Two dimensional phase unwrapping for distributed acoustic sensors

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1 Preparatory material

Calculus and mathematical modelling, numerical methods, signal processing. Some physics background would be useful.

2 Distributed acoustic sensing

Distributed acoustic sensing (DAS) is a relatively new technology used for measuring strain and vibration along a long cable installed on large linear infrastructures such as railways, roadways, pipelines, underground boreholes, and so forth. The sensor consists of a single-mode fibre optic cable installed along the length of structure under surveillance, and connected to a laser and photodetector. A pulse of light is sent down the fibre, and the reflections scattered back to the detector are analyzed to measure strain on the fibre. Depending on the length of the fibre, the laser may be fired many thousands of times per second, and the Rayleigh backscatter sampled to produce a measurement of the strain at numerous points on the fibre. Knowledge of the refractive index of the fibre allows one to use two-way travel-time to treat the fibre as a series of discrete sensors up to 40 km long, where each position acts somewhat like a two-beam interferometer.

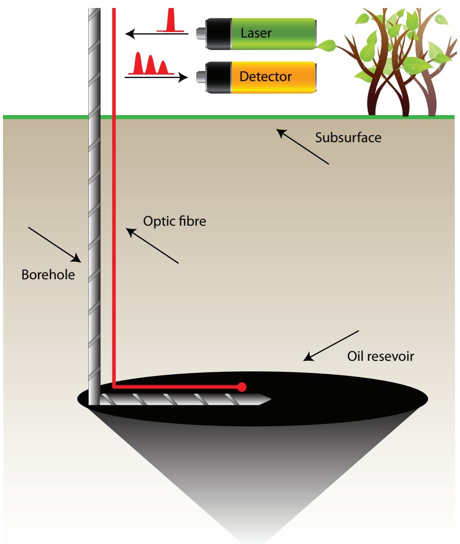

Figure 1: DAS with fibre in borehole.

In terms of typical data volumes collected, a 5 km fibre with a spatial sampling interval of 0.67 meters would translate to roughly 7462 fibre-positions being sampled 20,000 times per second. The signals can be treated as the output of a long string of individual sensors distributed along the cable, with a high frequency sampling rate.

A significant challenge in improving the sensitivity and accuracy of the DAS device is calculating the optical phase of the light signal travelling in the fibre from the measured intensities on the photodetector. Noise and phase wrapping are the main issues that make this difficult.

3 Problem Description

The details of the phase problem arises as follows. In distributed acoustic sensors, a laser pulse is sent along a fibre-optic cable, and the intensity of the backscattered light is measured as a function of time. This backscattered light is related to the elastic strain that the fibre experiences through measurement of the *wrapped* optical phase inside the fibre.

Assume the DAS response can be theoretically modelled as

$$\phi(x,t) = \phi_0(x) + s(x,t) + \eta$$

where $\phi(x,t)$ is the DAS response as a function of space x and time t, $\phi_0(x)$ is random in space, but constant in time, $s(x,t)$ is a continuous function, and η is a random variable. Let $\psi(x,t) = W[\phi](x,t)$ be the *measured* wrapped phase, where $W[\cdot]$ is the wrapping operator $W[\theta] = \theta + 2k\pi$, $k \in \mathbb{Z}$, such that $W[\theta]\ in[-pi, pi)$. The goal is then to recover $s(x,t)$ from $\psi(x,t)$.

The Itoh condition for phase unwrapping in one dimension is that

$$\Delta\phi = W[\Delta\psi]$$

where Δ is the discrete difference operator. Thus, ϕ can be recovered by writing

$$\phi_{i+1} = \phi_i + W[\Delta\psi].$$

Figure 2 shows a sine wave undergoing wrapping and then unwrapping, with and without additive noise. When there is no noise, and the phase is sampled with a sufficiently high sample rate, the phase unwrapping is exact. When the phase changes too much, too quickly, either due to noise or due to too low of a sample rate, the phase unwrapping fails and introduces spurious *jumps* in the data.

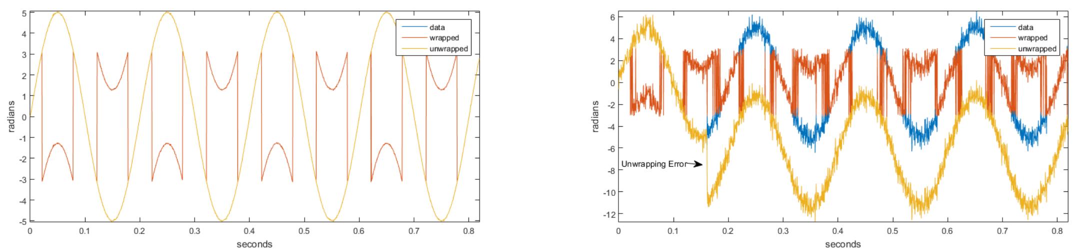

Figure 2: Wrapped and unwrapped phase with and without additive noise.

As distributed acoustic sensors create dense virtual sensors spaced $< 1m$ apart, it is appealing to seek some form of two-dimensional phase unwrapping algorithm that that takes neighbouring sensors into account in order to increase robustness and remove phase unwrapping errors such as those seen in Figure 2. The presence of the random offset of each sensor, represented by the term $\phi_0(x)$, makes this difficult however.

A relatively simple first step in recovering $s(x,t)$ is to estimate the mean of the wrapped phase for each location, subtract that mean-estimate, and then re-wrap the data. This produces

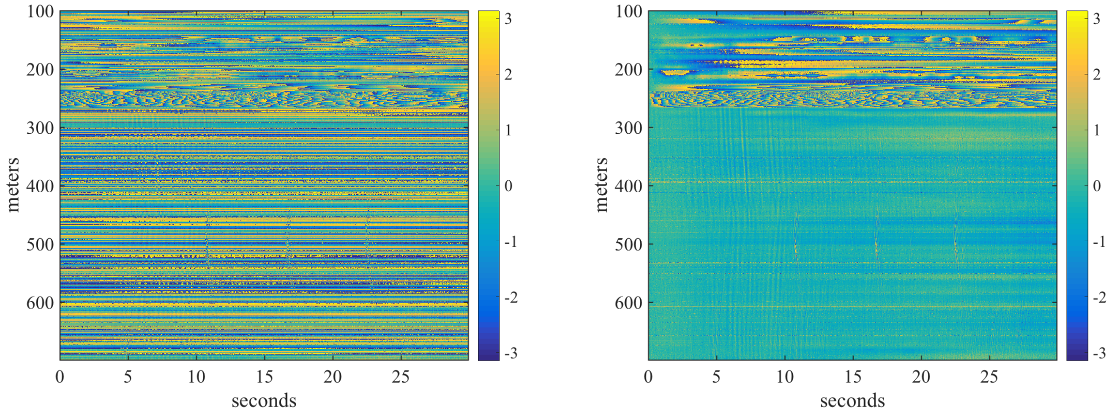

Figure 3: Wrapped 2D phase before and after mean subtraction and re-wrapping.

an estimate of $W[\phi(x,t) - \phi_0(x)] = W[s(x,t) + \eta]$ where small signals less than $\pm\pi$ radians are unwrapped. Figure 3 shows an example of a 2D wavefield recorded before and after subtracting an estimate of the mean of each location computed over 1001 time samples and re-wrapping.

Figure 4 shows a zoomed in region of the data in Figure 3 around a portion of the wavefield that is experiencing wrapping.

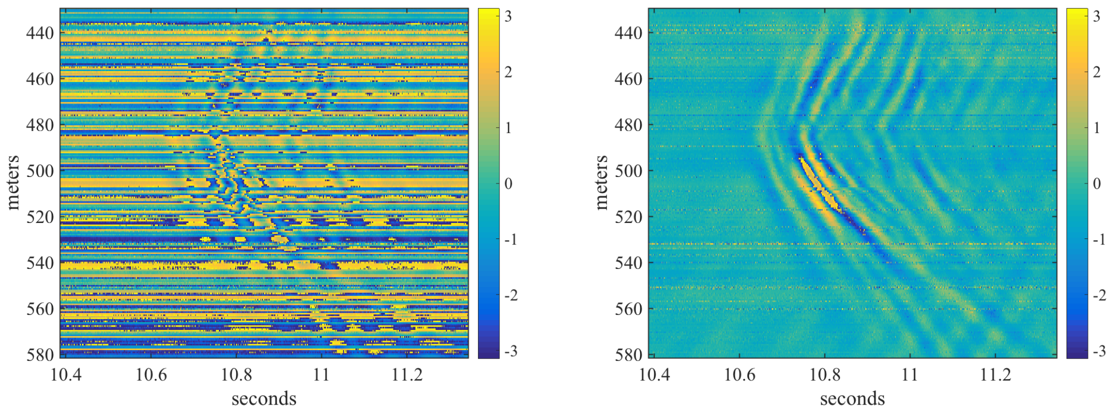

Figure 4: Wrapped 2D phase before and after mean subtraction and re-wrapping.

The general goal of the project is to develop a robust two-dimensional phase unwrapping algorithm, specific to distributed acoustic sensor data. It should be noted that two-dimensional phase unwrapping algorithms already exist for imaging of phase-wrapped surfaces. That is where both dimensions are space in a physical coordinate system. These techniques may prove useful, but the extension to space-time sampling of wavefields could provide further constraint.