PROBLEMS: p-ADIC HEIGHTS ON ELLIPTIC CURVES

JENNIFER BALAKRISHNAN

(1) Let E be the elliptic curve $y^2 = x^3 - 4x + 4$ over $\mathbb{Q}$.
  (a) Compute the Mordell-Weil rank of $E(\mathbb{Q})$.
  (b) Find the smallest good, ordinary prime p for E.
  (c) Using p from part (b) above, compute the cyclotomic p-adic height h of $P = (2, -2)$ and of $Q = (0, -2)$. Are $h(P)$ and $h(Q)$ related?

(2) Let E be the elliptic curve $y^2 + y = x^3 + x^2 - 2x$ (LMFDB label 389.a1).
  (a) What is the rank of $E(\mathbb{Q})$? Compute generators for the Mordell-Weil group.
  (b) Compute the p-adic regulator for good, ordinary primes $p < 100$. What do you notice about its valuation?
  (c) What is the valuation of the 16231-adic regulator?
  (d) Challenge (for those familiar with Sage development): check out the OMS code at `http://trac.sagemath.org/ticket/812` and see if you can compute Ш[16231].

(3) Find an example of an elliptic curve E, quadratic imaginary field K, prime p, and non-torsion point P such that the anticyclotomic p-adic height of P is 0.

Date: June 16, 2016.