The Gapped Consecutive-Ones Property

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Abstract

A binary matrix M has the linear-time decidable *Consecutive-Ones Property* (C1P) [Fulkerson and Gross, 1965] if there is an ordering of the columns of M such that for each row, the set of columns containing entry `1` in this row appear consecutively in this ordering. The C1P has been widely used in molecular biology, e.g., ancestral genome reconstruction [Alizadeh *et al.*, 1995; Chauve and Tannier, 2008]. However, a common problem in such applications is that matrices obtained from experiments do not have the C1P, often due to small errors in the data [Goldberg *et al.*, 1995; Chauve and Tannier, 2008].

To address this problem, we relax the condition of the consecutivity of the `1`'s in each row, by allowing gaps (of `0`'s), with a restriction on the nature of these gaps. Specifically, we define: a binary matrix M has the *Gapped* C1P, or the (k, δ)-C1P for integers k and δ if the columns of M can be ordered such that each row contains at most k blocks of `1`'s, and no two neighboring blocks of `1`'s are separated by a gap of more than δ `0`'s. The classical C1P is equivalent to the $(1, 0)$-C1P. Here we show that for every unbounded or bounded $k \geq 2, \delta \geq 1$, except when $(k, \delta) = (2, 1)$, deciding the (k, δ)-C1P is NP-complete.

Motivated by the fact that matrices obtained from experimental data are often sparse [Chauve and Tannier, 2008], we then introduce the following third parameter to the Gapped C1P. A binary matrix M has the (d, k, δ)-C1P if no row of M has more than d `1`'s, and M has the (k, δ)-C1P. Here, we show that the (d, k, δ)-C1P is polynomial-time decidable if all three parameters are fixed, while when δ is unbounded ($\delta = \infty$), then for every $d > k \geq 2$, deciding the (d, k, ∞)-C1P is NP-complete.

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