engaged in all the SMS activities and made valuable contributions (both questions and comments) during the lectures. The MSRI-funded students were also an excellent fit for the summer school. They also participated actively in the lectures and in discussions with the other summer school participants.

Outline of the Expenditures

Speakers: housing at the Terrace Royale hotel near the Université de Montréal, reimbursement of travel expenses and per diem meals.

SMS funded participants: 2 weeks at the Université de Montréal dormitories (CDN$ 380, non-refundable), plus support for travel expenses ranging from $250 CDN to $750 CDN depending on the distance to Montréal from the participant's location of study.

CMS scholarships: 2 at CDN $1000 each.

MSRI funded participants: 18 participants. MSRI support covered housing, meals and travel expenses.

Social activities:

- Daily breakfast and coffee breaks.
- One wine-and-cheese reception for the students and speakers (covered by the CDN $20 fee charged to each participant).
- Two dinners for the speakers at local restaurants (one per week due to some speakers not staying for the whole two weeks).

Acknowledgements

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Speakers

Nikhil Bansal (Eindhoven University, Institute of Technology).
Title: *Low discrepancy colorings and semidefinite programming.*

Hamed Hatami (McGill University).
Title: *Influences and sharp thresholds.*

Penny Haxell (University of Waterloo).
Title: *A topology-free topological method.*

James Lee (University of Washington).
Title: *Cover times and Gaussian measures.*

Colin McDiarmid (University of Oxford).
Title: *Colouring random graphs.*

Yuval Peres (Microsoft Research)
Title: *Markov chain mixing times and related topics.*

Alex Scott (University of Oxford)
Title: *Cliques, colourings and discrepancy.*

Perla Sousi (University of Cambridge)
Title: *Markov chain mixing times: bounds and asymptotics.*

Prasad Tetali (Georgia Institute of Technology)
Title: *Geometric and Functional Analysis on Discrete Spaces.*

Eric Vigoda (Georgia Institute of Technology)
Title: *Markov chains for graph colouring.*

Peter Winkler (Dartmouth)
Title: *Random walk on a graph.*

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THE SPRING SCHOOL IN GRAPH THEORY

In 2010, CARP member Bruce Reed organized the "First Montréal Spring School in Graph Theory". The 2010 school was supported by the CRM, Fields, PIMS, the ISM, and our CARP research group, and was a resounding success. This school was modelled on the PIMS summer schools in probability, which have taken place in 2004,2005,2008,2009, and 2010, and have played a large role in the development of an exceptionally strong community of young probabilists in North America and Europe.

The *Second* Montréal Spring School in Graph Theory (SSGT) took place in 2012. The first half of the second SSGT consisted of two courses, each consisting of ten 90-minute lectures over the course of two weeks. Bruce Reed gave a sequence of ten lectures on graph colouring and the probabilistic method; Louigi Addario-Berry gave a sequence of ten lectures on the use of Markov chain mixing, meeting, and covering times, and on random walks on random graphs. These two weeks provided an introduction to some of the probabilistic tools and techniques used in the research presented at the SMS. Approximately 40 of the 75 SMS participants also attended the two preliminary weeks of SSGT lectures.