

Pacific Institute *for the*  
Mathematical Sciences

# CT 2017: International Category Theory Conference



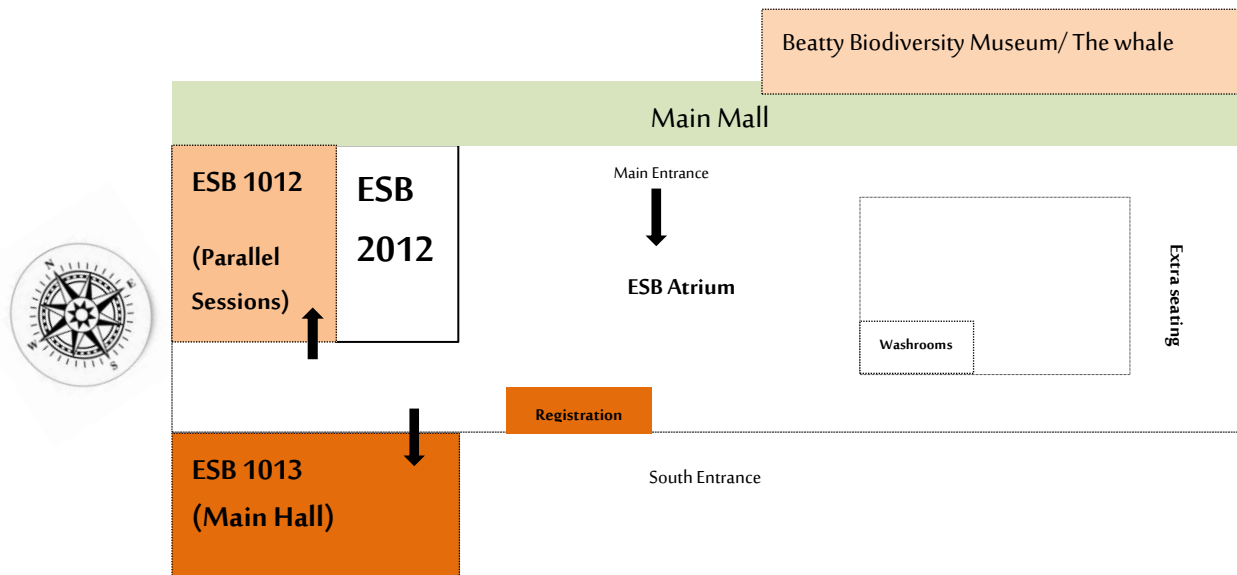
July 16 - 22, 2017  
University of British Columbia  
Vancouver, BC  
Canada

## Conference Program

# Program at a Glance

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday			
09:00-09:25	Opening	Lucyshyn-Wright	Adámek	Van der Linden	Hofmann	Paré			
09:30-09:55	Menni		Barr						
10:00-10:25		Campbell	Niefield	Goedecke	Clementino	Myers			
10:30-11:00	Break								
11:00-11:25	Marmolejo	Szyld	Pronk	Gran	Tholen	Bremner			
11:30-11:55	Pasquali	Descotte	Cockett	Jacqmin	Sousa	Riehl			
12:00-12:25	Emmenegger	Vasilakopoulou	Cruttwell	Rodelo	Frosoni	Rosebrugh			
12:30-14:00	Lunch		Excursion	Lunch					
14:00-14:25	Quijano	MacDonald		Cigoli	Jedrzejewicz				
14:30-14:55	Lima	Fieremans		García-Martínez	Janelidze				
15:00-15:30	Break			Break					
15:30-15:55	KAN seminar	Perrone		M.Ferreira	Gallagher		Moss	Scull	Yoshida
16:00-16:25		Ghosh		Lambert	Lemay		Aleiferi	Bayeh	Cicala
16:30-16:55		Discussion		MacAdam	DeWolf		Frey	Ríos	
17:00-17:25									

## Conference Room Guide: Earth Sciences Building



\*\* Not drawn to scale. See detailed UBC map on the last page

# Getting Started

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**Get connected:** Select the "ubcvisitor" wireless network on your wireless device. Open up a web browser, and you will be directed to the login page.

## FAQs

**Q: Where do I check in on the first day?** Check-in and package pick up can be done in the **Earth Sciences Building (ESB) Atrium**.

**Q: Where are the sessions?**

- All plenary sessions will be in the **ESB Room 1013**
- Breakout sessions on Tuesday, Thursday and Friday will be in **ESB 1013 and 1012**
- You will find a campus map at the end of the program.

**Q: Will the program change?** Program changes and updates will be announced at each session.

**Q: When should I wear my badge?** Please wear your name badges at all times on site so that PIMS Staff recognize you as a guest.

**Q: Where can I go for help on site?** If you need assistance or have a question during the conference, please connect with the conference organizers or with PIMS Staff.

**Q: Where can I get refreshments and meals?** For snacks or quick meals, please view the list of UBC eateries attached online at <http://www.food.ubc.ca/feed-me/>. Coffee breaks are provided each day of the workshop

**Q: Where can I get a cab to pick me up from the Venue?** You can call Yellow Cab (604-681-1111) and request to be picked up at the intersection of West Mall and Bio. Sciences Road. Use the south entrance and walk straight down to the intersection.

**Q: How can I get around?**

- **UBC Map link:** [Here](#)
- **Public Transit:** Feel free to search and plan your public transport rides by visiting <http://www.translink.ca/>, where directions, ticket costs and bus schedules are indicated.
- **Parking at UBC:** <http://www.parking.ubc.ca/visitor.html>

**Q: What emergency numbers should I know?**

- **Campus security (604-822-2222);**
- **General Emergencies (911);**
- **UBC hospital (604-822-7121).**

## Sunday July 16, 2017

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5:30pm - 7:00pm      **Optional Meet and Greet:**  
Light refreshments and nibbles served  
UBC Mahoney and Sons,  
5990 University Blvd, Vancouver, BC V6T 1Z3

## Monday July 17, 2017

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8:30am - 8:55am      **Registration and Check-in (ESB Atrium)**

9:00am - 9:25am      Opening

9:30am - 10:25am      **Matias Menni, Universidad Nacional de La Plata, Argentina**  
*On a problem in Objective Number Theory*

10:30am - 11:00am      Coffee Break (ESB Atrium)

11:00am - 11:25am      **Francisco Marmolejo, Universidad Nacional Autónoma de México**  
*The canonical intensive quality of a pre-cohesive topos*

11:30am - 11:55am      **Fabio Pasquali, University of Padova, Italy**  
*Quasi-toposes as elementary quotient completions*

12:00pm - 12:25pm      **Jacopo Emmenegger, Stockholms Universitet, Sweden**  
*On the local Cartesian closure of exact completions*

12:30pm - 2:00pm      Lunch- Own (See list of Campus eateries online at <http://www.food.ubc.ca/feed-me/>)

2:00pm - 2:25pm      **Juan Pablo Quijano, University of Lisbon, Portugal**  
*Functoriality and topos representations for quantales of coverable groupoids*

2:30pm - 2:55pm      **Guilherme Frederico Lima, University of Cambridge, UK**  
*Duality theorems for essential inclusions of Grothendieck toposes*

3:00pm - 3:30pm      Coffee Break (ESB Atrium)

3:30pm - 5:25pm      **Kan Extension Seminar** (organized by Emily Riehl)

## Tuesday July 18, 2017

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9:00am - 9:55am      **Rory Lucyshyn-Wright, Mount Allison University, Canada**  
*Algebraic duality and the abstract functional analysis of distribution monads*

10:00am - 10:25am      **Alexander Campbell, Macquarie University, Sydney, Australia**  
*Enriched algebraic weak factorization systems*

10:30am - 11:00am      Coffee Break (ESB Atrium)

11:00am - 11:25pm	<b>Martin Sztyld, Universidad de Buenos Aires – CONICET, Argentina</b> <i>A general limit lifting theorem for 2-dimensional monad theory</i>
11:30pm - 11:55pm	<b>Maria Emilia Descotte, Universidad de Buenos Aires – CONICET, Argentina</b> <i>On flat 2-functors</i>
12:00pm - 12:25pm	<b>Christina Vasilakopoulou, Université Libre de Bruxelles, Belgium</b> <i>Hopf categories as Hopf monads in enriched matrices</i>
12:30pm - 2:00pm	Lunch- Own (See list of Campus eateries online at <a href="http://www.food.ubc.ca/feed-me/">http://www.food.ubc.ca/feed-me/</a> )
2:00pm - 2:25pm	<b>Lauchie MacDonald, University of British Columbia, Vancouver, Canada</b> <i>Two dimensional algebra and natural distributive laws</i>
2:30pm - 2:55pm	<b>Timmy Fieremans, Vrije Universiteit Brussel, Belgium</b> <i>Frobenius and Hopf V-categories</i>
3:00pm - 3:30pm	Coffee Break (ESB Atrium)
3:30pm - 3:55pm	<b>Parallel Sessions</b> <ul style="list-style-type: none"> <li>• ESB 1012: <b>Paolo Perrone, Max Planck Institute for Mathematics in the Sciences, Leipzig, Germany</b> <i>The Wasserstein monad in categorical probability</i></li> <li>• ESB 1013 : <b>Nelson Martins-Ferreira, Polytechnic Institute of Leiria, Portugal</b> <i>Triangulations, triangulated surfaces and the multiplicative structure of internal groupoids</i></li> </ul>
4:00pm - 4:25pm	<b>Parallel Sessions</b> <ul style="list-style-type: none"> <li>• ESB 1012: <b>Partha Pratim Ghosh, University of South Africa, Gauteng, South Africa</b> <i>Internal neighbourhood spaces</i></li> <li>• ESB 1013: <b>Michael Lambert, Dalhousie University, Canada</b> <i>Generalized principal bundles</i></li> </ul>
4:30pm - 5:30pm	Discussion

## Wednesday July 19, 2017

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9:00am - 9:25am	<b>Jiri Adamek, Technical University Braunschweig, Germany</b> <i>Codensity and double-dualization monads</i>
9:30am - 9:55am	<b>Michael Barr, McGill University, Montreal, Canada</b> <i>Simplicial acyclic models</i>
10:00am - 10:25am	<b>Susan Niefield, Union College, New York, USA</b> <i>Topological groupoids and exponentiability</i>
10:30am - 11:00am	Coffee Break (ESB Atrium)
11:00am - 11:25am	<b>Dorette Pronk, Dalhousie University, Canada</b> <i>The orbifold construction for join restriction categories</i>

11:30am - 11:55am	<b>Robin Cockett, University of Calgary, Canada</b> <i>General Ehresmann connections and torsor bundles</i>
12:00pm - 12:25pm	<b>Geoffrey Cruttwell, Mount Allison University, Canada</b> <i>Differential equations in tangent categories</i>
12:30pm - 1:15pm	Lunch- Own (See list of Campus eateries online at <a href="http://www.food.ubc.ca/feed-me/">http://www.food.ubc.ca/feed-me/</a> )
1:15pm - 1:25pm	Excursion to Granville Island (Please assemble at the registration table <b>by 1:15pm</b> . We will then board <b>Lynch Buses</b> at 2175 West Mall)
5:30pm - 9:00pm	<b>Harbor Dinner Cruise</b> Boarding Vessel from Granville Island: Dock A, 1698 Duranleau St. Vancouver BC V6H 3S4 Point of contact: Maret Christiansen- 604-319-1448
9:15pm	Bus pick-up back to UBC conference venue

## Thursday July 20, 2017

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9:00am - 9:55am	<b>Tim Van der Linden, Université catholique de Louvain, Belgium</b> <i>Categorical-algebraic methods in group cohomology</i>
10:00am - 10:25am	<b>Julia Goedecke, University of Cambridge, UK</b> <i>Hopf formulae for Tor</i>
10:30am - 11:00am	Coffee Break (ESB Atrium)
11:00am - 11:25am	<b>Marino Gran, Université catholique de Louvain, Belgium</b> <i>A characterization of central extensions in the variety of quandles</i>
11:30am - 11:55am	<b>Pierre-Alain Jacqmin, Université catholique de Louvain, Belgium</b> <i>An embedding theorem for regular Mal'tsev categories</i>
12:00pm - 12:25pm	<b>Diana Rodelo, CMUC &amp; Universidade do Algarve, Faro, Portugal</b> <i>Stability properties for <math>n</math>-permutable categories</i>
12:30pm - 2:00pm	Lunch- Own (See list of Campus eateries online at <a href="http://www.food.ubc.ca/feed-me/">http://www.food.ubc.ca/feed-me/</a> )
2:00pm - 2:25pm	<b>Alan S. Cigoli, Université catholique de Louvain, Belgium</b> <i>A relative monotone-light factorization system for internal groupoids</i>
2:30pm - 2:55pm	<b>Xabier Garcia-Martinez, University of Santiago de Compostela, Spain</b> <i>A characterization of Lie algebras amongst alternating algebras</i>
3:00pm - 3:30pm	Coffee Break (ESB Atrium)
3:30pm - 3:55pm	<b>Parallel Sessions</b> <ul style="list-style-type: none"> <li>• <b>ESB 1012: Jonathon Gallagher, Dalhousie University, Canada</b> <i>Coherently closed tangent categories and the link between SDG and the differential <math>\lambda</math>-calculus</i></li> <li>• <b>ESB 1013: Sean Moss, University of Cambridge, UK</b> <i>The Diller-Nahm model of type theory</i></li> </ul>

4:00pm – 4:25pm

**Parallel Sessions**

- ESB 1012: **Jean-Simon Lemay, University of Calgary, Canada**  
*Integration in tangent categories*
- ESB 1013: **Evangelia Aleiferi, Dalhousie University, Canada**  
*Towards a characterization of the double category of spans*

4:30pm - 4:55pm

**Parallel Sessions**

- ESB 1012: **Ben MacAdam, University of Calgary, Canada**  
*Vector bundles and dependent linear logic in differential geometry*
- ESB 1013: **Darien DeWolf, Dalhousie University, Canada**  
*An element-based reformulation of restriction monads*

## Friday July 21, 2017

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9:00am – 9:55am

**Dirk Hofmann, Universidade de Aveiro, Portugal**

*Duality theory, convergence, and enriched categories*

10:00am – 10:25am

**Maria Manuel Clementino, Universidade de Coimbra, Portugal**

*On simple monads in ordered structures and the factorisations they induce*

10:30am – 11:00am

Coffee Break (ESB Atrium)

11:00am – 11:25am

**Walter Tholen, York University, Toronto, Canada**

*Topological theories*

11:30am – 11:55am

**Lurdes Sousa, CMUC, University of Coimbra & Polytechnic Institute of Viseu, Portugal**

*Aspects of algebras of KZ-monads*

12:00pm – 12:25pm

**Giulia Frosoni, University of Genoa, Italy**

*Properites of  $\Sigma\Sigma(-)$ -algebras in Equ*

12:30pm – 2:00pm

Lunch- Own (See list of Campus eateries online at <http://www.food.ubc.ca/feed-me/>)

2:00pm – 2:25pm

**Piotr Jedrzejewicz, Nicolaus Copernicus University, Toruń, Poland**

*Towards a categorification of integers*

2:30pm – 2:55pm

**George Janelidze, University of Cape Town, South Africa**

*Infinite addition, real numbers, and taut monads*

3:00pm – 3:30pm

Coffee Break (ESB Atrium)

3:30pm – 3:55pm

**Parallel Sessions**

- ESB 1012: **Laura Scull, Fort Lewis College, Colorado, USA**  
*Fundamental groupoids for orbifolds*
- ESB 1012: **Jun Yoshida, The University of Tokyo, Japan**  
*Graphical calculus in symmetric monoidal  $(\infty)$ -categories with duals*

4:00pm – 4:25pm

**Parallel Sessions**

- ESB 1012: **Marzieh Bayeh, Dalhousie University, Canada**  
*Orbit class and its application*
- ESB 1012: **Daniel Cicala, University of California, Riverside, USA**  
*Modeling graphical calculi with symmetric monoidal compact closed bicategories*

4:30 – 4:55pm

**Parallel Sessions**

- ESB 1012: **Jonas Frey, CMU Pittsburgh, USA**  
*Modelling homotopy type theory in Cartesian cubical sets*
- ESB 1012: **Francisco Rios, Dalhousie University, Canada**  
*A categorical model for a quantum circuit description language*

6:00pm

**CT 2017: Buffet Dinner:**

**University Golf Club**

5185 University Blvd, Vancouver, BC V6T 1X5

(15 min walk or 5 min bus ride on the #4/ #14 trolley buses)

Point of contact: Maret Christiansen- 604-319-1448

## Saturday July 22, 2017

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9:00am - 9:55am

**Robert Paré, Dalhousie University, Canada**

*Hypercategories*

10:00am - 10:25am

**David Jaz Myers, Oberlin College, USA**

*String diagrams for (virtual) proarrow equipments*

10:30am - 11:00am

Coffee Break (ESB Atrium)

11:00am - 11:25am

**Murray Bremner, University of Saskatchewan, Canada**

*Commutativity in double interchange semigroups*

11:30am - 11:55am

**Emily Riehl, John Hopkins University, Baltimore, USA**

*A synthetic theory of  $\infty$ -categories in homotopy type theory*

12:00pm - 12:25pm

**Robert Rosebrugh, Mount Allison University, Canada**

*Symmetric lenses and universality*

12:25pm - 12:30pm

Wrap- up



# ABSTRACTS

Jiří Adámek \*

Technical University Braunschweig, Germany

*Codensity and double-dualization monads*

It is known since 1970's that the codensity monad of the embedding of finite sets into *Set* is the ultrafilter monad. Leinster proved in [1] that the full embedding of finite-dimensional vector spaces into *K-Vec* has the codensity monad given by the double-dualization monad  $(-)^{**}$ . And he asked for generalizations covering the two examples above. We present a solution working in categories  $\mathcal{K}$  that are monoidal closed and have a strong cogenerator  $D$ . The functor  $(-)^* = [-, D]$  is left adjoint to its dual, and the resulting monad  $(-)^{**}$  is called the double-dualization monad.

**Example.** Varieties of algebras have a 'natural' tensor product, representing bimorphisms. Monoidal closedness means precisely that the variety (or, equivalently, its monad) is commutative, see [2]. Analogously, varieties of ordered algebras, presented by operations and inequations, are monoidal closed iff they are commutative.

**Definition.** By the **finite double-dualization monad** is meant the largest submonad of  $(-)^{**}$  whose unit has invertible components at all finitely presentable objects.

**Theorem.** *Let  $\mathcal{K}$  be a commutative variety of (possibly ordered) algebras. Let  $D$  be a strong cogenerator with  $D^n$  finitely presentable for all  $n \in \mathbb{N}$ . Then the finite double-dualization monad is the codensity monad of the full embedding of all finitely presentable objects into  $\mathcal{K}$ .*

**Examples.** (a)  $K$  is a strong cogenerator of *K-Vec*. Since for finitely-dimensional spaces the unit  $\eta_A : A \rightarrow A^{**}$  is invertible, we obtain Leinster's result that the codensity monad is all of  $(-)^{**}$ .

(b) The category *JSL* of join semilattices has the two-element chain as a strong cogenerator. Again, finite semilattices have invertible units, hence, the codensity monad of their embedding is also  $(-)^{**}$ .

(c) For *Set* the two-element set as a cogenerator yields  $X^* = \mathcal{P}X$ . The finite double-dualization monad is the ultrafilter monad.

(d) Analogously for *Pos*: take the two-element chain as a strong cogenerator. Then  $X^*$  is the poset  $\mathcal{P}^u X$  of all up-sets of  $X$ , ordered by the dual of inclusion. The finite double-dualization monad is the prime-filter monad on *Pos*.

**Remark.** We further study codensity monads of set functors. Every accessible functor possesses a codensity monad. The converse does not hold:

**Example.** (1) For the power-set functor  $\mathcal{P}$  the codensity monad assigns to  $X$  the product  $\prod_{Y \subseteq X} \mathcal{P}Y$ .

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\*Joint work with Lurdes Sousa.

- (2) For the subfunctor  $\mathcal{P}_0$  of all nonempty subsets the codensity monad is  $\mathcal{P}_0$  itself.
- (3) In contrast, the following modification  $\mathcal{P}'$  of  $\mathcal{P}$  does not possess a codensity monad: on objects  $\mathcal{P}'X = \mathcal{P}X$ , on morphisms  $f : X \rightarrow Y$ , for every  $M \subseteq X$  put  $\mathcal{P}'f(M) = \mathcal{P}f(M)$  in case  $f/M$  is monic, else  $\emptyset$ .

REFERENCES:

- [1] T. Leinster, Codensity and the ultrafilter monad. *Theory and Applications of Categories* 28 (2013), 332–370.
- [2] B. Banaschewski and N. Nelson, Tensor products and bimorphisms. *Canad. Math. Bull.* 19 (1976), 385–402.

Evangelia Aleiferi  
Dalhousie University

*Towards a characterization of the double category of spans*

In [1] it was shown that the bicategory of spans in a category with finite limits can be characterized as a Cartesian bicategory in which every comonad has an Eilenberg-Moore object and every left adjoint arrow is comonadic. Motivated by this result, we study whether or not a characterization of spans as a Cartesian double category is possible. In this talk, we will define a Cartesian double category to be a double category  $\mathbb{D}$  for which the diagonal double functor  $\Delta : \mathbb{D} \rightarrow \mathbb{D} \times \mathbb{D}$  and the unique double functor  $! : \mathbb{D} \rightarrow \mathbb{1}$  have right adjoints. We will describe some of their properties and we will specifically talk about Cartesian categories that are also fibrant. We will study the double category of comonads over a fibrant Cartesian double category that satisfies the Frobenius axiom and we will extend the theory of Eilenberg-Moore objects to double categories. It is worth mentioning that there are some results about the double category of spans already proven in [2], which will be very useful in our work.

REFERENCES:

- [1] S. Lack, R. F. C. Walters and R. J. Wood, Bicategories of spans as cartesian bicategories, *Theory and Applications of Categories* 24 (2010) 1–24.
- [2] S. Niefield, Span, Cospan, and other double categories, *Theory and Applications of Categories* 26 (2012) 729–742.

Michael Barr  
McGill University, Montreal, Canada

*Simplicial acyclic models*

In 1974 Kleisli published a paper on acyclic models for semi-simplicial complexes (also known as face complexes). This differed from the theorem for chain complexes in that the “presentation” mapping was required to commute with all face operators except  $d^0$ . I extend this to simplicial complexes, adding that the presentation commute with all degeneracies. I also show that the standard resolution of any cotriple satisfies these conditions with respect to the cotriple.

Marzieh Bayeh \*  
Dalhousie University

*Orbit class and its application*

In this talk we introduce a new concept to study topological spaces endowed with an action of a topological group. We call this concept orbit class and is often a good replacement for the well-known concept orbit type. We define a partial ordering on the set of all orbit classes. We apply the properties of orbit classes to define and study the equivariant LS-category and the invariant topological complexity. Furthermore, we consider the category of orbit classes. This is a progress report of an ongoing research topic.

REFERENCES:

- [1] M. Bayeh and S. Sarkar. Orbit class and remarks on invariant topological complexity. *Submitted* (2016).
- [2] H. Colman and M. Grant. Equivariant topological complexity. *Algebr. Geom. Topol.* 12 (2012) 2299–2316.
- [3] W. Lubawski and W. Marzantowicz. Invariant topological complexity. *Bull. Lond. Math. Soc.* 47 (2015) 101–117.

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\*Joint work with Soumen Sarkar.

Murray Bremner \*  
Department of Mathematics and Statistics,  
University of Saskatchewan, Canada

*Commutativity in double interchange semigroups*

We extend the work of Kock [1] and Bremner & Madariaga [2] on commutativity in double interchange semigroups (DIS) to 10 arguments, motivated by potential applications to double categories. Our methods involve algebraic operads: the free symmetric operad generated by two binary operations with no symmetry, its quotient by the two associative laws, its quotient by the interchange law, and its quotient by all three. We also consider a geometric realization of free double interchange magmas by rectangular partitions of the unit square  $I^2$ . We define morphisms between these structures which allow us to represent elements of free DIS both algebraically as tree monomials and geometrically as rectangular partitions. With these morphisms we reason diagrammatically about free DIS and prove our new commutativity relations.

REFERENCES:

- [1] J. Kock, Note on commutativity in double semigroups and two-fold monoidal categories, *Journal of Homotopy and Related Structures* 2 (2007) no. 2, 217–228.
- [2] M. Bremner and S. Madariaga, Permutation of elements in double semigroups, *Semigroup Forum* 92 (2016) 335–360.

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\*Joint work with Fatemeh Bagherzadeh. Research supported by a Discovery Grant from NSERC.

Alexander Campbell  
Macquarie University

*Enriched algebraic weak factorisation systems*

A modification of Garner’s small object argument shows that if  $\mathcal{V}$  is a monoidal model category in which every object is cofibrant, then any cofibrantly generated  $\mathcal{V}$ -enriched model category has a cofibrant replacement  $\mathcal{V}$ -comonad and a fibrant replacement  $\mathcal{V}$ -monad [4]. Conversely, an elementary argument shows that if a monoidal model category  $\mathcal{V}$  with cofibrant unit object has a cofibrant replacement  $\mathcal{V}$ -comonad, then every object of  $\mathcal{V}$  is cofibrant [2].

These results leave open the following question: what extra structure, if not an enrichment in the ordinary sense, is naturally possessed by the (co)fibrant replacement (co)monad of an enriched model category when not every object of the base monoidal model category is cofibrant? The purpose of this talk is to answer this question.

An analysis of the monoidal model category **2-Cat** of 2-categories (in which not every object is cofibrant, and which is monoidal under the Gray tensor product) suggests the decisive concept. For while the cofibrant replacement comonad **st** on **2-Cat**, which sends a 2-category  $A$  to its pseudofunctor classifier  $\mathbf{st}A$ , fails to extend to a **Gray**-comonad, it is nevertheless a monoidal closed comonad, and so comes equipped with pseudofunctors  $\mathbf{Gray}(A, B) \rightarrow \mathbf{Gray}(\mathbf{st}A, \mathbf{st}B)$  enriching **st** with the structure of a “locally weak **Gray**-comonad”. Generally, given a monoidal/closed comonad  $Q$  on a monoidal/closed category  $\mathcal{V}$ , one can define a 2-category of  $\mathcal{V}$ -categories, “locally  $Q$ -weak  $\mathcal{V}$ -functors”, and “locally  $Q$ -weak  $\mathcal{V}$ -natural transformations”.

Abstracting from these observations, I will introduce notions of monoidal, closed, and enriched algebraic weak factorisation systems (which are strengthenings of the notions of bi(co)lax morphisms of AWFS [3]) and demonstrate that the cofibrant replacement comonad  $Q$  for a monoidal/closed AWFS  $(L, R)$  on a monoidal/closed category  $\mathcal{V}$  is a monoidal/closed comonad on  $\mathcal{V}$ , and that the (co)fibrant replacement (co)monad for an  $(L, R)$ -enriched AWFS  $(H, M)$  on a  $\mathcal{V}$ -category  $\mathcal{A}$  is a locally  $Q$ -weak  $\mathcal{V}$ -(co)monad on  $\mathcal{A}$ , and moreover that the category of weak maps [1] for  $(H, M)$  is enriched over the skew-monoidal/closed category of weak maps for  $(L, R)$ .

REFERENCES:

- [1] John Bourke and Richard Garner. Algebraic weak factorisation systems II: Categories of weak maps. *J. Pure Appl. Algebra* 220 (2016), no. 1, 148–174.
- [2] Stephen Lack and Jiří Rosický. Homotopy locally presentable enriched categories. *Theory Appl. Categ.* 31 (2016), no. 25, 712–754.
- [3] Emily Riehl. Monoidal algebraic model structures. *J. Pure Appl. Algebra* 217 (2013), no. 6, 1069–1104.
- [4] Emily Riehl. *Categorical homotopy theory*, volume 24 of *New Mathematical Monographs*. Cambridge University Press, Cambridge, 2014.



Daniel Cicala  
University of California, Riverside

*Modeling graphical calculi with symmetric monoidal compact closed bicategories*

Compositionality is playing an increasingly large role in the study of complex systems. With this viewpoint, one studies a complex system by analyzing its smaller components and their connections. This is particularly useful for open systems admitting a graphical syntax. Two common features of such systems are the use of diagrams with ‘inputs’ and ‘outputs’, and an equality given by rewrite rules. In this talk, we introduce a framework in which these systems fit. In particular, we organize an open system into a symmetric monoidal and compact closed bicategory whose 0-cells are input and output types, 1-cells are the system’s diagrams, and 2-cells are their rewritings. We illustrate our framework by giving a bicategorical syntax for a commutative monoid.

Alan S. Cigoli <sup>\*</sup>  
Université catholique de Louvain

*A relative monotone-light factorization system for internal groupoids*

It is a well-known fact that a Barr-exact category  $\mathcal{C}$  can be seen as a reflective subcategory of the category  $\mathbf{Gpd}(\mathcal{C})$  of its internal groupoids:

$$\mathbf{Gpd}(\mathcal{C}) \begin{array}{c} \xrightarrow{\pi_0} \\ \perp \\ \xleftarrow{D} \end{array} \mathcal{C} \quad (1)$$

where  $D$  sends each object in  $\mathcal{C}$  to the corresponding discrete internal groupoid, and  $\pi_0$  is the connected components functor. This adjunction gives rise to an associated (reflective) factorization system  $(\mathcal{E}, \mathcal{M})$ , where  $\mathcal{E}$  is the class of internal functors inverted by  $\pi_0$ . As we will easily see, this factorization system does not admit an associated *monotone-light* factorization system in the sense of [1].

We will then restrict our attention to the case where  $\mathcal{C}$  is also a Mal'tsev category. As explained in [3], in this case the adjunction (1) presents  $\mathcal{C}$  as a Birkhoff subcategory of  $\mathbf{Gpd}(\mathcal{C})$  and the general theory of central extensions developed in [4] applies here. In particular, central extensions are characterized in [3] as regular epimorphic internal discrete fibrations. We will show that, together with the class of internal final functors, these form a *relative* monotone-light factorization system (in the sense of [2]) for regular epimorphic internal functors.

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<sup>\*</sup>Joint work with T. Everaert and M. Gran.

Maria Manuel Clementino \*  
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*On simple monads in ordered structures and the factorisations they induce*

We recall the notion of simple monad [2, 3] in order-enriched categories, that generalises the notion of simple reflection of Cassidy-Hébert-Kelly [1], and study the factorisations they induce. These factorisations are lax orthogonal, as defined in [2], and can be characterised by a cancellation property that, once again, includes the orthogonal case studied in [1].

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\*Joint work with Ignacio López Franco.

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University of Calgary

*General Erhesmann connections and torsor bundles*

In a tangent category [1,2] it is normal to define a connection on a differential bundle [3], however, there is a more general notion – originally explored in the classical case by Erhesmann – which works on an arbitrary bundle (that is an arbitrary map from  $E$  to  $M$ ). The purpose of this talk is to explore this more general notion and, in particular, to explore the theory of principal  $G$ -bundles expressed in a novel way using torsors. Of particular interest is when the torsor structure and the connection are “compatible”: this allows a re-expression of the data.

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\*Joint work with Geoff Cruttwell.

Geoffrey Cruttwell \*  
Mount Allison University

*Differential equations in tangent categories*

Tangent categories, first defined by Rosický [7], are an abstract setting for differential geometry. Recent work has shown that within their formalism one can define and work with many of the fundamental ideas of differential geometry such as the Lie bracket [3], vector bundles [4], connections [5], and de Rham cohomology [6]. A variety of models for the axioms have also been identified, ranging from examples in ordinary differential geometry to examples in algebraic geometry, synthetic differential geometry, and abelian functor calculus [2, 7, 1]

In this talk, we discuss how to define and work with solutions to ordinary differential equations in tangent categories. This requires several additions to the tangent category axioms. First of all, since solutions to differential equations need not be totally defined, we work in the more general setting of a tangent restriction category (described in [2]) in which maps need only be partially defined. Second, we assume the existence of a special “curve” object which translates vector fields into flows (that is, an object which “solves certain ordinary differential equations”). We will then discuss various consequences of these axioms in this general setting, such as the relationship between the Lie bracket of vector fields and the commutativity of their respective flows.

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\*Joint work with Robin Cockett and Rory Lucyshyn-Wright.

*On flat 2-functors*

The main theorem of the theory of flat functors ([1], [4]) states that  $A \xrightarrow{P} \mathcal{E}ns$  is flat if and only if  $P$  is a filtered colimit of representable functors, i.e. there is a filtered category  $I$  and a diagram  $I \xrightarrow{X} A$  such that  $P$  is the colimit of the composition  $I^{op} \xrightarrow{X} A^{op} \xrightarrow{h} Hom(A, \mathcal{E}ns)$  where  $h$  is the Yoneda embedding. For an arbitrary base category  $\mathcal{V}$  instead of  $\mathcal{E}ns$ , Kelly ([3]) has developed a theory of flat  $\mathcal{V}$ -enriched functors  $A \xrightarrow{P} \mathcal{V}$ , but there is no known generalization of the theorem above for any  $\mathcal{V}$  other than  $\mathcal{E}ns$ .

In [2] we have established a 2-dimensional version of this theorem, i.e. for a 2-functor  $\mathcal{A} \xrightarrow{P} \mathcal{C}at$ , where  $\mathcal{A}$  is a 2-category and  $\mathcal{C}at$  is the 2-category of categories. As it is usually the case for 2-categories, the  $\mathcal{C}at$ -enriched notions are not adequate for most purposes and the *relaxed* bi and pseudo notions are the important ones.

We define a 2-functor  $\mathcal{A} \xrightarrow{P} \mathcal{C}at$  to be *flat* when its *left bi-Kan extension*  $Hom_s(\mathcal{A}^{op}, \mathcal{C}at) \xrightarrow{P^*} \mathcal{C}at$  along the Yoneda 2-functor  $\mathcal{A} \xrightarrow{h} Hom_s(\mathcal{A}^{op}, \mathcal{C}at)$  is *left exact*.  $Hom_s(\mathcal{A}^{op}, \mathcal{C}at)$  denotes the 2-category of 2-functors, 2-natural transformations and modifications. By left bi-Kan extension we understand the bi-universal pseudonatural transformation  $P \implies P^*h$ , and by left exact we understand preservation of finite weighted bilimits. Let  $(\mathcal{A}, \Sigma)$  be a pair where  $\mathcal{A}$  is a 2-category and  $\Sigma$  a distinguished 1-subcategory. A  $\sigma$ -cone for a 2-functor  $\mathcal{A} \xrightarrow{F} \mathcal{B}$  is a lax cone such that the 2-cells corresponding to the distinguished arrows are invertible. The  $\sigma$ -limit of  $F$  is a universal  $\sigma$ -cone (characterized up to isomorphism). We introduce a notion of 2-filteredness of  $\mathcal{A}$  with respect to  $\Sigma$ , which we call  $\sigma$ -filtered. Our main result states the following:

A 2-functor  $\mathcal{A} \xrightarrow{P} \mathcal{C}at$  is flat if and only if there is a  $\sigma$ -filtered pair  $(\mathcal{I}^{op}, \Sigma)$  and a 2-diagram  $\mathcal{I} \xrightarrow{X} \mathcal{A}$  such that  $P$  is pseudo-equivalent to the  $\sigma$ -bicolimit of the composition  $\mathcal{I}^{op} \xrightarrow{X} \mathcal{A}^{op} \xrightarrow{h} Hom_s(\mathcal{A}, \mathcal{C}at)$ . As in the 1-dimensional case,  $X$  can be chosen as the 2-fibration associated to  $P$ .

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\*Joint work with E. Dubuc and M. Szyld.

Darien DeWolf  
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*An element-based reformulation of restriction monads*

Last year, I introduced restriction monads: monads in a bicategory equipped with a restriction 2-cell satisfying axioms reminiscent of those satisfied in a restriction category. This talk will give a reformulation of restriction monads in bicategories with an initial object. An immediate benefit of this reformulation is a pair of one-to-one correspondences between (i) small restriction categories and restriction monads in  $\text{Span}(\mathbf{Set})$  and (ii) small restriction categories and restriction monads in  $\mathbf{Set}\text{-Mat}$ . These correspondences form the motivation for defining internal restriction categories and restriction enriched categories, respectively.

Jacopo Emmenegger  
Stockholms Universitet

*On the local cartesian closure of exact completions*

Carboni and Rosolini have given in [1] a characterisation of (local) cartesian closure of exact completions in terms of a property of their projectives, but a recently discovered oversight in their argument entails that such characterisation is only valid when the projectives are internally projectives, i.e. closed under products (pullbacks for local cartesian closure).

We will introduce a different condition on a category with weak finite limits which alone implies that its exact completion is locally cartesian closed. This condition was inspired by an axiom in the context of constructive set theory and originally applied to a category defined from Martin-Löf type theory. However, we will see how this condition arises in the homotopy-theoretic context as well, where homotopy categories provide natural examples of categories with weak finite limits.

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Timmy Fieremans \*  
Vrije Universiteit Brussel

*Frobenius and Hopf  $\mathcal{V}$ -categories*

We define *Frobenius  $\mathcal{V}$ -categories*, for any monoidal category  $\mathcal{V}$ . We also recall basic notions of Hopf  $\mathcal{V}$ -categories as introduced in [1]. When  $\mathcal{V}$  is the category of modules over a commutative ring, we show that the classical Larson-Sweedler theorem can be generalised to this many-object setting by giving equivalent definitions of Frobenius  $k$ -linear categories in terms of Casimir elements and self-duality in the same style as ordinary Frobenius algebras.

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\*Joint work with Mitchell Buckley, Christina Vasilakopoulou and Joost Vercruyssen.

Jonas Frey \*  
CMU Pittsburgh

*Modelling homotopy type theory in cartesian cubical sets*

Starting from the observation that Voevodsky’s model [KL12] of homotopy type theory is not constructive, Coquand et al. [BCH14] developed a constructive model in a category of cubical sets, with the aim of solving the *canonicity problem*.

I will present work in progress on a variation of this model in the presheaf category of *cartesian cubical sets* [Awo16] where types are interpreted as uniform Kan complexes, and identity types are interpreted using an algebraic weak factorization system [BG16] based on a notion of path object given by exponentiation by an interval object.

A goal of our work is to construct a univalent universe that can be internalized in a topos with a small complete subcategory, such as Hyland’s effective topos [Hyl82]. This construction is based on recent work of Gambino and Sattler [GS17, Sat17].

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\*Joint work with Steve Awodey (CMU Pittsburgh) and Pieter Hofstra (University of Ottawa).

Giulia Frosoni \*  
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*Properties of  $\Sigma^{\Sigma^{(-)}}$ -algebras in Equ*

The category Equ of equilogical spaces, introduced in [2], provides a useful locally cartesian closed extension of the category  $\text{Top}_0$  of  $T_0$ -spaces and continuous maps; the embedding of  $T_0$ -spaces is full and preserves all the existing locally cartesian closed structure ([5, 6]). The Sierpinski space  $\Sigma$ , consisting of two elements, one open and one closed, is the open-subset classifier, *i.e.* given a  $T_0$ -space  $S$ , for every  $T_0$ -space  $X$ , a continuous map  $f: X \rightarrow \Sigma^S$  determines precisely an open subset of  $X \times S$ ; nevertheless,  $\Sigma^S$  is an equilogical space which need not be a topological space. In other words, Equ allows one to work with  $T_0$ -spaces as if they *were* a cartesian closed category.

The monad of the double power of  $\Sigma$  was considered in different settings in many papers, see for example [3, 4]. This led us to analyze the self-adjoint functor  $\Sigma^{(-)}: \text{Equ} \rightarrow \text{Equ}^{op}$  and the monad of the double power of  $\Sigma$  on the category of equilogical spaces. Interestingly, in [1], this double power monad on Equ gives an intrinsic description of the soberification of a  $T_0$ -space.

In this talk we investigate the category of the algebras for the double power monad of  $\Sigma$  on Equ, pointing out a connection with the category of frames and frame homomorphisms; in particular, we recall how the structure of  $\Sigma^{\Sigma^{(-)}}$ -algebra on an equilogical space gives rise to a frame on the set of its global sections. We then focus on some particular subcategories of Equ: the category of continuous lattices, the category of algebraic lattices and  $\text{Top}_0$  itself, restricting the double power monad to each of them and analyzing the algebras in each case. Finally, we introduce a full subcategory  $\text{REqu}$  of Equ, involving algebraic lattices and equivalence relations on them, and use an algebraic approach to determine the  $\Sigma^{\Sigma^{(-)}}$ -algebras in  $\text{REqu}$  and their relationship with spatial frames.

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\*Joint work with Giuseppe Rosolini.

Jonathan Gallagher \*  
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*Coherently closed tangent categories  
and the link between SDG and the differential  $\lambda$ -calculus*

Type theories for smooth maps have been independently studied by two schools of thought with different motivations. The first is synthetic differential geometry (SDG) [1, 2, 3]. Here, one uses the type theory of a topos to reason about microlinear spaces. The motivation is the development of a rigorous foundation for synthetic arguments used in differential geometry. The second is the differential  $\lambda$ -calculus, an explicit type theory for reasoning in smooth models of linear logic (Köthe sequence spaces, convenient vector spaces) [4, 5, 6, 7]. The motivation is to provide a syntax for resource sensitive proofs/computations [8].

The type theories are linked in a simple manner: categorical models of either are always tangent categories [9, 10]. Surprisingly, they are more intimately related as well. This talk will develop a direct relationship between Euclidean vector bundles in SDG, and the differential  $\lambda$ -calculus.

More generally, we will show that the differential bundles over a fixed base  $B$  (the analog of vector bundles in a tangent category) of any *coherent, locally cartesian closed tangent category* are a model of the differential  $\lambda$ -calculus. Thus, in SDG, the local reasoning in the category of vector bundles over  $B$  is captured by the differential  $\lambda$ -calculus. Having an explicit logic for vector bundles makes lifting certain parts of classical differential geometry, for example, Lagrangian systems and symplectic geometry, more direct.

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\*Joint work with Robin Cockett.

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Xabier García-Martínez \*  
University of Santiago de Compostela

*A characterisation of Lie algebras amongst alternating algebras*

The aim of this talk is to prove that, if a variety of alternating algebras—not necessarily associative, where  $xx = 0$  is a law—over an infinite field admits *algebraic exponents* in the sense of James Gray’s Ph.D. thesis [1], so when it is *locally algebraically cartesian closed* (or (LACC) for short), then it must necessarily be a variety of Lie algebras.

The number of examples of (LACC) semi-abelian categories currently known is very small, and almost all happen to consist of group objects in a cartesian closed category: groups, crossed modules, and cocommutative Hopf algebras over a field of characteristic zero being the principle ones. The only known exception is the category of Lie algebras over a commutative ring [2]. In the quest of finding new examples, we ended up showing that if a variety of alternating algebras is (LACC), then the Jacobi identity is amongst its laws.

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\*Joint work with Tim Van der Linden.

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*Internal neighbourhood spaces*

The talk generalises the construction of pretopological spaces and pseudotopological spaces to a context where the ground category of sets is replaced with an arbitrary finitely complete category equipped with a proper factorisation system and each lattice of *admissible subobjects* is a complete distributive lattice. It is shown that the categories of *internal weak neighbourhood spaces* and *internal pretopological spaces* are topological over the base category. The category of *internal weak neighbourhood spaces* is shown to be bireflective in the category of *internal pretopological spaces*. In the special case when each lattice of *admissible subobjects* is a pseudocomplemented complete distributive lattice and each change of base a homomorphism of pseudocomplemented complete lattices, the category of *internal pseudotopological spaces* is shown to contain the category of *internal pretopological spaces* bireflectively and is itself topological over the base category. There are *neighbourhood structures* over each object which are similar to the neighbourhoods obtained from a topology on a set. If every change of base is a homomorphism of pseudocomplemented complete lattices then the category of *internal neighbourhood spaces* is topological over the base category and is a bireflective full subcategory of the category of *internal weak neighbourhood spaces*. The special *neighbourhood structures* on an object whose *open subsets* make a topology give rise to *topological structures* on the object. In the special case when each lattice of *admissible subobjects* is a frame and each change of base is a homomorphism of pseudocomplemented complete lattices the category of *internal topological spaces* is isomorphic to the category of *internal neighbourhood spaces* and hence is topological over the base category. Thus, in particular, the classical case for the context of sets and functions is obtained as a special case of the results presented in a more general context in this talk.

Julia Goedecke \*  
University of Cambridge

*Hopf formulae for Tor*

A Hopf formula expresses a homology object in terms of a projective presentation, its kernel and certain (generalised) commutators. The first such formula, for second group homology, was given by Hopf in 1942. Over the last 13 years or so, Everaert, Gran, Van der Linden and others have developed Hopf formulae in more general categorical contexts. One of these general contexts is that of a semi-abelian category with a Birkhoff subcategory where the reflector factors through a protoadditive functor. In that generality, some elements of the Hopf formula are necessarily very abstract. With Tim Van der Linden and Guram Donadze, I am studying the special situation of subvarieties of categories of  $R$ -modules. It can be seen using properties of algebraic theories that every such subvariety is again a category of modules. Here we can find explicit and easy formulations of the generalised commutators. Since the reflector in this situation turns out to be tensoring, the resulting homology functors are Tor functors. Through these fairly simple formulations we obtain new ways of calculating, for example, homology of Lie algebras, and Hochschild homology of an associative unital algebra. More generally, our results apply to any abelian Birkhoff subcategory of any semi-abelian variety, using a factorisation through the abelian core.

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\*Joint work with Tim Van der Linden and Guram Donadze.



Marino Gran \*  
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*A characterization of central extensions in the variety of quandles*

During the last years some categorical properties of the category  $\mathbf{Qnd}$  of *quandles* have been investigated [1, 2, 3]. We shall first recall some of these results that will be useful for the present work [4], and then consider the subvariety  $\mathbf{SymQnd}$  of  $\mathbf{Qnd}$  consisting in *symmetric quandles*. This latter is a Mal'tsev variety, whose subvariety  $\mathbf{AbSymQnd}$  of abelian symmetric quandles turns out to be an admissible subcategory (in the sense of categorical Galois theory) in the larger category  $\mathbf{Qnd}$ . We shall give an algebraic description of the quandle extensions that are central for the adjunction between the variety of quandles and its subvariety of abelian symmetric quandles.

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\*Joint work with Valérian Even and Andrea Montoli.

Dirk Hofmann  
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*Duality theory, convergence, and enriched categories*

The principal aim of this talk is to combine the three keywords of the title in a suitable way. It is probably impossible to talk about duality theory without mentioning Stone’s famous duality results for Boolean algebras and distributive lattices, which motivated many other duality results, typically involving some kind of lattices. Our first goal is to develop a systematic method, based on enriched category theory, for extending these duality theorems to categories including all compact Hausdorff spaces. Keeping in mind that ordered sets can be viewed as categories enriched in the two-element quantale, our thesis is that *the passage from the two-element space to the compact Hausdorff space  $[0, 1]$  (a cogenerator of the category of compact Hausdorff spaces) on one side of the duality should be matched by a move from ordered structures to categories enriched in  $[0, 1]$  on the other side*. Accordingly, we present duality theory for ordered compact Hausdorff spaces and monoids of categories enriched in the quantale  $[0, 1]$  with finite weighted colimits. One should think of these monoids as “[0, 1]-enriched lattices”.

However, doing so is somehow inconsequential, as we still consider *ordered* compact Hausdorff spaces. Our next step aims at an extension of these results to compact Hausdorff spaces equipped with a quantale-enriched category structure, which constitute a generalisation of Nachbin’s ordered spaces (see [6, 7]) and are closely related to Hermida’s representable multi-categories [4]. Arguably, these spaces are best studied within the framework of “quantale-enriched topological spaces”; that is, lax algebras for the ultrafilter monad *à la* Barr’s description of topological spaces as relational algebras [1]. We use this opportunity to recall the setting of monad-quantale enriched categories [5] and in particular the important notion of distributor. We sharpen some results on Cauchy-completeness presented earlier, and give a more systematic study of enriched compact Hausdorff spaces. If time permits, we will also consider the case of an enrichment in a symmetric monoidal closed category (see [2]).

Finally, already Halmos [3] observed that it is often beneficial to study duality theory “at a slightly more general level than might appear relevant at first sight”, and proved that the category of Boolean spaces and Boolean *relations* is dually equivalent to the category of Boolean algebras and maps preserving finite suprema

$$\text{BoolRel} \simeq \text{FinSup}_{\text{boo}}^{\text{op}};$$

here  $\text{BoolRel}$  is actually the Kleisli category of the Vietoris monad, and the latter category we describe as the full subcategory of the category of finitely complete ordered sets defined by Boolean algebras. Using again the theory of monad-quantale enriched categories, we introduce and study enriched versions of the classical Vietoris monad. With these tools at our disposal, we develop duality theory for  $[0, 1]$ -enriched compact Hausdorff spaces and distributors on one side, and categories enriched in the quantale

$[0, 1]$  with finite weighted colimits on the other side. These results entail the duality results mentioned before; surprisingly or not, the general theory seems to work better in this setting. We also use these results to show that the dual of the category of partially ordered compact Hausdorff spaces is a  $\aleph_1$ -ary quasivariety and give a partial description of its algebraic theory, which is sufficient to identify also the dual of the category of Vietoris coalgebras as a  $\aleph_1$ -ary quasivariety.

This talk is based on joint work with Maria Manuel Clementino, Renato Neves, Pedro Nora, Carla Reis, Isar Stubbe and Walter Tholen.

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*An embedding theorem for regular Mal'tsev categories*

Barr's embedding theorem for regular categories [1] provides, for each small regular category  $\mathcal{C}$ , a full and faithful embedding  $\mathcal{C} \hookrightarrow \mathbf{Set}^{\mathcal{P}}$  preserving finite limits and regular epimorphisms into a presheaf category. In the abelian context, Lubkin's embedding theorem [6] states that any small abelian category  $\mathcal{A}$  admits a faithful conservative exact embedding  $\mathcal{A} \hookrightarrow \mathbf{Ab}$  into the category of abelian groups. The aim of this talk is to present similar embedding theorems in the non-abelian context, and in particular for regular Mal'tsev categories.

A regular category is a Mal'tsev category when the composition of equivalence relations on each object is commutative, or equivalently, when each reflexive relation is an equivalence relation [2, 3]. I shall show a construction of a particular regular Mal'tsev locally finitely presentable category  $\mathcal{M}$  in terms of (partial) operations and identities. This category can be thought of as a 'representing Mal'tsev category' in the sense that the following embedding theorem holds [4]: each small regular Mal'tsev category  $\mathcal{C}$  admits a faithful conservative embedding  $\mathcal{C} \hookrightarrow \mathcal{M}^n$  which preserves finite limits and regular epimorphisms. Here,  $n$  is the (cardinal) number of subobjects of the terminal object  $1$  of  $\mathcal{C}$ . This embedding theorem allows one to prove results about finite limits and regular epimorphisms for regular Mal'tsev categories using elements and operations in an (essentially) algebraic way.

Similar embedding theorems also hold for regular unital, regular strongly unital, regular subtractive and  $n$ -permutable categories [5].

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\*Joint work with Zurab Janelidze.

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*Infinite addition, real numbers, and taut monads*

Classically, in a category  $\mathbf{C}$ , finite coproducts canonically coincide with finite products if and only if  $\mathbf{C}$  admits addition of morphisms satisfying well-known standard conditions [3]. In fact having such an addition is the same as having an enrichment in the symmetric monoidal closed category of commutative monoids, and that classical observation of Mac Lane has a straightforward infinite counterpart, which can also be formulated for all infinite cardinals separately. In particular, the countable case is considered in [2] (referring to [1] and other papers); the countable counterparts of commutative monoids are called series monoids there.

In this talk we recall some results of the first four sections of [2], and add new ones. In particular, we shall say more about what was called the series monoid of paradoxical positive reals in [2]; it turns out that its construction, out of the ordinary monoid of positive reals, can be extended to algebras over any taut monad on an extensive category with pullbacks. Note that, say,  $0.999\dots \neq 1$  in the “paradoxical” context, which excludes the existence of negation, and this agrees with the fact that the (abelian) group monad is not taut in contrast to the (commutative) monoid monad.

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\*Joint work with Ross Street.

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*Towards a categorification of integers*

The motivation comes from Stephen Schanuel’s question:

“Where are negative sets?”

Though ill-posed, the question is suggestive; a good answer should complete the diagram

$$\begin{array}{ccc} \mathbb{S} & \hookrightarrow & \mathbb{E} \\ \downarrow & & \downarrow \\ \mathbb{N} & \hookrightarrow & \mathbb{Z} \end{array}$$

where  $\mathbb{S}$  is the category of finite sets; we seek an enlargement  $\mathbb{E}$ , the isomorphism classes of which should give rise to all integers, rather than just natural numbers ([4]).”

We would like to present a background for constructing a positive answer to the above question, based on generalized multisets. A multiset is a set with multiple elements. The first known observation that one can define a generalized multiset with arbitrary integer (positive, negative or zero) multiplicities, belongs to Hassler Whitney ([5]). Systematic studies in this field started with the works of Wolfgang Reisig ([3], ch. 9), Wayne D. Blizard ([1]) and Daniel Loeb ([2]).

When we restrict multiplicities to: 1, 0,  $-1$ , we obtain a generalized set which is a pair of disjoint sets  $(A, B)$ , where  $A$  is the positive part and  $B$  is the negative one. Generalized union and intersection are defined by max and min of multiplicities, respectively, so

$$(A, B) \overset{\text{g}}{\cup} (C, D) = (A \cup C, B \cap D), \quad (A, B) \overset{\text{g}}{\cap} (C, D) = (A \cap C, B \cup D).$$

Inclusion is defined by inequality between multiplicities, so

$$(A, B) \overset{\text{g}}{\subset} (C, D) \Leftrightarrow A \subset C, D \subset B.$$

If  $A$  and  $B$  are finite disjoint sets, we put  $|A| - |B|$  to be the generalized cardinality of  $(A, B)$ . Natural candidates for a direct sum and a direct product of  $(A, B)$  and  $(C, D)$  are:

$$(A \sqcup C, B \sqcup D), \quad (A \times C \sqcup B \times D, A \times D \sqcup B \times C).$$

Now, we can precise Schanuel’s question if it is possible to define in some natural way maps between finite generalized sets in order to obtain a category extending the category of finite sets. It may be also interesting to look for some similar constructions in other categories, where two pairs of objects  $(A, B)$  and  $(C, D)$  are isomorphic if and only if  $A \oplus D$  and  $B \oplus C$  are isomorphic in the initial category.

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Michael Lambert  
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*Generalized principal bundles*

A “generalized principal bundle” for an ordinary 1-category will be defined as a certain category-valued pseudo-bimodule that, roughly speaking, is a filtered category in each of its total fibres. This definition provides a possible generalization of a version of the classical definition found in [1].

For any pseudo-bimodule, an explicit construction of a pointwise Kan extension will be given. This gives a concrete computation of certain weighted pseudo-colimits. The Kan extension is expressible as a pseudo-coequalizer and admits a right calculus of fractions under certain further hypotheses. The main result is that a bimodule is a generalized principal bundle if, and only if, its induced Kan extension preserves finite weighed pseudo-limits in a suitable sense.

Finally, we will discuss the extent to which 2-categories of indexed categories can be seen as classifying categories for generalized principal bundles.

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*Integration in tangent categories*

Since the turn of the 21<sup>st</sup> century, the theory of differential categories has led to significant progress in the abstract understanding of differentiation in a variety of settings. In particular, tangent categories [5, 3], which come equipped with a tangent functor, provide an axiomatic setting for differential geometry, while cartesian differential categories [2], which come equipped with a differential combinator, axiomatizes the directional derivative. Recently there has been effort put into studying the axiomatization of integration and antiderivatives in the various differential category settings [1]. In this talk we will introduce the notion of integration in a tangent category, which involves integrating linear bundle morphisms between differential bundles [4]. We will also discuss integration for cartesian differential categories and show the relation with tangent category integration. With this, we will be able to formalize a number of properties of integration, such as Fubini's theorem, the Fundamental Theorems of Calculus, integration of forms, and Stoke's theorem for tangent categories.

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\*Joint work with Robin Cockett and Geoff Cruttwell.

*Duality theorems for essential inclusions of Grothendieck toposes*

An inclusion of toposes is said to be essential if the inverse image functor has an extra left adjoint. In their paper of 1989 entitled ‘On the Complete Lattice of Essential Localizations’ [1], Kelly and Lawvere gave a characterisation of essential inclusions of Grothendieck toposes, and also established a duality between essential inclusions of presheaf toposes and idempotent ideals on the respective base category. In the talk we will see extensions of both of these results, which appear in the speaker’s Ph.D. thesis [2].

We shall analyse the cases where the extra left adjoint of an essential inclusion has specific exactness properties, such as preserving finite limits or preserving finite products, and exhibit the corresponding characterisations. We shall give a final answer to the question posed in the 100th PSSL in Cambridge regarding the stability of essential inclusions under pullback, and explain how it relates to their stability under the inclusion-surjection factorisation.

We shall also generalise the aforementioned duality result of Kelly and Lawvere from presheaf toposes to general Grothendieck toposes, and show how when applied to the special case of localic toposes one can find another proof for the result of Johnstone and Moerdijk [3] which characterises local geometric morphisms between toposes over a topological space.

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Rory Lucyshyn-Wright  
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*Algebraic duality and the abstract functional analysis of distribution monads*

Given a commutative ring  $S$  in a suitable category  $\mathcal{V}$ , the familiar process of dualization of  $S$ -modules leads to a form of abstract functional analysis, in terms of which certain measure and distribution monads can be studied [5, 6]. Generalizing from  $S$ -modules to  $\mathcal{T}$ -algebras for a suitable  $\mathcal{V}$ -enriched algebraic theory  $\mathcal{T}$  on a system of arities  $\mathcal{J}$  [4], we arrive at the notions of *functional-analytic context* and *functional distribution monad* [1], which capture several kinds of measures, probability measures, distributions, and filters, as well as certain hyperspaces of closed subsets.

In this talk, we study a notion of dualization with respect to a given object  $S$  of an arbitrary  $\mathcal{J}$ -algebraic  $\mathcal{V}$ -category  $\mathcal{A}$ , leading to a general study of dualities between algebraic categories. Building on an insight of Freyd, we show that every dual adjunction  $\Delta \dashv \nabla : \mathcal{B}^{op} \rightarrow \mathcal{A}$  between  $\mathcal{J}$ -algebraic  $\mathcal{V}$ -categories is given by dualizing with respect to a *bifold algebra*  $S$ , i.e. an object of  $\mathcal{V}$  equipped with a pair of commuting algebra structures for specified  $\mathcal{J}$ -theories  $\mathcal{T}$  and  $\mathcal{S}$ . Calling such adjunctions  *$\mathcal{J}$ -algebraic dualities*, we characterize those whose inducing bifold algebra  $S$  exhibits  $\mathcal{T}$  and  $\mathcal{S}$  as *commutants* of each other [2, 3], leading to the notion of *stable  $\mathcal{J}$ -algebraic duality*. This yields an equivalent formulation of functional-analytic contexts as certain stable  $\mathcal{J}$ -algebraic dualities. We discuss several examples of  $\mathcal{J}$ -algebraic dualities, functional-analytic contexts, and functional distribution monads.

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*Vector bundles and dependent linear logic in differential geometry*

Multicategories provide a categorical semantics for multi-linear maps in linear algebra, and Hermida showed that representable multicategories are equivalent to monoidal categories [1]. Blute-Cockett-Seely introduced systems of linear maps to provide a language for multilinear maps in categories of “smooth” maps, and described when these multilinear maps gave rise to a representable multicategory or a storage comonad [2]. Topological vector bundles - epimorphisms  $q : E \rightarrow B$  so that for every  $b \in B$ ,  $q^{-1}(b)$  is a vector space - give a model of *local* linear structure which is a basic building block in differential geometry.

In this talk, indexed systems of linear maps are developed to model the fibrewise linearity of topological vector bundles. Indexed systems of linear maps gives rise to fiberwise notions of monoidal representability and storage, which in turn gives rise to an indexed monoidal category and the categorical semantics of dependent linear logic [3][4]. This structure is then applied to the differential bundle fibration in a tangent category [5], which was first explored by Cockett and Cruttwell [6], to cleanly express the basic concepts of differential forms and symplectic geometry in a tangent category.

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\*Joint work with Robin Cockett and Jonathan Gallagher.

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*Two dimensional algebra and natural distributive laws*

Our structures of interest here are general two dimensional monads, algebras and adjunctions and the natural distributive laws that show up within.

*The canonical intensive quality of a pre-cohesive topos*

In the context of Lawvere’s Axiomatic Cohesion [2], an essential and local geometric morphism  $p : \mathcal{E} \rightarrow \mathcal{S}$  between toposes is *cohesive* if

- i)  $p_! : \mathcal{E} \rightarrow \mathcal{S}$  preserves finite products.
- ii) (“Continuity”) for every  $E \in \mathcal{E}$  and  $S \in \mathcal{S}$  the induced morphism  $p_!(E^{(p^*S)}) \rightarrow (p_!E)^S$  is an isomorphism.
- iii) (“Nullstellensatz”) the canonical map  $\theta : p_* \rightarrow p_!$  is epi.

Without the continuity condition ii), we refer to  $p : \mathcal{E} \rightarrow \mathcal{S}$  as *pre-cohesive* [4]. For any pre-cohesive  $p : \mathcal{E} \rightarrow \mathcal{S}$ , [2] constructs the associated canonical intensive quality as the full subcategory  $\mathcal{L}$  of  $\mathcal{E}$  of those objects  $X$  for which  $\theta_X : p_*X \rightarrow p_!X$  is an isomorphism. We call  $\mathcal{L}$  the Leibniz category associated to  $p$ .

In this talk we will review some of the basic properties of the category  $\mathcal{L}$ , we will give elementary constructions of the left and right adjoints of the inclusion functor  $\mathcal{L} \rightarrow \mathcal{E}$ , and we will determine sufficient conditions for a pieces preserving geometric morphism [3]  $g : \mathcal{F} \rightarrow \mathcal{E}$  between two pre-cohesive toposes over  $\mathcal{S}$  to restrict to a geometric morphism between the corresponding Leibniz categories.

Furthermore, we will produce a subcanonical site for the Leibniz category determined by the cohesive site over sets of piecewise linear functions constructed in [5].

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\*Joint work with Matías Menni.

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*Triangulations, triangulated surfaces  
and the multiplicative structure of internal groupoids*

A triangulation [1] is a straightforward generalization of a directed graph. In the same way as a directed graph, internal to an arbitrary category, consists of two objects and two parallel morphisms between them, a triangulation consists of two objects (the object of triangles and the object of vertices) and three parallel morphisms between the two objects.

Every triangulated surface gives rise to a collection of triangles and hence a triangulation. Another example of a triangulation is obtained from the multiplicative structure of an internal groupoid, or an internal category.

In this talk we will see how to detect whether a given triangulation is the structure of a triangulated surface or the structure of an internal groupoid.

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Matias Menni

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*On a problem in Objective Number Theory*

I will sketch the proof of a result extending some of the work by Schanuel, Lawvere, Blass and Gates cited below.

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*The Diller-Nahm model of type theory*

Gödel's Dialectica interpretation is a proof interpretation of Heyting arithmetic into a system of computable functionals of finite type. De Paiva [1], Hyland [2] and others have worked on the idea of a semantic version of Dialectica: starting with a category of types and a fibration of predicates over it, a new structured category is built whose morphisms correspond to the Dialectica interpretation of logical implication. Recently, von Glehn [3] has adapted this idea for the original Dialectica interpretation to categorical models of dependent type theory. I will discuss how we can build categorical models of dependent type theory based on other variants of Dialectica, including the Diller-Nahm variant.

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David Jaz Myers  
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*String diagrams for (virtual) proarrow equipments*

String diagrams for monoidal categories make computations tactile and intuitive affairs. Complicated diagram chases can be expressed in a few pictures and rediscovered with a shoelace. In this talk, I will extend the usual string diagrams for monoidal categories to (virtual) proarrow equipments with the hopes of bringing the diagrammatic method to formal category theory. I will then give some applications of the diagrams.

The proof that the string diagrams for equipments have invariant meaning under deformation builds off the analogous proofs of Joyal and Street [2] for monoidal categories, together with the work of Dawson and Paré [3] on tile orders and Dawson [4] on composition in double categories.

In his paper [5] on enriched category theory, Lawvere mentions that not only are the common objects of mathematics organized into categories, but they are often enriched categories in their own right. Using the diagrams, I will embed any virtual equipment into the virtual equipment of categories enriched in it. This extends Lawvere's claim by showing that as long as our objects of interest are organized into a virtual equipment, they are enriched categories of a sort.

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*Topological groupoids and exponentiability*

We consider exponentiable objects and morphisms in the 2-category  $\text{Gpd}(\mathcal{C})$  of internal groupoids in a category  $\mathcal{C}$  with finite coproducts when  $\mathcal{C}$  is: (1) finitely complete, (2) cartesian closed, and (3) locally cartesian closed. The examples of interest include (1) topological spaces, (2) compactly generated spaces, and (3) sets, respectively. It is well known that if  $\mathcal{C}$  is the category of sets or any topos, then  $G \rightarrow B$  is exponentiable in  $\text{Gpd}(\mathcal{C})/B$  if and only if it is a fibration. We will see that the sufficiency of this condition extends to the case when  $\mathcal{C}$  is merely finitely complete if each  $G_i \rightarrow B_i$  is exponentiable in  $\mathcal{C}$ , where the  $G_i$  and  $B_i$  are the objects of objects, objects of morphisms, and objects of composable pairs, for  $i = 0, 1, 2$ , respectively. When  $\mathcal{C}$  is the category of compactly generated spaces, this includes the case where each  $B_i$  is weakly Hausdorff.

We will also consider pseudo-exponentiable morphisms in the pseudo-slice categories  $\text{Gpd}(\mathcal{C})//B$ . Since the latter is the Kleisli category of a monad  $T$  on the strict slice over  $B$ , we can apply a general theorem from [1] which states that if  $TY$  is exponentiable in a 2-category  $\mathcal{K}$ , then  $Y$  is pseudo-exponentiable in the Kleisli category  $\mathcal{K}_T$ . Consequently, we will see that  $\text{Gpd}(\mathcal{C})//B$  is pseudo-cartesian closed, when  $\mathcal{C}$  is the category of compactly generated spaces and each  $B_i$  is weakly Hausdorff, and  $\text{Gpd}(\mathcal{C})$  is locally pseudo-cartesian closed when  $\mathcal{C}$  is the category of sets or any locally cartesian closed category.

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\*Joint work with Dorette Pronk.

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*A generalized Hochschild-Kostant-Rosenberg theorem*

The Hochschild-Kostant-Rosenberg theorem relates the modules of differential forms for a smooth commutative algebra to its Hochschild homology. Consequently, geometric properties of the affine scheme associated to the algebra may be interpreted in terms of its Hochschild homology. This is particularly interesting for those working in differential categories, where the modules of differential forms and related objects are salient.

In this talk we look at a generalization of the HKR theorem, which utilizes category theoretic language to extend its purview to not only commutative algebras, but associative algebras as well. In particular, from this perspective we have that for any associative algebra modality - a monad whose free  $T$ -algebras inherit an associative algebra structure - there is an associated HKR-type theorem. The upshot of this is a framework for the investigation of smoothness in a variety of contexts; here we focus on the HKR-type theorem associated to the free associative algebra monad, which demonstrably holds for the noncommutative smooth algebras of Kontsevich and Rosenberg.

Robert Paré  
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*Hypercategories*

Hypercategories are just **Cat**-indexed categories. A small hypercategory is (represented by) a small strict double category. Small weak double categories can also be viewed as hypercategories and these are essentially small, i.e. weakly equivalent to small ones. 2-categories are double categories whose vertical arrows are identities and thus may be considered as hypercategories. Even large 2-categories produce hypercategories with small homs. This was our motivation for appropriating the old name for 2-categories, “hypercategories”.

Another source of hypercategories is the indexing of **Cat** by itself. There is the standard way, via slice categories, but there are several other possibilities and sorting them out poses interesting questions. In general, 2-categories of categories with extra structure will give examples of large hypercategories.

Derivators are also hypercategories and provide a wealth of examples. We will comment on some of the implications they have for hypercategories and vice versa.

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*Quasi-toposes as elementary quotient completions*

In [11] the notion of *elementary quotient completion* of an elementary doctrine<sup>1</sup> is introduced as a generalization of the notion of the exact completion of a category with finite products and weak equalizers, presented in [4], see also [2, 10, 13] for other examples.

Such a completion is the free elementary doctrine with stable effective quotients of equivalence relations (in the sense of the doctrine). In general the base category of the completion need not be exact though the exact completion of a category with finite limits turns out to be an instance of this construction.

In this talk we focus on the special class of Lawvere’s elementary doctrines called *triposes*, introduced in [7], to build elementary toposes by means of what is now known as the tripos-to-topos construction, see [5]. We characterize those triposes whose elementary quotient completion is an arithmetic quasi-topos—*i.e.* a quasi-topos equipped with a natural number object—as base category.

To obtain the characterization, we extend some known results about exact completions such as Carboni and Vitale’s characterization of exact completions in terms of its projective objects in [4], Menni’s characterization of the exact completions which are toposes in [12] and Carboni and Rosolini’s characterization of the locally cartesian closed exact completions [3]. In particular, we show that

- an elementary doctrine  $P : \mathbb{C}^{op} \rightarrow \mathbf{InfSL}$  closed under effective quotients is the elementary quotient completion of the doctrine determined by the restriction of  $P$  to the full subcategory of  $\mathbb{C}$  on its projective objects;
- the base category of the elementary quotient completion of  $P$  turns weak universal properties of  $\mathbb{C}$  into (strong) universal properties of the base of the elementary quotient completion. Those include binary co-products, a natural number object, a parametrized list object, a subobject classifier, a cartesian closed structure, a locally cartesian structure.

We conclude by pointing out some relevant examples of arithmetic quasi-toposes arising as non-exact elementary quotient completions. Most notably they include the category of equilogical spaces of [14, 15, 1], that of assemblies over a partial combinatory algebra (see [6, 16]), and the category of total setoids, in the style of E. Bishop, over Coquand and Paulin’s Calculus of Inductive Constructions which is the theory at the base of the proof-assistant Coq.

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\*Joint work with M. E. Maietti (University of Padova) and G. Rosolini (University of Genova).

<sup>1</sup>Following Lawvere [8, 9], an elementary doctrine is a functor  $P : \mathbb{C}^{op} \rightarrow \mathbf{InfSL}$  from a category  $\mathbb{C}$  with finite products to the category of inf-semilattices such that maps of the form  $P(\langle id_A, id_A \rangle)$  have a left adjoint satisfying Beck-Chevalley condition.

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*The Wasserstein monad in categorical probability*

In existing approaches to categorical probability theory, one works with a suitable category of measurable spaces and equips it with a monad, which associates to every space  $X$  the space of probability measures on  $X$ . This applies e.g. to the Giry monad [1] or the Radon monad [2]; see also [3] for a more general setup. These monads constitute an extra piece of structure that needs to be put in by hand. Here, we introduce another such monad—the *Wasserstein monad*—and prove that it arises from a colimit construction on the underlying category  $\mathbf{CMS}$  (compact metric spaces).

Besides the utility of this colimit characterization, an advantage of the Wasserstein monad over the existing ones is as follows. Deriving quantitative bounds on approximations is a standard tool in probability theory. Therefore we also expect that working with metric spaces will allow us more easily to find categorical proofs and perhaps generalizations of probability theory’s basic results, such as the law of large numbers, or similarly the Glivenko-Cantelli theorem on the convergence of the empirical distribution.

Another advantage is that the Wasserstein monad is a monoidal monad with respect to the closed monoidal structure on  $\mathbf{CMS}$  given by adding the distances [4, Section 2]; as one would expect, the monoidal structure encodes the formation of product distributions. The Giry monad on the category of measurable spaces does not have both properties: the category of measurable spaces is not cartesian closed; and while there is another monoidal structure with respect to which the category is closed, in this one the Giry monad does not even permit a strength [5], and therefore it lacks an essential piece of structure needed for probability theory.

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\*Joint work with Tobias Fritz.



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*The orbifold construction for join restriction categories*

It is natural to describe a geometric object by an atlas in a variety of contexts. We are most familiar with manifolds; other examples include orbifolds [2] and foliations. A drawback of this approach is the difficulty of defining maps between the objects: for manifolds, for instance, one needs to first find a suitable refinement of the atlas on the domain object and then follow this by a map of atlases. A similar approach can be used to define maps between orbifolds, but here it is further complicated by the difficulty in determining whether two such maps are the same or not, since unlike the case for manifolds, two distinct maps between orbifolds may induce the same map on the underlying spaces.

Grandis [1] introduced an elegant way around the need for refinements for manifolds. His idea was to view a manifold as a type of diagram of charts with partial maps between them, indexed by a chaotic category (with precisely one arrow between any two objects). Maps between such diagrams are then given by a matrix of partial maps satisfying certain properties. This makes the category of manifolds easier to work with, and also allows us to define manifold objects for any join restriction category.

We generalize this construction to a more orbifold-like context where one needs a more complicated indexing category, with parallel arrows and non-identity automorphisms. We introduce an orbifold construction for join restriction categories. We define orbifolds using inverse categories as indexing categories, and then defining orbifold objects as linking functors from our index category into a given join restriction category  $B$ . Maps between these orbifolds will be (isomorphism classes of) a particular type of modules over the base category  $B$ . With this approach, we can define a category of orbifold objects for  $B$  which is again a join restriction category. We will show that this construction defines a monad on the category of join restriction categories, and discuss how our construction relates to the standard orbifolds.

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\*Joint work with Robin Cockett and Laura Scull.

*Functoriality and topos representations for quantales of coverable groupoids*

In [3] it has been seen that for a well behaved open localic groupoid  $G$  (a *coverable* groupoid) there is a strong form of embedding of the quantale  $\mathcal{O}$  of  $G$  into the quantale  $Q$  of an étale groupoid  $\hat{G}$  that covers  $G$  in the sense that there is a surjective morphism  $J : \hat{G} \rightarrow G$  which restricts to an isomorphism  $\hat{G}_0 \cong G_0$ . For instance, any locally compact Hausdorff groupoid in the sense of harmonic analysis [2], regarded as a localic groupoid, is of this kind, and so, in particular, Lie groupoids are coverable. Let us refer to such a pair  $(Q, \mathcal{O})$  as a *quantal pair*. The main motivation in [3] has been to provide a quantale-theoretic description of (at least some) open groupoids which, similarly to the situation with étale groupoids, does not require the multiplicativity axiom.

The purpose of this talk is to give an overview of new results that improve our understanding of coverable groupoids and quantal pairs. One set of results concerns the functoriality of the quantal pair associated to a coverable groupoid: an appropriate notion of action for quantal pairs yields an equivalence of categories  $G\text{-Loc} \cong (Q, \mathcal{O})\text{-Loc}$ , where  $(Q, \mathcal{O})$  is the quantal pair associated to  $G$ , and based on this we obtain quantale-theoretic descriptions of equivariant sheaves on groupoids, principal bundles, Hilsu–Skandalis maps and Morita equivalence in a way that extends the functoriality results for quantales of étale groupoids developed in [7, 6, 4].

Another set of results concerns global element representations of groupoid quantales. For an étale groupoid  $G$  the domain map  $d : G_1 \rightarrow G_0$  equipped with the left  $G$ -action given by multiplication is regarded as an object  $\mathbf{G}$  of the classifying topos  $BG$ , and the quantale  $Q$  of  $G$  is isomorphic to the quantale of global sections of the internal quantale of binary relations  $P(\mathbf{G} \times \mathbf{G})$ . This has been previously mentioned in [5] and a written proof appeared in the work of Simon Henry [1]. A reasonable generalization of this for general open groupoids is unlikely to exist, but for a coverable groupoid  $G$ , if we now write  $\mathbf{G}$  for the domain map  $d : G_1 \rightarrow G_0$  regarded as an internal locale in  $BG$ , the internal sup-lattice tensor product  $\mathbf{G} \otimes \mathbf{G}$  yields an internal quantale in  $BG$  whose quantale of global elements is isomorphic to the quantale of  $G$ .

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\*Joint work with Pedro Resende.

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*A synthetic theory of  $\infty$ -categories in homotopy type theory*

One of the observations that launched homotopy type theory is that the rule of identity-elimination in Martin-Löf’s identity types automatically generates the structure of an  $\infty$ -groupoid. In this way, homotopy type theory can be viewed as a “synthetic theory of  $\infty$ -groupoids.” It is natural to ask whether there is a similar *directed* type theory that describes a “synthetic theory of  $(\infty, 1)$ -categories,” but on account of a number of technical obstructions, this has long proven elusive.

In this talk, we propose foundations for a synthetic theory of  $(\infty, 1)$ -categories in homotopy type theory [1] motivated by the model of homotopy type theory in the category of Reedy fibrant simplicial spaces [2], which contains as a full subcategory the  $\infty$ -cosmos of Rezk spaces (aka complete Segal spaces) [3], a well-known model of  $(\infty, 1)$ -categories whose category theory can be developed synthetically [4]. We introduce simplices and cofibrations into homotopy type theory to probe the internal categorical structure of types, and define *Segal types*, in which binary composites exist uniquely up to homotopy, and *Rezk types*, in which the categorical isomorphisms are equivalent to the type-theoretic identities — a “local univalence” condition. In the simplicial spaces model these correspond exactly to the Segal and Rezk spaces. We then demonstrate that these simple definitions suffice to develop the synthetic theory of  $(\infty, 1)$ -categories. So far this includes functors, natural transformations, co- and contravariant type families with discrete ( $\infty$ -groupoid) fibers, a “dependent” Yoneda lemma that looks like “directed identity-elimination,” and the theory of coherent adjunctions closely resembling [5].

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\*Joint work with Michael Shulman.

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*A categorical model for a quantum circuit description language*

Quipper is a practical programming language for describing families of quantum circuits. In this talk, we formalize a small, but useful fragment of Quipper called Proto-Quipper-M. Unlike its parent Quipper, this language is type-safe and has a formal denotational and operational semantics. Proto-Quipper-M is also more general than Quipper, in that it can describe families of morphisms in any symmetric monoidal category, of which quantum circuits are but one example. We design Proto-Quipper-M from the ground up, by first giving a general categorical model of parameters and states. After finding some interesting categorical structures in the model, we then define the programming language to fit the model. We cement the connection between the language and the model by proving type safety, soundness, and adequacy properties.

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\*Joint work with Peter Selinger.

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*Stability properties for  $n$ -permutable categories*

The purpose of this talk is two-fold. A first and more concrete aim is to characterise  $n$ -permutable categories through certain stability properties of regular epimorphisms. These characterisations allow us to recover the ternary terms and the  $(n + 1)$ -ary terms describing  $n$ -permutable varieties of universal algebras.

A second and more abstract aim is to explain two proof techniques, by using the above characterisation as an opportunity to provide explicit new examples of their use:

- an *embedding theorem* for  $n$ -permutable categories which allows us to follow the varietal proof to show that an  $n$ -permutable category has certain properties;
- the theory of *unconditional exactness properties* which allows us to remove the assumption of the existence of colimits, in particular when we use the *approximate co-operations* approach to show that a regular category is  $n$ -permutable.

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\*Joint work with Pierre-Alain Jacqmin.

Robert Rosebrugh \*  
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*Symmetric lenses and universality*

A *lens* between two domains of model states is an important example of what is known in Computer Science as a *bidirectional transformation* (*BX*). A *symmetric lens* has both state synchronization data and operations to restore synchronization after state change. They have applications in model-driven engineering. An *asymmetric lens* has only one-way synchronization data and restoration operations. They define a strategy to lift a state change (update) in the target model domain back through the one-way synchronization, and for databases to solve *view update problem*.

If the domains of model states are categories, lenses are called *delta-(or d-)lenses*. Earlier we showed that spans of asymmetric d-lenses represent symmetric d-lenses. The one-way synchronization for an asymmetric d-lens is a functor. In the special case that we named (asymmetric) *c-lenses* the update lifting satisfies a universal property. This makes c-lenses what the BX community calls *least change* (and makes the functor exactly a split op-fibration). We might define spans of c-lenses to be symmetric d-lenses with the hope that they characterize those symmetric d-lenses satisfying a least change universal property. However, we will explain why we now do not expect this. Instead, motivated by applications to database interoperation, we consider *cospan*s of c-lenses. We show that such cospan>s do indeed generate symmetric d-lenses with a universal property. We also consider how to characterize those symmetric d-lenses that arise from cospan>s of c-lenses.

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\*Joint work with Michael Johnson.

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*Fundamental groupoids for orbifolds*

In equivariant topology, we often study  $G$ -spaces by regarding them as a diagram of fixed sets  $X^H = \{x \in X | hx = x\}$  for various subgroups  $H$ . This diagram is indexed over the orbit category  $O_G$ , and various topological invariants can be defined by thinking of  $G$ -spaces as functors from  $O_G$  to  $Top$ . One such invariant is tom Dieck's fundamental groupoid, a category defined by taking the fundamental groupoid functor  $\Pi : O_G \rightarrow Gpoid$  defined by  $\Pi(X^H)$ , and then combining these using a Grothendieck colimit construction  $\Pi_G(X) = \int_{O_G} \Pi(X^-)$  [3, 5].

Orbifolds are locally modelled by group actions, but can be created from charts carrying the action of many different groups, so it is not immediately clear how to create a category to play the role of  $O_G$  and organize the fixed point data. Additionally, the orbifold structure can be modeled locally by group actions and globally by groupoids, but this representation is not unique, but only defined up to Morita equivalence. So creating an analogous category for orbifolds presents some challenges.

The category defined by Haefliger [1, 2] incorporates some but not all of the information captured by the tom Dieck construction. It includes some of the internal jumps present in the Grothendieck colimit, but does not include the stratification of the fixed point sets. This category is equivalent to Thurston's deck transformations of the universal cover [5], and to the fundamental group of the classifying space  $B\mathcal{G}$  [3]. In this talk, I will expand on the relationship between this category and the tom Dieck category, and discuss ways that the various definitions could lead to an orbifold definition of a tom Dieck fundamental groupoid category.

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\*Joint work with Dorette Pronk and Courtney Thatcher.



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*Aspects of algebras of KZ-monads*

We investigate interesting categories between the Kleisli category and the Eilenberg-Moore category for a Kock-Zöberlein monad on an order-enriched category, namely, the idempotent split completion and the (weighted) limit completion of the free algebras, for an appropriate base category. The first completion was shown to be equivalent to the category of split Eilenberg-Moore algebras in [2], and we give a characterization of those split algebras which are indeed free algebras. Numerous examples of KZ-monads have algebras characterized by a colimit-construction. In [1], the authors introduced the notion of completion KZ-monad for capturing this typical behaviour. The downset monad over posets, whose algebras are posets with all suprema and maps preserving them, is a simple example of a completion KZ-monad. In contrast, the filter, the proper and the prime filter KZ-monads over topological spaces are not; however, their algebras have a certain completion behaviour. For these special three monads we give a concrete description of the idempotent split and the limit completions. For that we make use of the notion of regular cogenerator in an order-enriched sense. In any order-enriched category the existence of a such cogenerator and weighted limits assures the existence of weighted colimits. In particular, for the filter monad, the idempotent split completion of the Kleisli category has as objects the algebraic lattices whose subposet of compact elements form a frame.

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\*Joint work with Dirk Hofmann.

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*A general limit lifting theorem for 2-dimensional monad theory*

An important question for monad theory is the possibility of lifting limits along the forgetful functor of the category of algebras. The article I will present [1] deals with this subject within the theory of 2-categories. For strict morphisms of algebras, it is well known [2] that all limits lift. However, as it is usually the case, the relevant notions for its applications are the weaker pseudo and lax notions, and for these notions it is no longer the case that all limits lift. There are in the literature significant lifting results [3], [4] for the 2-categories of pseudo and lax morphisms of algebras.

Let  $T$  be a 2-monad on a 2-category  $\mathcal{K}$ , and let  $\Omega$  be a family of 2-cells of  $\mathcal{K}$ . We consider in [1] the notion of a lax morphism such that its structural 2-cell is in  $\Omega$ . There are three *distinguished* families of 2-cells which can be considered in any 2-category, and by doing so we recover the notions of lax, pseudo and strict  $T$ -algebra morphisms. We also consider a notion of weak limit which is a *weighted* version of Gray's cartesian quasi-limits [5], and define what it means for such a limit to be compatible with another family of 2-cells. These concepts allow to state and prove a limit lifting theorem which unifies and generalizes the results of [2], [3], [4] above.

Another result of [1], which simplifies the proof of our theorem by allowing to consider only conical limits, is the following: every (weighted) weak limit can be expressed as a conical weak limit, with respect to the same family of 2-cells. By considering the three distinguished families of 2-cells as above, our result yields previously known weighted-as-conical results for lax limits,  $\sigma$ -limits [6] and Street's result [7], expressing any strict limit as a cartesian quasi-limit.

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*Topological theories*

In his 2007 paper [1], Hofmann provided a notion of topological theory, involving a **Set**-monad  $\mathcal{T}$ , a (commutative and unital) quantale  $\mathcal{V}$ , and a (lax)  $\mathcal{T}$ -algebra structure on  $\mathcal{V}$  that makes the operations of the **Sup**-enriched monoid  $\mathcal{V}$  (lax)  $\mathcal{T}$ -homomorphisms; in addition, the  $\mathcal{T}$ -structure must satisfy a certain compatibility condition with suprema, which proves to be essential in applications, but does not appear to be well aligned with the other conditions of the notion. Furthermore, in its current form, the notion does not seem to lend itself to generalization, beyond the **Set**-based and quantalic context.

The aim of this talk is to frame Hofmann's notion in the context of a *lax* version of one of the cornerstones of monad theory. In its strict form, given two monads  $\mathcal{T}, \mathcal{P}$  on any category  $\mathcal{C}$ , it describes the interaction and equivalence of the following four algebraic gadgets: distributive laws of  $\mathcal{T}$  over  $\mathcal{P}$ ; extensions of  $\mathcal{T}$  to the Kleisli category of  $\mathcal{P}$ ; liftings of  $\mathcal{P}$  to the Eilenberg-Moore category of  $\mathcal{T}$ ; composite monad structures for  $\mathcal{P}\mathcal{T}$ . In the case at hand,  $\mathcal{T}$  may be any **Set**-monad, but is normally assumed to satisfy the Beck-Chevalley condition, and  $\mathcal{P}$  is the  $\mathcal{V}$ -powerset (or presheaf) monad, the Kleisli category of which is the (dual of) the category  $\mathcal{V}\text{-Rel}$  of sets and  $\mathcal{V}$ -relations.

We show how the various conditions of Hofmann's notion can be made to fit within this framework and to naturally lead to generalizations beyond its current context, as alluded to in part in [2]. Time permitting, we will also discuss examples in the generalized context.

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*Categorical-algebraic methods in group cohomology*

In the article [8], Janelidze introduced the concept of a *double central extension* in order to analyse the Hopf formula for the third integral homology of a group [2]. Later it turned out that this “double extension” viewpoint on group homology may be extended to higher degrees, and at the same time generalised to the framework of semi-abelian categories [5]. Indeed, categorical Galois theory gives rise to the concept of an *n-fold central extension* ( $n \geq 1$ ), which is such that the higher Hopf formulae of [2, 3], suitably reinterpreted in terms of these higher central extensions, give an explicit description of the derived functors of any reflector from a semi-abelian variety to one of its subvarieties. In the particular case of the abelianisation reflection from the category of groups to the category of abelian groups, the Hopf formulae for integral group homology are thus regained.

Central extensions do however also appear in group cohomology, in the interpretation of the second cohomology group with coefficients in a trivial  $\mathbb{Z}$ -module  $A$ , which is one of the derived functors of the functor  $\text{Hom}(-, A)$ . This result extends to semi-abelian categories [7] and to non-trivial coefficients (via the concept of a torsor [1]). On the other hand, in the abelian case there is Yoneda’s classical interpretation of these derived functors via classes of exact sequences of a certain fixed length [10]. In Barr-exact categories, the higher-dimensional torsors of [4] play essentially the same role.

The aim of this talk is to explain how, in a semi-abelian context, these two developments are related. Through an equivalence between higher torsors (with trivial coefficients) and higher central extensions we obtain a duality, in a certain sense, between homology and cohomology [9, 6]. Even in the case of groups this viewpoint is new, but it is automatically valid as well for other non-abelian algebraic structures such as Lie algebras, crossed modules, associative algebras, and so on.

In its most general version, the theory depends on some non-trivial recent developments in categorical algebra. Part of the talk focuses on these categorical-algebraic aspects: how questions in homological algebra naturally lead to categorical conditions and results. The need for further development of categorical algebra becomes particularly apparent in the case of cohomology with non-trivial coefficients. This case is much more complicated, because here the techniques of categorical Galois theory are no longer available.

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Université Libre de Bruxelles

*Hopf categories as Hopf monads in enriched matrices*

Hopf categories, as many-object generalizations of Hopf algebras, were introduced in [1]. In this talk, we present a framework for viewing them as Hopf monads in the bicategory of  $\mathcal{V}$ -matrices [2]. We also explore a double categorical perspective for such structures, involving a notion of a Hopf monad in fibrant double categories a.k.a. proarrow equipments.

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\*Joint work with Mitchell Buckley, Timmy Fieremans and Joost Vercruysse.

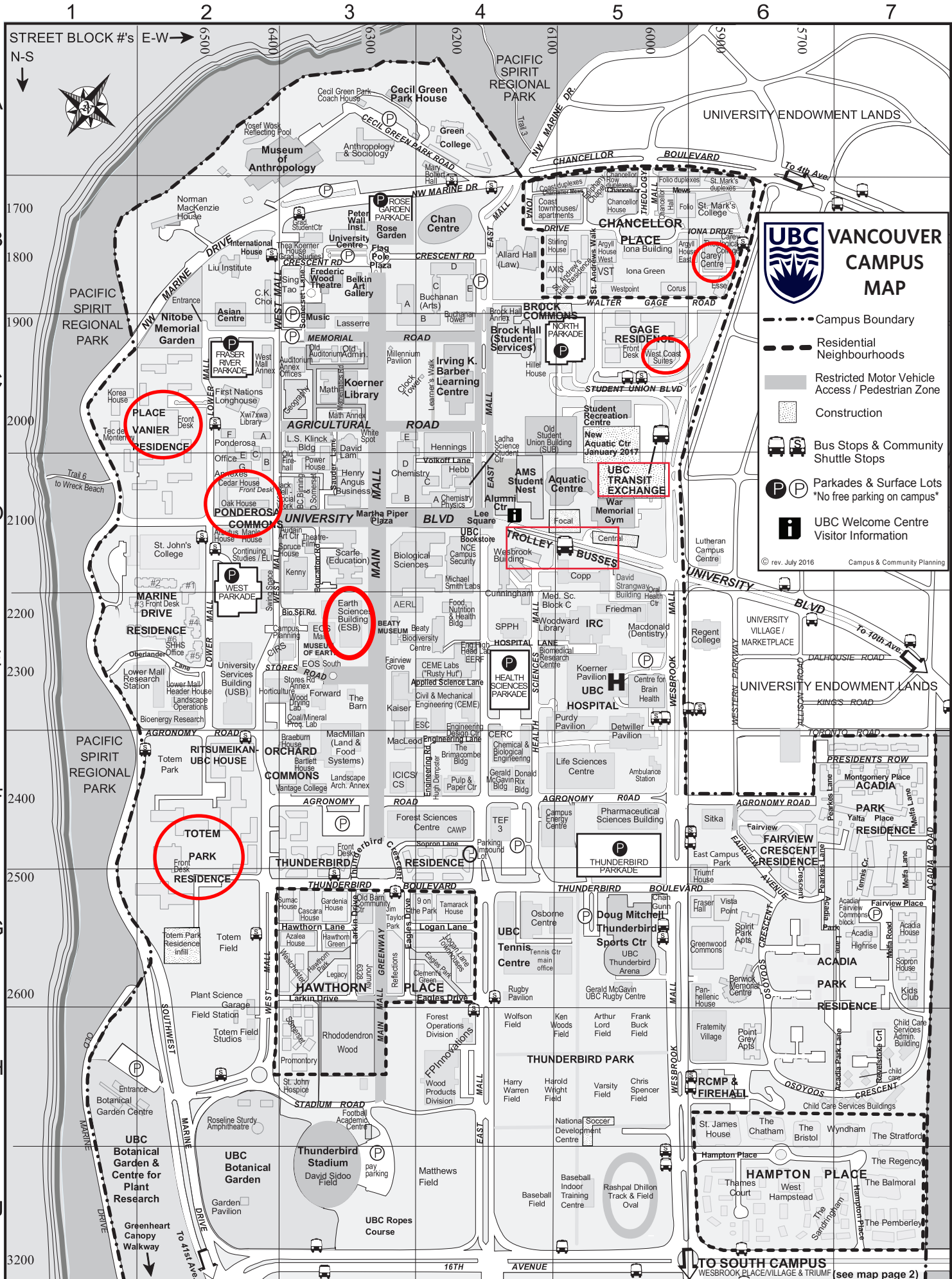
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Graduate School of Mathematical Sciences,  
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*Graphical calculus in symmetric monoidal ( $\infty$ -) categories with duals*

Graphical calculi are a sort of techniques to compute morphisms in monoidal categories, and a really general and geometric formalization was given by Joyal and Street [3]. In this talk, we focus on that in symmetric monoidal categories with duals. They are examples of pivotal categories, and it is vaguely believed by researchers in quantum representation theory that pivotal categories are described by a calculus of planar tangles (see [2] for example). We give a purely geometric description for this calculus and, using the Cobordism Hypothesis [1] (proved by Lurie [4]), show every symmetric monoidal category admits a graphical calculus in a coherent way so that we can extend it to the  $\infty$ -contexts. This also gives an extension of [5].

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**UBC VANCOUVER CAMPUS MAP**

- Campus Boundary
- Residential Neighbourhoods
- Restricted Motor Vehicle Access / Pedestrian Zone
- Construction
- Bus Stops & Community Shuttle Stops
- Parkades & Surface Lots "No free parking on campus"
- UBC Welcome Centre Visitor Information

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1 2 3 4 5 6 7

STREET BLOCK #'s E-W →

N-S ↓

A

1700

B 1800

C 1900

D 2000

E 2100

F 2200

G 2300

H 2400

I 2500

J 2600

2700

2800

2900

3000

3100

3200

3300

3400

3500

3600

3700

3800

3900

4000

4100

4200

4300

4400

1 2 3 4 5 6 7

TO SOUTH CAMPUS  
WESBROOK PLACE VILLAGE & TRIUMF (see map page 2)



# Map Directory

Site or Building Name & Address	Grid
Abdul Ladhia Science Student Ctr, 2055 East Mall.....	D4
Acadia/Fairview Commonsblock & Front Desk, 2707 Tennis Cres.....	G7
Acadia House, 2700-2720 Acadia Rd.....	G7
Acadia Park Residence (Student Family Housing).....	F/H-67
Acadia Park Highrise, 2725 Meifa Rd.....	G7
Allard Hall [Faculty of Law], 1822 East Mall.....	B4
<b>Alumni Centre (Robert H. Lee), 6163 University Blvd.....</b>	<b>D4</b>
<b>AMS Student Nest (new student union building), 6133 University Blvd.....</b>	<b>D4</b>
Anthropology & Sociology (ANSOC) Bldg, 6303 NW Marine Dr.....	A3
<b>Aquatic Centre (New - opening Jan. 2017), 6080 Student Union Blvd.....</b>	<b>C5</b>
<b>Aquatic Centre (Old), 6121 University Blvd.....</b>	<b>D5</b>
Aquatic Ecosystems Research Lab (AERL), 2202 Main Mall.....	E3
Asian Centre, 1871 West Mall.....	B2
Audain Art Centre (in Ponderosa Commons), 6398 University Blvd.....	D3
Auditorium Annex Offices A & B, 1924 West Mall.....	C3
<b>Barn ("Ow") child care, 2323 Main Mall.....</b>	<b>E3</b>
Baseball Indoor Training Centre, 3085 West Mall.....	J5
B.C. Binning Studios, 6373 University Blvd.....	D3
<b>Beaty Biodiversity Centre &amp; Museum, 2212 Main Mall.....</b>	<b>E3/4</b>
<b>Belkin (Morris &amp; Helen) Art Gallery, 1825 Main Mall.....</b>	<b>B3</b>
Berwick Memorial Centre, 2765 Osoyoos Cres.....	G6
Bioenergy Research & Demonstration Facility (BRDF), 2337 Lower Mall.....	E2
Biological Sciences Bldg, 6270 University Blvd.....	D3
Biomedical Research Ctr, 2222 Health Sciences Mall.....	E4
Bollert (Mary) Hall, 6253 NW Marine Dr.....	A4
Bookstore, 6200 University Blvd.....	D4
<b>Botanical Garden/Gatehouse, 6804 SW Marine Dr.....</b>	<b>H1</b>
Botan. Gard. Greenhouses/ Workshops, 3929 Wesbrook Mall.....	South Campus
Brimacombe Building, 2355 East Mall.....	F4
Brook Commons - Tallwood House (construction), 6088 Walter Gage Rd.....	B4
<b>BROCK HALL: Student Services &amp; Welcome Centre, 1874 East Mall.....</b>	<b>C4</b>
Brook Hall Annex, 1874 East Mall.....	C4
Buchanan Building (Blocks A, B, C, D, & E) [Arts], 1866 Main Mall.....	B3/4
Buchanan Tower, 1873 East Mall.....	C4
Building Ops Nursery/Greenhouses, 6029 Nurseries Rd.....	South Campus
<b>C.K. Choi Building for the Institute of Asian Research, 1855 West Mall.....</b>	<b>B2</b>
Campus & Community Planning, 2210 West Mall.....	E3
Campus Energy Centre, 6130 Agronomy Rd.....	F5
Campus Security, 2133 East Mall.....	D4
Carey Centre / Theological College, 5920 Iona Drive/1815 Wesbrook Mall.....	B6
Cecil Green Park Coach House, 6323 Cecil Green Park Rd.....	A3
Cecil Green Park House, 6251 Cecil Green Park Rd.....	A3
Centre for Brain Health (Djavad Mowafaghian), 2215 Wesbrook Mall.....	E5
Centre for Comparative Medicine (CCM), 4145 Wesbrook Mall.....	South Campus
<b>Chan Centre for the Performing Arts, 6265 Crescent Rd.....</b>	<b>B4</b>
Chan Gunn Pavilion (new sports med. construction), 2553 Wesbrook Mall.....	G5
Chemical & Biological Engineering Bldg, 2360 East Mall.....	F4
Chemistry A Block - Chemistry Physics Building, 6221 University Blvd.....	D4
Chemistry B.C.D & E Blocks, 2036 Main Mall.....	D3
Child Care Services Administration Bldg, 2881 Acadia Rd.....	H7
Child Care Services Bldgs, Osoyoos Crescent and Revelstoke Crt.....	H7
CIRS (Centre for Interactive Research on Sustainability), 2260 West Mall.....	E3
Civil & Mechanical Engineering Bldg (CEME), 6250 Applied Science Lane.....	E4
Civil & Mechanical Eng. Labs ("Rusty Hut"), 2275 East Mall.....	E4
Coal & Mineral Processing Lab, 2332 West Mall.....	E3
Continuing Studies Bldg [English Language Institute], 2121 West Mall.....	D2
Copp (D.H.) Building, 2146 Health Sciences Mall.....	D5
Cunningham (George) Building, 2146 East Mall.....	E4
David Lam Learning Centre, 6326 Agricultural Rd.....	C3
David Lam Management Research Ctr, 2033 Main Mall.....	C3
David Strangway Building, 5950 University Blvd.....	D5
Donald Rix Building, 2389 Health Sciences Mall.....	F4
<b>Doug Mitchell Thunderbird Sports Centre, 6066 Thunderbird Blvd.....</b>	<b>G5</b>
<b>Dorothy Somerset Studios, 6361 University Blvd.....</b>	<b>D3</b>
Earth Sciences Building (ESB), 2207 Main Mall.....	E3
Earth & Ocean Sciences (EOS) - Main and South, 6339 Stores Rd.....	E3
Earthquake Engineering Research Facility (EERF), 2235 East Mall.....	E4
Engineering High Head Room Lab, 2225 East Mall.....	E4
Engineering Student Centre, 2335 Engineering Road.....	E4
English Language Institute (E.L.I.) — see <i>Continuing Studies Building</i>	
Environmental Services Facility, 6025 Nurseries Rd.....	South Campus
Fairview Crescent Residence, 2600-2804 Fairview Cres.....	F6
Fire Hall, 2992 Wesbrook Mall.....	H6
First Nations Longhouse, 1985 West Mall.....	C2
Flag Pole Plaza (Main Mall & Crescent Rd).....	B3
Food, Nutrition and Health Bldg, 2205 East Mall.....	E4
Forest Sciences Centre [Faculty of Forestry], 2424 Main Mall.....	F4
Forward (Frank) Building, 6350 Stores Rd.....	E3
FPIInnovations, 2601 & 2665 East Mall.....	H4
Fraser Hall, 2550 Wesbrook Mall.....	G6
Fraternity Village, 2880 Wesbrook Mall.....	H6
<b>Frederic Wood Theatre, 6354 Crescent Rd.....</b>	<b>B3</b>
Friedman Bldg, 2177 Wesbrook Mall.....	E5
Gage (Walter H.) Residence, 5959 Student Union Blvd.....	C5
Geography Building, 1984 West Mall.....	C3
Gerald McGavin Building, 2386 East Mall.....	F4
Gerald McGavin UBC Rugby Centre, 2765 Wesbrook Mall.....	G5
Graduate Student Centre — see <i>Thea Koerner House</i>	
Green College, 6201 Cecil Green Park Rd.....	A4
Hebb Building, 2045 East Mall.....	D4
Hennings Building, 6224 Agricultural Rd.....	C4
Henry Angus Building [Sauder School of Business], 2053 Main Mall.....	D3
Hillel House, 6145 Student Union Blvd.....	E4
Horticulture Building/Greenhouse, 6394 Stores Rd.....	E2/3

Site or Building Name & Address	Grid
High Dempster Pavilion, 6245 Agronomy Rd.....	F4
ICICS/CS (Institute for Computing, Information & Cognitive Systems/Computer Science), 2366 Main Mall.....	F4
Instructional Resources Centre (IRC), 2194 Health Sciences Mall.....	E5
International House, 1783 West Mall.....	B2
In-Vessel Composting Facility, 6035 Nurseries Road.....	South Campus
<b>Irvig K. Barber Learning Centre, 1961 East Mall.....</b>	<b>C4</b>
Jack Bell Building for the School of Social Work, 2080 West Mall.....	D3
Kaiser (Fred) Building [Faculty of Applied Science], 2332 Main Mall.....	E3
Kenny (Douglas T) Building, [Psychology] 2136 West Mall.....	D3
Kids Club, 2855 Acadia Rd.....	G7
Klinck (Leonard S.) Bldg, 6356 Agricultural Rd.....	C3
Koerner (Walter C.) Library, 1958 Main Mall.....	C3
Landscape Architecture Annex, 2371 Main Mall.....	F3
Lasserre (Frederic) Building, 6333 Memorial Rd.....	C3
Library Preservation Archives (PARC), 6049 Nurseries Rd.....	South Campus
Life Sciences Centre, 2350 Health Sciences Mall.....	F5
Liu Institute for Global Issues, 6476 NW Marine Dr.....	B2
Lower Mall Research Station, 2259 Lower Mall.....	E2
Macdonald (J.B.) Building [Dentistry], 2199 Wesbrook Mall.....	E5
MacLeod (Hector) Building, 2356 Main Mall.....	F3
MacMillan (H.R.) Bldg [Faculty of Land & Food Systems], 2357 Main Mall.....	F3
Marine Drive Residence (Front Desk in Bldg #3), 2205 Lower Mall.....	E2
Material Recovery Facility, 6055 Nurseries Rd.....	South Campus
Mathematics Annex, 1986 Mathematics Rd.....	C3
Mathematics Building, 1984 Mathematics Rd.....	C3
Medical Sciences Bldg C, 2176 Health Sc. Mall.....	E4
Michael Smith Laboratories, 2185 East Mall.....	D4
<b>Museum of Anthropology (MOA), 6393 NW Marine Dr.....</b>	<b>A2/3</b>
Music Building, 6361 Memorial Rd.....	B/C3
National Soccer Development Centre, 3065 Wesbrook Mall.....	H5
Networks of Ctrs of Excellence (NCE), 2125 East Mall.....	D4
Nitobe Memorial Garden, 1895 Lower Mall.....	B/C2
Nobel Biocare Oral Health Centre, 2151 Wesbrook Mall.....	E5
Norman MacKenzie House, 6565 NW Marine Dr.....	B2
NRC Institute for Fuel Cell Innovation, 4250 Wesbrook Mall.....	South Campus
Old Administration Building, 6328 Memorial Rd.....	C3
Old Auditorium, 6344 Memorial Rd.....	C3
Old Ban Community Centre, 6308 Thunderbird Blvd.....	D3
Old Firehall, 2038 West Mall.....	G3
Orchard Commons, 6363 Agronomy Rd.....	J3
Osborne (Robert F.) Centre/Gym, 6108 Thunderbird Blvd.....	G4
<b>Pacific Museum of Earth (in EOS-Main), 6339 Stores Rd.....</b>	<b>E3</b>
Panhellene House, 2770 Wesbrook Mall.....	G6
Peter Wall Institute for Advanced Studies (PWIAS), 6331 Crescent Rd.....	B3
Pharmaceutical Sciences Building, 2405 Wesbrook Mall.....	F5
Place Vanier Residence, 1935 Lower Mall.....	C/D2
Plant Science Field Station & Garage, 2613 West Mall.....	H2
Point Grey Apartments, 2875 Osoyoos Cres.....	H6
Police (RCMP) & Fire Department, 2990/2992 Wesbrook Mall.....	H6
PONDEROSA COMMONS, University Blvd & West Mall.....	D2/3
Arbutus & Maple Houses, 6488 University Blvd.....	D2
Cedar House (Ponderosa Commons Front Desk), 2075 West Mall.....	D2
Oak House, 6445 University Blvd.....	D2
Spruce House, 2118 West Mall.....	D3

Site or Building Name & Address	Grid
Ponderosa Office Annexes: A, B, & C, 2011-2029 West Mall.....	C/D2
Ponderosa Office Annexes: E, F & G, 2008-2044 Lower Mall.....	C/D2
Power House, 2040 West Mall.....	D3
Pulp and Paper Centre, 2385 East Mall.....	F4
Ritsumeikan-UBC House, 6460 Agronomy Rd.....	F2
Rose Garden.....	B3
Rugby Pavilion, 2584 East Mall.....	G4
Scarfe (Neville) Building [Education], 2125 Main Mall.....	D3
School of Population & Public Health (SPPH), 2206 East Mall.....	E4
SERC (Staging Environmental Research Ctr), 6045 Nurseries Rd.....	South Campus
Sing Tao Building, 6388 Crescent Rd.....	B3
Sopron House, 2730 Acadia Rd.....	G7
South Campus Warehouse, 6116 Nurseries Rd.....	South Campus
Spirit Park Apartments, 2705-2725 Osoyoos Cres.....	G8
St. Andrew's Hall/Residence, 6040 Iona Dr.....	B5
St. John Hospice, 6389 Stadium Road.....	H3
St. John's College, 2111 Lower Mall.....	D2
St. Mark's College, 5935 Iona Dr.....	B6
Stores Road Annex, 6368 Stores Rd.....	E3
Student Family Housing (Acadia Park Residence).....	F/H-67
Student Recreation Centre, 6000 Student Union Blvd.....	C5
<b>Student Union Bldg (old) (Old SUB), 6138 Student Union Blvd.....</b>	<b>C4</b>
TEF3 (Technology Enterprise Facility 3), 6190 Agronomy Rd.....	F4
Thea Koerner House [Faculty of Graduate Studies], 6371 Crescent Rd.....	B3
Theatre-Film Production Bldg, 6358 University Blvd.....	D3
Thunderbird Residence, 6335 Thunderbird Cres.....	F3/4
Thunderbird Arena (in Doug Mitchell Centre), 2555 Wesbrook Mall.....	G5
Thunderbird Stadium, 6288 Stadium Rd.....	J3
Totem Field Studios, 2613 West Mall.....	H2
Totem Park Residence, 2925 West Mall.....	F/G2
TRIUMF, 4004 Wesbrook Mall.....	South Campus
Triumph House (TRIUMF Visitors' Residence), 5835 Thunderbird Blvd.....	G6
<b>UBC Bookstore, 6200 University Blvd.....</b>	<b>D4</b>
<b>UBC Farm, 3461 Ross Drive.....</b>	<b>South Campus</b>
UBC Football Academic Centre, 6298 Stadium Rd.....	H3
UBC Hospital, 2211 Wesbrook Mall.....	E5
UBC Parking Impound Lot, 2451 East Mall.....	F4
UBC Tennis Centre, 6160 Thunderbird Blvd.....	G4
University Centre (Leon & Thea Koerner), 6331 Crescent Rd.....	B3
University Services Building (USB), 2329 West Mall.....	E2
Vancouver School of Theology (VST), 6015 Walter Gage Rd.....	B5
Vantage College (in Orchard Commons, Fall 2016), 6363 Agronomy Rd.....	F3
<b>War Memorial Gymnasium, 6081 University Blvd.....</b>	<b>D5</b>
Wayne & William White Engineering Design Ctr, 2345 East Mall.....	E4
Wesbrook Bldg, 6174 University Blvd.....	D4
Wesbrook Community Centre, 5998 Berton Ave.....	South Campus
Wesbrook Village commercial centre.....	South Campus
West Mall Annex, 1933 West Mall.....	C2
West Mall Swing Space Bldg, 2175 West Mall.....	D2
Wood Drying Laboratory, 2324 West Mall.....	E3
Woodward IRC, 2194 Health Sciences Mall.....	E4/5
Woodward Library, 2198 Health Sciences Mall.....	E4/5

## SOUTH CAMPUS MAP

Local Traffic Only

### Map Information

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