Group Field Theory

Răzvan Gurău

UBC, 2009

Space-time and Scales

Space-time and Scales

Space-time is one of the most fundamental notions is physics. In many theories (e.g. quantum mechanics) it appears as a fixed background. The distances and lapses of time are measured with respect to this fixed background.

Space-time and Scales

Space-time is one of the most fundamental notions is physics. In many theories (e.g. quantum mechanics) it appears as a fixed background. The distances and lapses of time are measured with respect to this fixed background.

In this setting, one has well defined distance (energy) scales. Physics at large distance scales (low energy) is determined by the physics at shorter scales (high energy).

Space-time and Scales

Space-time is one of the most fundamental notions is physics. In many theories (e.g. quantum mechanics) it appears as a fixed background. The distances and lapses of time are measured with respect to this fixed background.

In this setting, one has well defined distance (energy) scales. Physics at large distance scales (low energy) is determined by the physics at shorter scales (high energy). Quantum field theory, through the renormalization group flow, gives the precise relationship between the fundamental (UV) theories and effective (IR) theories.

Space-time and Scales

Space-time is one of the most fundamental notions is physics. In many theories (e.g. quantum mechanics) it appears as a fixed background. The distances and lapses of time are measured with respect to this fixed background.

In this setting, one has well defined distance (energy) scales. Physics at large distance scales (low energy) is determined by the physics at shorter scales (high energy). Quantum field theory, through the renormalization group flow, gives the precise relationship between the fundamental (UV) theories and effective (IR) theories.

General relativity promotes the metric to a dynamical variable. The scales themselves become dynamical! One cannot distingush between fundamental and effective phenomena!

Space-time and Scales

Space-time is one of the most fundamental notions is physics. In many theories (e.g. quantum mechanics) it appears as a fixed background. The distances and lapses of time are measured with respect to this fixed background.

In this setting, one has well defined distance (energy) scales. Physics at large distance scales (low energy) is determined by the physics at shorter scales (high energy). Quantum field theory, through the renormalization group flow, gives the precise relationship between the fundamental (UV) theories and effective (IR) theories.

General relativity promotes the metric to a dynamical variable. The scales themselves become dynamical! One cannot distingush between fundamental and effective phenomena! Naive solution: define the scales with some background metric, and study the fluctuations around this metric.

Space-time and Scales

Space-time is one of the most fundamental notions is physics. In many theories (e.g. quantum mechanics) it appears as a fixed background. The distances and lapses of time are measured with respect to this fixed background.

In this setting, one has well defined distance (energy) scales. Physics at large distance scales (low energy) is determined by the physics at shorter scales (high energy). Quantum field theory, through the renormalization group flow, gives the precise relationship between the fundamental (UV) theories and effective (IR) theories.

General relativity promotes the metric to a dynamical variable. The scales themselves become dynamical! One cannot distingush between fundamental and effective phenomena! Naive solution: define the scales with some background metric, and study the fluctuations around this metric. It does not work! Gravity coupled to matter is non renormalizable around a flat vacuum!

Space-time and Scales

Space-time is one of the most fundamental notions is physics. In many theories (e.g. quantum mechanics) it appears as a fixed background. The distances and lapses of time are measured with respect to this fixed background.

In this setting, one has well defined distance (energy) scales. Physics at large distance scales (low energy) is determined by the physics at shorter scales (high energy). Quantum field theory, through the renormalization group flow, gives the precise relationship between the fundamental (UV) theories and effective (IR) theories.

General relativity promotes the metric to a dynamical variable. The scales themselves become dynamical! One cannot distingush between fundamental and effective phenomena! Naive solution: define the scales with some background metric, and study the fluctuations around this metric. It does not work! Gravity coupled to matter is non renormalizable around a flat vacuum!

Thus two major questions arise:

Space-time and Scales

Space-time is one of the most fundamental notions is physics. In many theories (e.g. quantum mechanics) it appears as a fixed background. The distances and lapses of time are measured with respect to this fixed background.

In this setting, one has well defined distance (energy) scales. Physics at large distance scales (low energy) is determined by the physics at shorter scales (high energy). Quantum field theory, through the renormalization group flow, gives the precise relationship between the fundamental (UV) theories and effective (IR) theories.

General relativity promotes the metric to a dynamical variable. The scales themselves become dynamical! One cannot distingush between fundamental and effective phenomena! Naive solution: define the scales with some background metric, and study the fluctuations around this metric. It does not work! Gravity coupled to matter is non renormalizable around a flat vacuum!

Thus two major questions arise:

- How to define background independent scales ?

Space-time and Scales

Space-time is one of the most fundamental notions is physics. In many theories (e.g. quantum mechanics) it appears as a fixed background. The distances and lapses of time are measured with respect to this fixed background.

In this setting, one has well defined distance (energy) scales. Physics at large distance scales (low energy) is determined by the physics at shorter scales (high energy). Quantum field theory, through the renormalization group flow, gives the precise relationship between the fundamental (UV) theories and effective (IR) theories.

General relativity promotes the metric to a dynamical variable. The scales themselves become dynamical! One cannot distingush between fundamental and effective phenomena! Naive solution: define the scales with some background metric, and study the fluctuations around this metric. It does not work! Gravity coupled to matter is non renormalizable around a flat vacuum!

Thus two major questions arise:

- How to define background independent scales ?
- How to obtain the usual space-time as an effective, IR phenomenon?

A Non Exhaustive List of Approaches

A Non Exhaustive List of Approaches

String Theory: Postulate new matter fields. Replacing point particles by extended objects (strings) one can in principle regularize the divergencies.

A Non Exhaustive List of Approaches

String Theory: Postulate new matter fields. Replacing point particles by extended objects (strings) one can in principle regularize the divergencies.

Canonical approach: Formulate a diffeomorphic covariant quantization scheme. The evolution (Wheeler-deWitt) equations become constraints which one needs to implement in the quantum theory.

A Non Exhaustive List of Approaches

String Theory: Postulate new matter fields. Replacing point particles by extended objects (strings) one can in principle regularize the divergencies.

Canonical approach: Formulate a diffeomorphic covariant quantization scheme. The evolution (Wheeler-deWitt) equations become constraints which one needs to implement in the quantum theory.

Spectral triple: Start with the algebra of observables. Postulate a spectral action and identify the space time only later.

A Non Exhaustive List of Approaches

String Theory: Postulate new matter fields. Replacing point particles by extended objects (strings) one can in principle regularize the divergencies.

Canonical approach: Formulate a diffeomorphic covariant quantization scheme. The evolution (Wheeler-deWitt) equations become constraints which one needs to implement in the quantum theory.

Spectral triple: Start with the algebra of observables. Postulate a spectral action and identify the space time only later.

Non-trivial fixed point: Bypass the puzzle of non renormalizability by postulating the existence of a non-trivial ultraviolet fixed point. Flow away from this fixed point in IR, and recover the usual gravity plus matter theory.

A Non Exhaustive List of Approaches

String Theory: Postulate new matter fields. Replacing point particles by extended objects (strings) one can in principle regularize the divergencies.

Canonical approach: Formulate a diffeomorphic covariant quantization scheme. The evolution (Wheeler-deWitt) equations become constraints which one needs to implement in the quantum theory.

Spectral triple: Start with the algebra of observables. Postulate a spectral action and identify the space time only later.

Non-trivial fixed point: Bypass the puzzle of non renormalizability by postulating the existence of a non-trivial ultraviolet fixed point. Flow away from this fixed point in IR, and recover the usual gravity plus matter theory.

Discrete approaches: Build space-time out of discrete blocks, "space time quanta", and recover the usual gravity in the continuum limit.

Scales and Blocking in Discrete Approaches

Scales and Blocking in Discrete Approaches

Historically Wilson's renormalization group was first understood as a blocking procedure averaging degrees of freedom on "small blocks" to obtain effective behavior over "large blocks".

Scales and Blocking in Discrete Approaches

Historically Wilson's renormalization group was first understood as a blocking procedure averaging degrees of freedom on "small blocks" to obtain effective behavior over "large blocks". A blocking procedure automatically defines scales: "small block" correspond to UV scales and "large blocks" correspond to IR scales.

Scales and Blocking in Discrete Approaches

Historically Wilson's renormalization group was first understood as a blocking procedure averaging degrees of freedom on "small blocks" to obtain effective behavior over "large blocks". A blocking procedure automatically defines scales: "small block" correspond to UV scales and "large blocks" correspond to IR scales.

A "good" blocking procedure can be recast into a path integral formulation. One then combines UV graphs with many vertices to obtain renormalized graphs with few vertices.

Scales and Blocking in Discrete Approaches

Historically Wilson's renormalization group was first understood as a blocking procedure averaging degrees of freedom on "small blocks" to obtain effective behavior over "large blocks". A blocking procedure automatically defines scales: "small block" correspond to UV scales and "large blocks" correspond to IR scales.

A "good" blocking procedure can be recast into a path integral formulation. One then combines UV graphs with many vertices to obtain renormalized graphs with few vertices.

Discrete approaches to gravity have built in scales. "Large blocks" have many space time quanta whereas "small blocks" have few.

Scales and Blocking in Discrete Approaches

Historically Wilson's renormalization group was first understood as a blocking procedure averaging degrees of freedom on "small blocks" to obtain effective behavior over "large blocks". A blocking procedure automatically defines scales: "small block" correspond to UV scales and "large blocks" correspond to IR scales.

A "good" blocking procedure can be recast into a path integral formulation. One then combines UV graphs with many vertices to obtain renormalized graphs with few vertices.

Discrete approaches to gravity have built in scales. "Large blocks" have many space time quanta whereas "small blocks" have few.

Tasks:

Scales and Blocking in Discrete Approaches

Historically Wilson's renormalization group was first understood as a blocking procedure averaging degrees of freedom on "small blocks" to obtain effective behavior over "large blocks". A blocking procedure automatically defines scales: "small block" correspond to UV scales and "large blocks" correspond to IR scales.

A "good" blocking procedure can be recast into a path integral formulation. One then combines UV graphs with many vertices to obtain renormalized graphs with few vertices.

Discrete approaches to gravity have built in scales. "Large blocks" have many space time quanta whereas "small blocks" have few.

Tasks:

- Find a procedure to discretize space time and associate weights to discretizations.

Scales and Blocking in Discrete Approaches

Historically Wilson's renormalization group was first understood as a blocking procedure averaging degrees of freedom on "small blocks" to obtain effective behavior over "large blocks". A blocking procedure automatically defines scales: "small block" correspond to UV scales and "large blocks" correspond to IR scales.

A "good" blocking procedure can be recast into a path integral formulation. One then combines UV graphs with many vertices to obtain renormalized graphs with few vertices.

Discrete approaches to gravity have built in scales. "Large blocks" have many space time quanta whereas "small blocks" have few.

Tasks:

- Find a procedure to discretize space time and associate weights to discretizations.
- Formulate a path integral for the theory of all discretizations.

Scales and Blocking in Discrete Approaches

Historically Wilson's renormalization group was first understood as a blocking procedure averaging degrees of freedom on "small blocks" to obtain effective behavior over "large blocks". A blocking procedure automatically defines scales: "small block" correspond to UV scales and "large blocks" correspond to IR scales.

A "good" blocking procedure can be recast into a path integral formulation. One then combines UV graphs with many vertices to obtain renormalized graphs with few vertices.

Discrete approaches to gravity have built in scales. "Large blocks" have many space time quanta whereas "small blocks" have few.

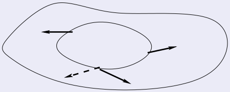
Tasks:

- Find a procedure to discretize space time and associate weights to discretizations.
- Formulate a path integral for the theory of all discretizations.
- Find a well defined transformation from finer to rougher discretizations.

Metric and Holonomies

Metric and Holonomies

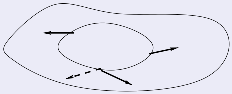
Let a manifold M, a closed curve $\gamma(T) = \gamma(0) \in M$, and $\mathbf{X}$ the vector field solution of the parallel transport equation



$$\frac{d\gamma^\nu}{dt}\partial_\nu \mathbf{X}^\mu + \Gamma^\mu_{\nu\sigma}\mathbf{X}^\nu \frac{d\gamma^\sigma}{dt} = 0 \ , \quad \mathbf{X}(0) = \mathbf{X}_0$$

Metric and Holonomies

Let a manifold M, a closed curve $\gamma(T) = \gamma(0) \in M$, and $\mathbf{X}$ the vector field solution of the parallel transport equation

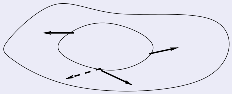


$$\frac{d\gamma^\nu}{dt}\partial_\nu \mathbf{X}^\mu + \Gamma^\mu_{\nu\sigma}\mathbf{X}^\nu \frac{d\gamma^\sigma}{dt} = 0 \ , \quad \mathbf{X}(0) = \mathbf{X}_0$$

Then $\mathbf{X}(T) = g\mathbf{X}_0$ for some $g \in GL(TM_{\gamma(0)})$. g (independent of $\mathbf{X}_0$) is the Green function of the parallel transport equation and is called the holonomy along the curve γ. The information about the metric of M is encoded in the holonomies along all curves.

Metric and Holonomies

Let a manifold M, a closed curve $\gamma(T) = \gamma(0) \in M$, and $\mathbf{X}$ the vector field solution of the parallel transport equation



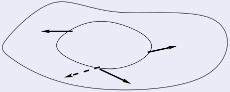
$$\frac{d\gamma^\nu}{dt}\partial_\nu \mathbf{X}^\mu + \Gamma^\mu_{\nu\sigma}\mathbf{X}^\nu \frac{d\gamma^\sigma}{dt} = 0 \, , \quad \mathbf{X}(0) = \mathbf{X}_0$$

Then $\mathbf{X}(T) = g\mathbf{X}_0$ for some $g \in GL(TM_{\gamma(0)})$. g (independent of $\mathbf{X}_0$) is the Green function of the parallel transport equation and is called the holonomy along the curve γ. The information about the metric of M is encoded in the holonomies along all curves.

Question: To what do I associate holonomies in a discretization of the manifold M?

Metric and Holonomies

Let a manifold M, a closed curve $\gamma(T) = \gamma(0) \in M$, and $\mathbf{X}$ the vector field solution of the parallel transport equation



$$\frac{d\gamma^\nu}{dt}\partial_\nu \mathbf{X}^\mu + \Gamma^\mu_{\nu\sigma}\mathbf{X}^\nu \frac{d\gamma^\sigma}{dt} = 0 \,, \quad \mathbf{X}(0) = \mathbf{X}_0$$

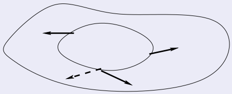
Then $\mathbf{X}(T) = g\mathbf{X}_0$ for some $g \in GL(TM_{\gamma(0)})$. g (independent of $\mathbf{X}_0$) is the Green function of the parallel transport equation and is called the holonomy along the curve γ. The information about the metric of M is encoded in the holonomies along all curves.

Question: To what do I associate holonomies in a discretization of the manifold M?

We choose flat discrete n dimensional blocks. Their boundary ($n-1$ dimensional) is also flat. The curvature is located at the "joints" of these blocks, that is at $n-2$ dimensional blocks.

Metric and Holonomies

Let a manifold M, a closed curve $\gamma(T) = \gamma(0) \in M$, and $\mathbf{X}$ the vector field solution of the parallel transport equation



$$\frac{d\gamma^\nu}{dt}\partial_\nu \mathbf{X}^\mu + \Gamma^\mu_{\nu\sigma}\mathbf{X}^\nu \frac{d\gamma^\sigma}{dt} = 0 \,, \quad \mathbf{X}(0) = \mathbf{X}_0$$

Then $\mathbf{X}(T) = g\mathbf{X}_0$ for some $g \in GL(TM_{\gamma(0)})$. g (independent of $\mathbf{X}_0$) is the Green function of the parallel transport equation and is called the holonomy along the curve γ. The information about the metric of M is encoded in the holonomies along all curves.

Question: To what do I associate holonomies in a discretization of the manifold M?

We choose flat discrete n dimensional blocks. Their boundary ($n-1$ dimensional) is also flat. The curvature is located at the "joints" of these blocks, that is at $n-2$ dimensional blocks.

We associate a holonomy g to a discrete block of codimension 2, the same for all curves γ which encircle it!

Discretized Surfaces

Discretized Surfaces

First consider M a two dimensional surface, and fix a triangulation of M.

Discretized Surfaces

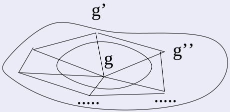
First consider M a two dimensional surface, and fix a triangulation of M.

The holonomy group of a surface is $G = U(1)$.

Discretized Surfaces

First consider M a two dimensional surface, and fix a triangulation of M.

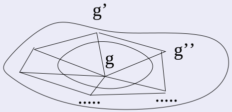
The holonomy group of a surface is $G = U(1)$. To all vertices (points) in our triangulation we associate a holonomy g, representing the transformation of a vector under parallel transport along a small curve encircling the vertex.



Discretized Surfaces

First consider M a two dimensional surface, and fix a triangulation of M.

The holonomy group of a surface is $G = U(1)$. To all vertices (points) in our triangulation we associate a holonomy g, representing the transformation of a vector under parallel transport along a small curve encircling the vertex.

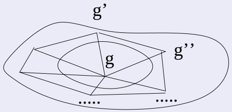


The surface and its metric are specified by the connectivity of the triangles and the holonomies g. The weight associated to a surface is a function $F(g, g', \dots)$

Discretized Surfaces

First consider M a two dimensional surface, and fix a triangulation of M.

The holonomy group of a surface is $G = U(1)$. To all vertices (points) in our triangulation we associate a holonomy g, representing the transformation of a vector under parallel transport along a small curve encircling the vertex.



The surface and its metric are specified by the connectivity of the triangles and the holonomies g. The weight associated to a surface is a function $F(g, g', \dots)$

A "good" blocking procedure should combine the weights corresponding to all metrics compatible with a triangulation and then combine all triangulations.

The Dual Graph

The Dual Graph

The discrete information about the triangulation and metric of the surface can be encoded in a ribbon (stranded) graph.

The Dual Graph

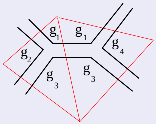
The discrete information about the triangulation and metric of the surface can be encoded in a ribbon (stranded) graph.

The ribbon vertices of the graph are dual to triangles and the ribbon lines of the graph are dual to edges.

The Dual Graph

The discrete information about the triangulation and metric of the surface can be encoded in a ribbon (stranded) graph.

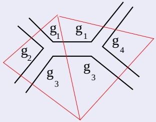
The ribbon vertices of the graph are dual to triangles and the ribbon lines of the graph are dual to edges.



The Dual Graph

The discrete information about the triangulation and metric of the surface can be encoded in a ribbon (stranded) graph.

The ribbon vertices of the graph are dual to triangles and the ribbon lines of the graph are dual to edges.

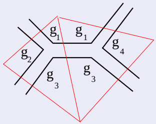


In the dual graph the group elements g are associated to the sides of the ribbons, also called strands. We distribute them on all ribbon vertices sharing the same strand.

The Dual Graph

The discrete information about the triangulation and metric of the surface can be encoded in a ribbon (stranded) graph.

The ribbon vertices of the graph are dual to triangles and the ribbon lines of the graph are dual to edges.



In the dual graph the group elements g are associated to the sides of the ribbons, also called strands. We distribute them on all ribbon vertices sharing the same strand.

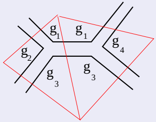
Suppose that weight function F factors into contributions of dual vertices and dual lines like

$$F = \prod_V V(g_1, g_2, g_3) \prod_L K(g_1, g_2)$$

The Dual Graph

The discrete information about the triangulation and metric of the surface can be encoded in a ribbon (stranded) graph.

The ribbon vertices of the graph are dual to triangles and the ribbon lines of the graph are dual to edges.



In the dual graph the group elements g are associated to the sides of the ribbons, also called strands. We distribute them on all ribbon vertices sharing the same strand.

Suppose that weight function F factors into contributions of dual vertices and dual lines like

$$F = \prod_V V(g_1, g_2, g_3) \prod_L K(g_1, g_2)$$

Then a stranded graph is a Feynman graph with fixed internal group elements of a matrix model. Its weight is the integrand of the associated Feynman amplitude.

A First Example of a Group Field Theory

A First Example of a Group Field Theory

The associated matrix model action is ($\phi(g_1, g_2) = \phi^*(g_2, g_1)$)

$$S = \frac{1}{2} \int_{G\times G} \phi(g_1, g_2) K^{-1}(g_1, g_2) \phi^*(g_1, g_2)$$
$$+\lambda \int_{G\times G\times G} V(g_1, g_2, g_3) \phi(g_1, g_2) \phi(g_2, g_3) \phi(g_3, g_1) \ ,$$

A First Example of a Group Field Theory

The associated matrix model action is $(\phi(g_1, g_2) = \phi^*(g_2, g_1))$

$$
\begin{aligned}
S = & \frac{1}{2} \int_{G \times G} \phi(g_1, g_2) K^{-1}(g_1, g_2) \phi^*(g_1, g_2) \\
& + \lambda \int_{G \times G \times G} V(g_1, g_2, g_3) \phi(g_1, g_2) \phi(g_2, g_3) \phi(g_3, g_1) \ ,
\end{aligned}
$$

and the complete correlation function is

$$
< \phi(g_1, g_2) \dots \phi(g_{2n-1}, g_{2n}) >= \int [d\phi] e^{-S} \phi(g_1, g_2) \dots \phi(g_{2n-1}, g_{2n}) \ .
$$

A First Example of a Group Field Theory

The associated matrix model action is $(\phi(g_1, g_2) = \phi^*(g_2, g_1))$

$$S = \frac{1}{2}\int_{G\times G} \phi(g_1,g_2)K^{-1}(g_1,g_2)\phi^*(g_1,g_2)$$
$$+\lambda \int_{G\times G\times G} V(g_1,g_2,g_3)\phi(g_1,g_2)\phi(g_2,g_3)\phi(g_3,g_1)\ ,$$

and the complete correlation function is

$$< \phi(g_1,g_2)\dots\phi(g_{2n-1},g_{2n}) >= \int [d\phi]e^{-S}\phi(g_1,g_2)\dots\phi(g_{2n-1},g_{2n})\ .$$

The insertions $\phi(g_1, g_2)$ fix a boundary triangulation and metric.

A First Example of a Group Field Theory

The associated matrix model action is $(\phi(g_1, g_2) = \phi^*(g_2, g_1))$

$$S = \frac{1}{2} \int_{G\times G} \phi(g_1, g_2) K^{-1}(g_1, g_2) \phi^*(g_1, g_2)$$
$$+\lambda \int_{G\times G\times G} V(g_1, g_2, g_3) \phi(g_1, g_2) \phi(g_2, g_3) \phi(g_3, g_1) \ ,$$

and the complete correlation function is

$$< \phi(g_1, g_2) \dots \phi(g_{2n-1}, g_{2n}) >= \int [d\phi] e^{-S} \phi(g_1, g_2) \dots \phi(g_{2n-1}, g_{2n}) \ .$$

The insertions $\phi(g_1, g_2)$ fix a boundary triangulation and metric.

Each Feynman graph of the correlation function fixes a bulk triangulation.

A First Example of a Group Field Theory

The associated matrix model action is $(\phi(g_1, g_2) = \phi^*(g_2, g_1))$

$$S = \frac{1}{2}\int_{G\times G} \phi(g_1, g_2)K^{-1}(g_1, g_2)\phi^*(g_1, g_2)$$
$$+\lambda \int_{G\times G\times G} V(g_1, g_2, g_3)\phi(g_1, g_2)\phi(g_2, g_3)\phi(g_3, g_1)\ ,$$

and the complete correlation function is

$$< \phi(g_1, g_2)\dots\phi(g_{2n-1}, g_{2n}) >= \int [d\phi]e^{-S}\phi(g_1, g_2)\dots\phi(g_{2n-1}, g_{2n})\ .$$

The insertions $\phi(g_1, g_2)$ fix a boundary triangulation and metric.

Each Feynman graph of the correlation function fixes a bulk triangulation. The amplitude of a graph is the sum over all bulk metrics compatible with the fixed bulk triangulation.

A First Example of a Group Field Theory

The associated matrix model action is $(\phi(g_1, g_2) = \phi^*(g_2, g_1))$

$$S = \frac{1}{2} \int_{G\times G} \phi(g_1, g_2) K^{-1}(g_1, g_2) \phi^*(g_1, g_2)$$
$$+\lambda \int_{G\times G\times G} V(g_1, g_2, g_3) \phi(g_1, g_2) \phi(g_2, g_3) \phi(g_3, g_1) \; ,$$

and the complete correlation function is

$$< \phi(g_1, g_2) \dots \phi(g_{2n-1}, g_{2n}) >= \int [d\phi] e^{-S} \phi(g_1, g_2) \dots \phi(g_{2n-1}, g_{2n}) \; .$$

The insertions $\phi(g_1, g_2)$ fix a boundary triangulation and metric.

Each Feynman graph of the correlation function fixes a bulk triangulation. The amplitude of a graph is the sum over all bulk metrics compatible with the fixed bulk triangulation. The complete correlation function automatically performs both sums!

Scales in Matrix Models

Scales in Matrix Models

Take $K, V = 1$ and develop ϕ in Fourier series

$$\phi(g_1, g_2) = \sum_{\mathbb{Z}\times\mathbb{Z}} e^{\imath m g_1} e^{-\imath n g_2} \phi_{mn} \ .$$

Scales in Matrix Models

Take $K, V = 1$ and develop ϕ in Fourier series

$$\phi(g_1, g_2) = \sum_{\mathbb{Z}\times\mathbb{Z}} e^{\imath m g_1} e^{-\imath n g_2} \phi_{mn} \ .$$

The action takes the more familiar form in Fourier space

$$S = \frac{1}{2} \sum_{mn} \phi_{mn} \phi^*_{mn} + \lambda \sum_{mnk} \phi_{mn} \phi_{nk} \phi_{km} \ .$$

Scales in Matrix Models

Take $K, V = 1$ and develop ϕ in Fourier series

$$\phi(g_1, g_2) = \sum_{\mathbb{Z}\times\mathbb{Z}} e^{\imath m g_1} e^{-\imath n g_2} \phi_{mn} \ .$$

The action takes the more familiar form in Fourier space

$$S = \frac{1}{2} \sum_{mn} \phi_{mn}\phi^*_{mn} + \lambda \sum_{mnk} \phi_{mn}\phi_{nk}\phi_{km} \ .$$

The scales are then defined as follows: high scales, UV, correspond to large values of matrix indices ($|m|$) whereas low scales, IR, correspond to low values of matrix indices. In two dimensions our program is achieved...

Scales in Matrix Models

Take $K, V = 1$ and develop ϕ in Fourier series

$$\phi(g_1, g_2) = \sum_{\mathbb{Z}\times\mathbb{Z}} e^{\imath m g_1} e^{-\imath n g_2} \phi_{mn} \ .$$

The action takes the more familiar form in Fourier space

$$S = \frac{1}{2} \sum_{mn} \phi_{mn}\phi^*_{mn} + \lambda \sum_{mnk} \phi_{mn}\phi_{nk}\phi_{km} \ .$$

The scales are then defined as follows: high scales, UV, correspond to large values of matrix indices ($|m|$) whereas low scales, IR, correspond to low values of matrix indices. In two dimensions our program is achieved...

But one needs to consider all the Feynman graphs (and there dual topological spaces) generated by the matrix model action!

Higher Dimensions

Higher Dimensions

- Define a discretization of a n dimensional manifold.

Higher Dimensions

- Define a discretization of a n dimensional manifold.
- Build the class of dual graphs $\mathcal{G}$.

Higher Dimensions

- Define a discretization of a n dimensional manifold.
- Build the class of dual graphs $\mathcal{G}$.
- Find an action S whose Feynman graphs contain $\mathcal{G}$.

Higher Dimensions

- Define a discretization of a n dimensional manifold.
- Build the class of dual graphs $\mathcal{G}$.
- Find an action S whose Feynman graphs contain $\mathcal{G}$.
- Identify scales.

Higher Dimensions

- Define a discretization of a n dimensional manifold.
- Build the class of dual graphs $\mathcal{G}$.
- Find an action S whose Feynman graphs contain $\mathcal{G}$.
- Identify scales.
- Analyze and classify all graphs generated by S.

Higher Dimensions

- Define a discretization of a n dimensional manifold.
- Build the class of dual graphs $\mathcal{G}$.
- Find an action S whose Feynman graphs contain $\mathcal{G}$.
- Identify scales.
- Analyze and classify all graphs generated by S.
- Define a renormalization transformation.

GFT Action in Three Dimensions

GFT Action in Three Dimensions

In three dimensions we choose the holonomy group $G = SU(2)$.

GFT Action in Three Dimensions

In three dimensions we choose the holonomy group $G = SU(2)$. Group elements are associated to edges in the triangulation (codimension 2).

GFT Action in Three Dimensions

In three dimensions we choose the holonomy group $G = SU(2)$. Group elements are associated to edges in the triangulation (codimension 2). The field ϕ is associated to the faces of a tetrahedron (triangles), therefore it has three arguments.

GFT Action in Three Dimensions

In three dimensions we choose the holonomy group $G = SU(2)$. Group elements are associated to edges in the triangulation (codimension 2). The field ϕ is associated to the faces of a tetrahedron (triangles), therefore it has three arguments. The (real) quadratic part is

$$\frac{1}{2}\int_{G^3} \phi(g_a, g_b, g_c) K^{-1}(g_a, g_b, g_c) \phi^*(g_a, g_b, g_c)$$

GFT Action in Three Dimensions

In three dimensions we choose the holonomy group $G = SU(2)$. Group elements are associated to edges in the triangulation (codimension 2). The field ϕ is associated to the faces of a tetrahedron (triangles), therefore it has three arguments. The (real) quadratic part is

$$\frac{1}{2}\int_{G^3} \phi(g_a, g_b, g_c) K^{-1}(g_a, g_b, g_c) \phi^*(g_a, g_b, g_c)$$

ϕ is invariant under even permutations and transforms into its conjugate under odd permutations.

GFT Action in Three Dimensions

In three dimensions we choose the holonomy group $G = SU(2)$. Group elements are associated to edges in the triangulation (codimension 2). The field ϕ is associated to the faces of a tetrahedron (triangles), therefore it has three arguments. The (real) quadratic part is

$$\frac{1}{2}\int_{G^3} \phi(g_a, g_b, g_c) K^{-1}(g_a, g_b, g_c) \phi^*(g_a, g_b, g_c)$$

ϕ is invariant under even permutations and transforms into its conjugate under odd permutations. The vertex is dual to a tetrahedron. A tetrahedron is bounded by four triangles therefore the vertex is a ϕ^4 term

$$\lambda \int_{G^6} V(g, \ldots, g) \phi(g_{03}, g_{02}, g_{01}) \phi(g_{01}, g_{13}, g_{12}) \phi(g_{12}, g_{02}, g_{23}) \phi(g_{23}, g_{13}, g_{03})$$

GFT Action in Three Dimensions

In three dimensions we choose the holonomy group $G = SU(2)$. Group elements are associated to edges in the triangulation (codimension 2). The field ϕ is associated to the faces of a tetrahedron (triangles), therefore it has three arguments. The (real) quadratic part is

$$\frac{1}{2}\int_{G^3} \phi(g_a, g_b, g_c) K^{-1}(g_a, g_b, g_c) \phi^*(g_a, g_b, g_c)$$

ϕ is invariant under even permutations and transforms into its conjugate under odd permutations. The vertex is dual to a tetrahedron. A tetrahedron is bounded by four triangles therefore the vertex is a ϕ^4 term

$$\lambda \int_{G^6} V(g, \dots, g) \phi(g_{03}, g_{02}, g_{01}) \phi(g_{01}, g_{13}, g_{12}) \phi(g_{12}, g_{02}, g_{23}) \phi(g_{23}, g_{13}, g_{03})$$

The scales are again defined using the representations of $SU(2)$. UV scales are high values of j and IR scales are low values of j.

The Tetrahedron and the GFT Vertex

The Tetrahedron and the GFT Vertex

In order to understand the combinatorics of the arguments in the GFT vertex one needs to carefully label all the elements of a tetrahedron.

The Tetrahedron and the GFT Vertex

In order to understand the combinatorics of the arguments in the GFT vertex one needs to carefully label all the elements of a tetrahedron.

We label the vertices (points) 0 to 3.

The Tetrahedron and the GFT Vertex

In order to understand the combinatorics of the arguments in the GFT vertex one needs to carefully label all the elements of a tetrahedron.

We label the vertices (points) 0 to 3. Edges connect two vertices and we label them by the couple of labels of there endpoints.

The Tetrahedron and the GFT Vertex

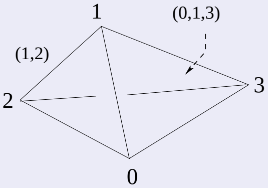
In order to understand the combinatorics of the arguments in the GFT vertex one needs to carefully label all the elements of a tetrahedron.

We label the vertices (points) 0 to 3. Edges connect two vertices and we label them by the couple of labels of there endpoints. Triangles have three vertices and we label them by the corresponding triple of labels.

The Tetrahedron and the GFT Vertex

In order to understand the combinatorics of the arguments in the GFT vertex one needs to carefully label all the elements of a tetrahedron.

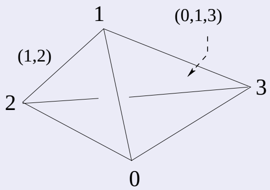
We label the vertices (points) 0 to 3. Edges connect two vertices and we label them by the couple of labels of there endpoints. Triangles have three vertices and we label them by the corresponding triple of labels.



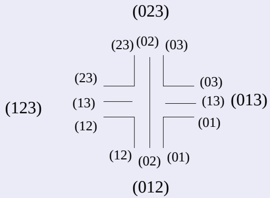
The Tetrahedron and the GFT Vertex

In order to understand the combinatorics of the arguments in the GFT vertex one needs to carefully label all the elements of a tetrahedron.

We label the vertices (points) 0 to 3. Edges connect two vertices and we label them by the couple of labels of there endpoints. Triangles have three vertices and we label them by the corresponding triple of labels.



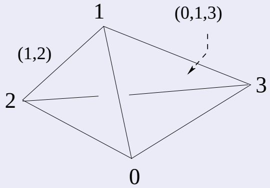
The tetrahedron is dual to a vertex, its triangles are dual to halflines and its edges are dual to strands. The fully labeled GFT vertex is



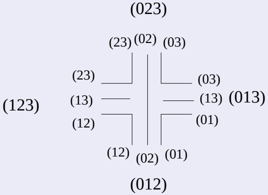
The Tetrahedron and the GFT Vertex

In order to understand the combinatorics of the arguments in the GFT vertex one needs to carefully label all the elements of a tetrahedron.

We label the vertices (points) 0 to 3. Edges connect two vertices and we label them by the couple of labels of there endpoints. Triangles have three vertices and we label them by the corresponding triple of labels.



The tetrahedron is dual to a vertex, its triangles are dual to halflines and its edges are dual to strands. The fully labeled GFT vertex is

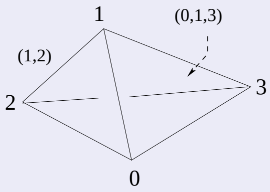


The GFT lines connect two vertices, thus are formed of three strands with an arbitrary permutation.

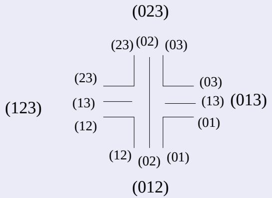
The Tetrahedron and the GFT Vertex

In order to understand the combinatorics of the arguments in the GFT vertex one needs to carefully label all the elements of a tetrahedron.

We label the vertices (points) 0 to 3. Edges connect two vertices and we label them by the couple of labels of there endpoints. Triangles have three vertices and we label them by the corresponding triple of labels.



The tetrahedron is dual to a vertex, its triangles are dual to halflines and its edges are dual to strands. The fully labeled GFT vertex is



The GFT lines connect two vertices, thus are formed of three strands with an arbitrary permutation. The graph built with such vertices and lines is called a stranded graph.

Bubbles and Projection

Bubbles and Projection

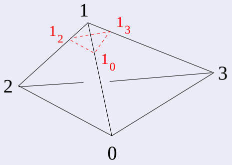
Question: what is dual to a vertex (point) of the tetrahedron?

Bubbles and Projection

Question: what is dual to a vertex (point) of the tetrahedron? The vertex is dual to some three dimensional region. We call the two dimensional surface bounding such a region a bubble.

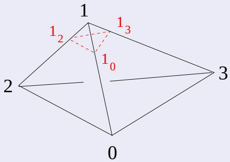
Bubbles and Projection

Question: what is dual to a vertex (point) of the tetrahedron? The vertex is dual to some three dimensional region. We call the two dimensional surface bounding such a region a bubble. To identify a bubble we carve out the corners of all tetrahedra to which the vertex belongs.



Bubbles and Projection

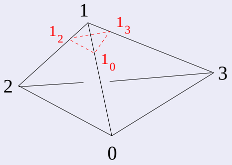
Question: what is dual to a vertex (point) of the tetrahedron? The vertex is dual to some three dimensional region. We call the two dimensional surface bounding such a region a bubble. To identify a bubble we carve out the corners of all tetrahedra to which the vertex belongs.



The tetrahedron (0123) will "project" into the red triangle $1_2 1_0 1_3$.

Bubbles and Projection

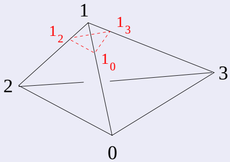
Question: what is dual to a vertex (point) of the tetrahedron? The vertex is dual to some three dimensional region. We call the two dimensional surface bounding such a region a bubble. To identify a bubble we carve out the corners of all tetrahedra to which the vertex belongs.



The tetrahedron (0123) will "project" into the red triangle $1_2 1_0 1_3$. The triangle (012) will "project" into the edge $1_2 1_0$.

Bubbles and Projection

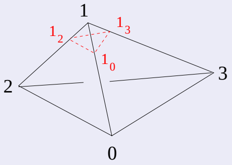
Question: what is dual to a vertex (point) of the tetrahedron? The vertex is dual to some three dimensional region. We call the two dimensional surface bounding such a region a bubble. To identify a bubble we carve out the corners of all tetrahedra to which the vertex belongs.



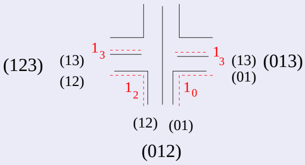
The tetrahedron (0123) will "project" into the red triangle $1_2 1_0 1_3$. The triangle (012) will "project" into the edge $1_2 1_0$. The edge (12) will "project" into the point 1_2.

Bubbles and Projection

Question: what is dual to a vertex (point) of the tetrahedron? The vertex is dual to some three dimensional region. We call the two dimensional surface bounding such a region a bubble. To identify a bubble we carve out the corners of all tetrahedra to which the vertex belongs.

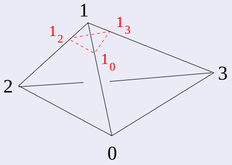


The tetrahedron (0123) will “project” into the red triangle $1_2 1_0 1_3$. The triangle (012) will “project” into the edge $1_2 1_0$. The edge (12) will “project” into the point 1_2.

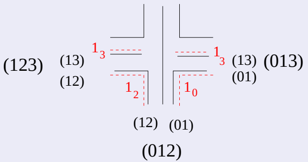


Bubbles and Projection

Question: what is dual to a vertex (point) of the tetrahedron? The vertex is dual to some three dimensional region. We call the two dimensional surface bounding such a region a bubble. To identify a bubble we carve out the corners of all tetrahedra to which the vertex belongs.



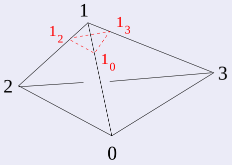
The tetrahedron (0123) will "project" into the red triangle $1_2 1_0 1_3$. The triangle (012) will "project" into the edge $1_2 1_0$. The edge (12) will "project" into the point 1_2.



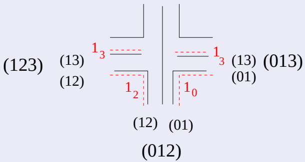
The strand (12) will project in the strand 1_2.

Bubbles and Projection

Question: what is dual to a vertex (point) of the tetrahedron? The vertex is dual to some three dimensional region. We call the two dimensional surface bounding such a region a bubble. To identify a bubble we carve out the corners of all tetrahedra to which the vertex belongs.



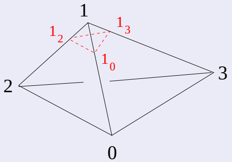
The tetrahedron (0123) will "project" into the red triangle $1_2 1_0 1_3$. The triangle (012) will "project" into the edge $1_2 1_0$. The edge (12) will "project" into the point 1_2.



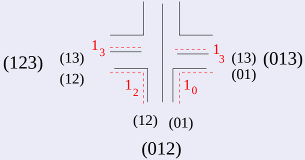
The strand (12) will project in the strand 1_2. The halfline (012) will project in the halfline $1_2 1_0$.

Bubbles and Projection

Question: what is dual to a vertex (point) of the tetrahedron? The vertex is dual to some three dimensional region. We call the two dimensional surface bounding such a region a bubble. To identify a bubble we carve out the corners of all tetrahedra to which the vertex belongs.



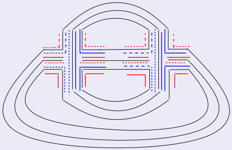
The tetrahedron (0123) will "project" into the red triangle $1_2 1_0 1_3$. The triangle (012) will "project" into the edge $1_2 1_0$. The edge (12) will "project" into the point 1_2.



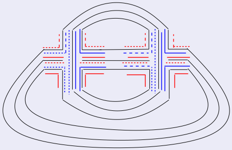
The strand (12) will project in the strand 1_2. The halfline (012) will project in the halfline $1_2 1_0$. The vertex (0123) will project in the red vertex $1_2 1_0 1_3$.

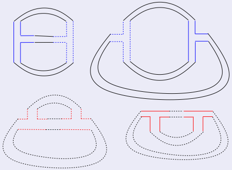
Examples of Stranded Graphs

Examples of Stranded Graphs

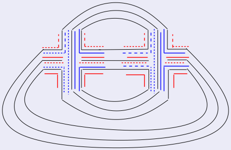


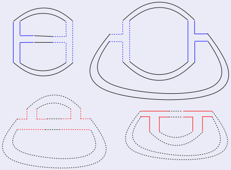
Examples of Stranded Graphs





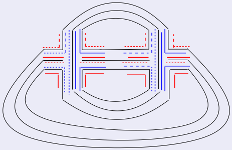
Examples of Stranded Graphs

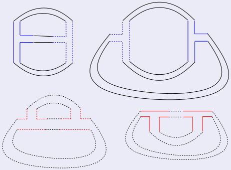




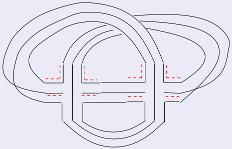
All the bubble graphs are planar: the bubbles are $\mathbb{S}^2$. The regions bounded by bubbles are solid balls $\mathbb{B}^3$. The dual of this graph is a closed manifold.

Examples of Stranded Graphs

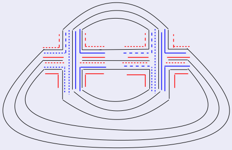


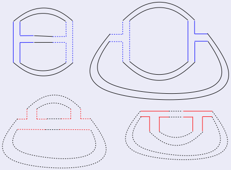


All the bubble graphs are planar: the bubbles are $\mathbb{S}^2$. The regions bounded by bubbles are solid balls $\mathbb{B}^3$. The dual of this graph is a closed manifold.

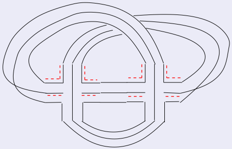


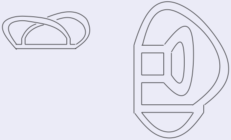
Examples of Stranded Graphs



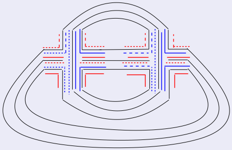


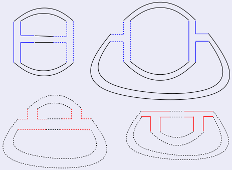
All the bubble graphs are planar: the bubbles are $\mathbb{S}^2$. The regions bounded by bubbles are solid balls $\mathbb{B}^3$. The dual of this graph is a closed manifold.



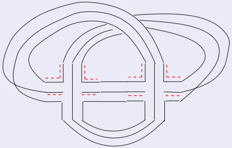


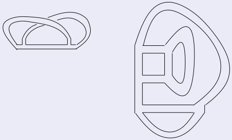
Examples of Stranded Graphs





All the bubble graphs are planar: the bubbles are $\mathbb{S}^2$. The regions bounded by bubbles are solid balls $\mathbb{B}^3$. The dual of this graph is a closed manifold.





There can exist bubbles whose graphs are non planar (eg a torus $\mathbb{T}^2$). The region bounded by this bubble cannot be a ball! The dual of this graph is therefore not a manifold, but a pseudomanifold.

Singular Graphs

Singular Graphs

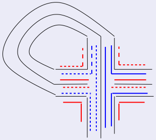
The vertices, lines and faces (closed strands) of graphs are always well defined.

Singular Graphs

The vertices, lines and faces (closed strands) of graphs are always well defined. But there exist graphs generated by the GFT action for which the bubbles are ill defined! For instance

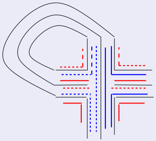
Singular Graphs

The vertices, lines and faces (closed strands) of graphs are always well defined. But there exist graphs generated by the GFT action for which the bubbles are ill defined! For instance



Singular Graphs

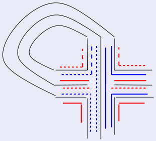
The vertices, lines and faces (closed strands) of graphs are always well defined. But there exist graphs generated by the GFT action for which the bubbles are ill defined! For instance



The dual of this graph has an extended singularity. Therefore

Singular Graphs

The vertices, lines and faces (closed strands) of graphs are always well defined. But there exist graphs generated by the GFT action for which the bubbles are ill defined! For instance

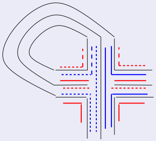


The dual of this graph has an extended singularity. Therefore

- We have a good discretization procedure in 3D.

Singular Graphs

The vertices, lines and faces (closed strands) of graphs are always well defined. But there exist graphs generated by the GFT action for which the bubbles are ill defined! For instance

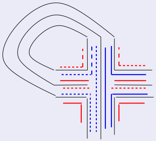


The dual of this graph has an extended singularity. Therefore

- We have a good discretization procedure in 3D.
- The resulting discretized spaces are dual to graphs. Scales are built in.

Singular Graphs

The vertices, lines and faces (closed strands) of graphs are always well defined. But there exist graphs generated by the GFT action for which the bubbles are ill defined! For instance

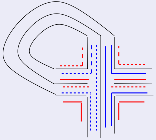


The dual of this graph has an extended singularity. Therefore

- We have a good discretization procedure in 3D.
- The resulting discretized spaces are dual to graphs. Scales are built in.
- The graphs can be generated by a path integral. The correlation functions sum over triangulations and metrics with appropriate weights

Singular Graphs

The vertices, lines and faces (closed strands) of graphs are always well defined. But there exist graphs generated by the GFT action for which the bubbles are ill defined! For instance

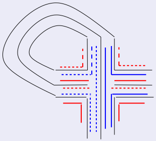


The dual of this graph has an extended singularity. Therefore

- We have a good discretization procedure in 3D.
- The resulting discretized spaces are dual to graphs. Scales are built in.
- The graphs can be generated by a path integral. The correlation functions sum over triangulations and metrics with appropriate weights
- We know how to identify vertices lines faces and bubbles in the graphs.

Singular Graphs

The vertices, lines and faces (closed strands) of graphs are always well defined. But there exist graphs generated by the GFT action for which the bubbles are ill defined! For instance



The dual of this graph has an extended singularity. Therefore

- We have a good discretization procedure in 3D.
- The resulting discretized spaces are dual to graphs. Scales are built in.
- The graphs can be generated by a path integral. The correlation functions sum over triangulations and metrics with appropriate weights
- We know how to identify vertices lines faces and bubbles in the graphs.
- But we generate many singular, nonphysical graphs!

The Colored Model

The Colored Model

We can build a new GFT action whose graphs are dual only to manifolds or pseudomanifolds!

The Colored Model

We can build a new GFT action whose graphs are dual only to manifolds or pseudomanifolds!

Color the triangles of the tetrahedron with colors $0, 1, 2, 3$.

The Colored Model

We can build a new GFT action whose graphs are dual only to manifolds or pseudomanifolds!

Color the triangles of the tetrahedron with colors $0, 1, 2, 3$. This translates into having four different fields ψ^p, one for each triangle.

The Colored Model

We can build a new GFT action whose graphs are dual only to manifolds or pseudomanifolds!

Color the triangles of the tetrahedron with colors $0, 1, 2, 3$. This translates into having four different fields ψ^p, one for each triangle.

Introduce positive and negative oriented tetrahedra.

The Colored Model

We can build a new GFT action whose graphs are dual only to manifolds or pseudomanifolds!

Color the triangles of the tetrahedron with colors $0, 1, 2, 3$. This translates into having four different fields ψ^p, one for each triangle.

Introduce positive and negative oriented tetrahedra. This means choose two interaction terms, one involving ψ^p the other involving $\bar{\psi}^p$.

The Colored Model

We can build a new GFT action whose graphs are dual only to manifolds or pseudomanifolds!

Color the triangles of the tetrahedron with colors $0, 1, 2, 3$. This translates into having four different fields ψ^p, one for each triangle.

Introduce positive and negative oriented tetrahedra. This means choose two interaction terms, one involving ψ^p the other involving $\bar{\psi}^p$.

Only glue triangles of the same color on tetrahedra with opposite orientations.

The Colored Model

We can build a new GFT action whose graphs are dual only to manifolds or pseudomanifolds!

Color the triangles of the tetrahedron with colors $0, 1, 2, 3$. This translates into having four different fields ψ^p, one for each triangle.

Introduce positive and negative oriented tetrahedra. This means choose two interaction terms, one involving ψ^p the other involving $\bar{\psi}^p$.

Only glue triangles of the same color on tetrahedra with opposite orientations. That is connect only ψ^p with $\bar{\psi}^p$ in the quadratic form.

The Colored Model

We can build a new GFT action whose graphs are dual only to manifolds or pseudomanifolds!

Color the triangles of the tetrahedron with colors $0, 1, 2, 3$. This translates into having four different fields ψ^p, one for each triangle.

Introduce positive and negative oriented tetrahedra. This means choose two interaction terms, one involving ψ^p the other involving $\bar{\psi}^p$.

Only glue triangles of the same color on tetrahedra with opposite orientations. That is connect only ψ^p with $\bar{\psi}^p$ in the quadratic form.

$$S = \frac{1}{2} \int_{G^3} \sum_p \bar{\psi}^p K \psi^p + \int_{G^6} V(\psi^0\psi^1\psi^2\psi^3 + \bar{\psi}^0\bar{\psi}^1\bar{\psi}^2\bar{\psi}^3)$$

where the ψ's have no symmetry properties.

The Colored Model

We can build a new GFT action whose graphs are dual only to manifolds or pseudomanifolds!

Color the triangles of the tetrahedron with colors $0, 1, 2, 3$. This translates into having four different fields ψ^p, one for each triangle.

Introduce positive and negative oriented tetrahedra. This means choose two interaction terms, one involving ψ^p the other involving $\bar{\psi}^p$.

Only glue triangles of the same color on tetrahedra with opposite orientations. That is connect only ψ^p with $\bar{\psi}^p$ in the quadratic form.

$$S = \frac{1}{2} \int_{G^3} \sum_p \bar{\psi}^p K \psi^p + \int_{G^6} V(\psi^0 \psi^1 \psi^2 \psi^3 + \bar{\psi}^0 \bar{\psi}^1 \bar{\psi}^2 \bar{\psi}^3)$$

where the ψ's have no symmetry properties. For Fermionic fields we have an unexpected $SU(n+1)$ color symmetry!

Colored Graphs vs. Stranded Graphs

Colored Graphs vs. Stranded Graphs

All the lines in the graphs of this model have a color index.

Colored Graphs vs. Stranded Graphs

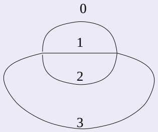
All the lines in the graphs of this model have a color index. Vertices fall in two categories, clockwise and anticlockwise turning.

Colored Graphs vs. Stranded Graphs

All the lines in the graphs of this model have a color index. Vertices fall in two categories, clockwise and anticlockwise turning. Lines connect opposing vertices.

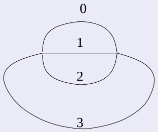
Colored Graphs vs. Stranded Graphs

All the lines in the graphs of this model have a color index. Vertices fall in two categories, clockwise and anticlockwise turning. Lines connect opposing vertices. Schematically a graph can be represented as a colored graph



Colored Graphs vs. Stranded Graphs

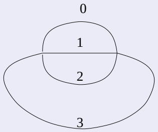
All the lines in the graphs of this model have a color index. Vertices fall in two categories, clockwise and anticlockwise turning. Lines connect opposing vertices. Schematically a graph can be represented as a colored graph



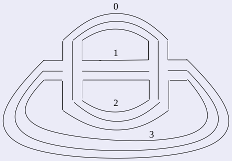
which is a simplified representation of a stranded graph

Colored Graphs vs. Stranded Graphs

All the lines in the graphs of this model have a color index. Vertices fall in two categories, clockwise and anticlockwise turning. Lines connect opposing vertices. Schematically a graph can be represented as a colored graph

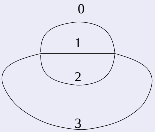


which is a simplified representation of a stranded graph

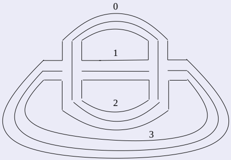


Colored Graphs vs. Stranded Graphs

All the lines in the graphs of this model have a color index. Vertices fall in two categories, clockwise and anticlockwise turning. Lines connect opposing vertices. Schematically a graph can be represented as a colored graph



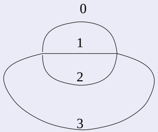
which is a simplified representation of a stranded graph



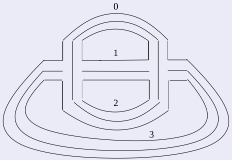
The colored graph encodes completely the structure of the stranded graph

Colored Graphs vs. Stranded Graphs

All the lines in the graphs of this model have a color index. Vertices fall in two categories, clockwise and anticlockwise turning. Lines connect opposing vertices. Schematically a graph can be represented as a colored graph



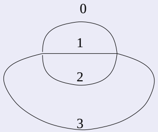
which is a simplified representation of a stranded graph



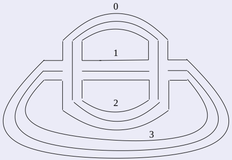
The colored graph encodes completely the structure of the stranded graph To obtain the stranded graph: expand the vertices into stranded vertices and the lines into stranded lines with parallel strands

Colored Graphs vs. Stranded Graphs

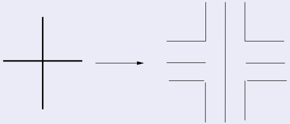
All the lines in the graphs of this model have a color index. Vertices fall in two categories, clockwise and anticlockwise turning. Lines connect opposing vertices. Schematically a graph can be represented as a colored graph



which is a simplified representation of a stranded graph



The colored graph encodes completely the structure of the stranded graph To obtain the stranded graph: expand the vertices into stranded vertices and the lines into stranded lines with parallel strands



Boundary Operator and Graph Homology

Boundary Operator and Graph Homology

The bubbles in a colored graph are always well defined and consists of all maximal connected colored subgraphs with three fixed colors.

Boundary Operator and Graph Homology

The bubbles in a colored graph are always well defined and consists of all maximal connected colored subgraphs with three fixed colors.

The faces (closed strands) are always well defined and consist of the maximal connected subgraphs with two colors.

Boundary Operator and Graph Homology

The bubbles in a colored graph are always well defined and consists of all maximal connected colored subgraphs with three fixed colors.

The faces (closed strands) are always well defined and consist of the maximal connected subgraphs with two colors.

The lines are the connected subgraphs with one color, whereas the vertices are the subgraphs with zero colors.

Boundary Operator and Graph Homology

The bubbles in a colored graph are always well defined and consists of all maximal connected colored subgraphs with three fixed colors.

The faces (closed strands) are always well defined and consist of the maximal connected subgraphs with two colors.

The lines are the connected subgraphs with one color, whereas the vertices are the subgraphs with zero colors.

We define then a *p*-cell of our graph $\mathcal{B}^{\mathcal{C}}_{\mathcal{V}} \in \mathfrak{B}^p$ as the subgraph with set of vertices $\mathcal{V} = \{v_1, \dots v_n\}$ and ordered set of colors $\mathcal{C} = \{i_1, \dots i_p\} \subset \{0, 1, 2, 3\}$,

Boundary Operator and Graph Homology

The bubbles in a colored graph are always well defined and consists of all maximal connected colored subgraphs with three fixed colors.

The faces (closed strands) are always well defined and consist of the maximal connected subgraphs with two colors.

The lines are the connected subgraphs with one color, whereas the vertices are the subgraphs with zero colors.

We define then a *p*-cell of our graph $\mathcal{B}^{\mathcal{C}}_{\mathcal{V}} \in \mathfrak{B}^p$ as the subgraph with set of vertices $\mathcal{V} = \{v_1, \dots v_n\}$ and ordered set of colors $\mathcal{C} = \{i_1, \dots i_p\} \subset \{0, 1, 2, 3\}$, and the boundary operator d_p

$$d_p(\mathcal{B}^{\mathcal{C}}_{\mathcal{V}}) = \sum_q (-)^{q+1} \sum_{\mathcal{B}'^{\mathcal{C}'}_{\mathcal{V}'} \in \mathfrak{B}^{p-1},\ \mathcal{V}' \subset \mathcal{V}, \mathcal{C}' = \mathcal{C} \setminus i_q} \mathcal{B}'^{\mathcal{C}'}_{\mathcal{V}'} .$$

Boundary Operator and Graph Homology

The bubbles in a colored graph are always well defined and consists of all maximal connected colored subgraphs with three fixed colors.

The faces (closed strands) are always well defined and consist of the maximal connected subgraphs with two colors.

The lines are the connected subgraphs with one color, whereas the vertices are the subgraphs with zero colors.

We define then a p-cell of our graph $\mathcal{B}^{\mathcal{C}}_{\mathcal{V}} \in \mathfrak{B}^p$ as the subgraph with set of vertices $\mathcal{V} = \{v_1, \dots v_n\}$ and ordered set of colors $\mathcal{C} = \{i_1, \dots i_p\} \subset \{0, 1, 2, 3\}$, and the boundary operator d_p

$$d_p(\mathcal{B}^{\mathcal{C}}_{\mathcal{V}}) = \sum_q (-)^{q+1} \sum_{\mathcal{B}'^{\mathcal{C}'}_{\mathcal{V}'} \in \mathfrak{B}^{p-1},\ \mathcal{V}' \subset \mathcal{V}, \mathcal{C}' = \mathcal{C} \setminus i_q} \mathcal{B}'^{\mathcal{C}'}_{\mathcal{V}'} .$$

Any graph becomes a cellular complex. We can then compute homology groups of graphs!

Some Graphs and There Homology

Some Graphs and There Homology

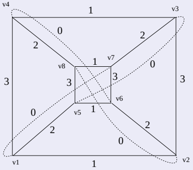
For connected closed graphs we have $H_0 = \mathbb{Z}$, $H_3 = \mathbb{Z}$, $\mathrm{Ker}(d_1) = \bigoplus_{L-N+1} \mathbb{Z}$, $\mathrm{Im}(d_3) = \bigoplus_{B-1} \mathbb{Z}$.

Some Graphs and There Homology

For connected closed graphs we have $H_0 = \mathbb{Z}$, $H_3 = \mathbb{Z}$, $\mathrm{Ker}(d_1) = \bigoplus_{L-N+1} \mathbb{Z}$, $\mathrm{Im}(d_3) = \bigoplus_{B-1} \mathbb{Z}$. The only operator which distinguishes between graphs is d_2. For example

Some Graphs and There Homology

For connected closed graphs we have $H_0 = \mathbb{Z}$, $H_3 = \mathbb{Z}$, $\mathrm{Ker}(d_1) = \bigoplus_{L-N+1} \mathbb{Z}$, $\mathrm{Im}(d_3) = \bigoplus_{B-1} \mathbb{Z}$. The only operator which distinguishes between graphs is d_2. For example

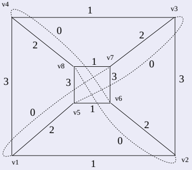


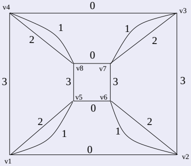
$$
\begin{aligned}
H_0 &= \mathbb{Z} \\
H_1 &= \mathbb{Z}_2 \\
H_2 &= 0 \\
H_3 &= \mathbb{Z}
\end{aligned}
$$

Consistent with $\mathbb{R}P^3$

Some Graphs and There Homology

For connected closed graphs we have $H_0 = \mathbb{Z}$, $H_3 = \mathbb{Z}$, $\mathrm{Ker}(d_1) = \bigoplus_{L-N+1} \mathbb{Z}$, $\mathrm{Im}(d_3) = \bigoplus_{B-1} \mathbb{Z}$. The only operator which distinguishes between graphs is d_2. For example



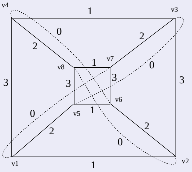


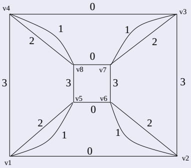
$$
\begin{aligned}
H_0 &= \mathbb{Z} \\
H_1 &= \mathbb{Z}_2 \\
H_2 &= 0 \\
H_3 &= \mathbb{Z}
\end{aligned}
\qquad\qquad
\begin{aligned}
H_0 &= \mathbb{Z} \\
H_1 &= 0 \\
H_2 &= 0 \\
H_3 &= \mathbb{Z}
\end{aligned}
$$

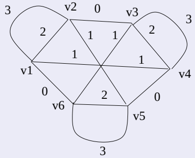
Consistent with $\mathbb{R}P^3$ Consistent with $\mathbb{S}^3$

Some Graphs and There Homology

For connected closed graphs we have $H_0 = \mathbb{Z}$, $H_3 = \mathbb{Z}$, $\mathrm{Ker}(d_1) = \bigoplus_{L-N+1} \mathbb{Z}$, $\mathrm{Im}(d_3) = \bigoplus_{B-1} \mathbb{Z}$. The only operator which distinguishes between graphs is d_2. For example







$$\begin{aligned} H_0 &= \mathbb{Z} \\ H_1 &= \mathbb{Z}_2 \\ H_2 &= 0 \\ H_3 &= \mathbb{Z} \end{aligned}$$

Consistent with $\mathbb{R}P^3$

$$\begin{aligned} H_0 &= \mathbb{Z} \\ H_1 &= 0 \\ H_2 &= 0 \\ H_3 &= \mathbb{Z} \end{aligned}$$

Consistent with $\mathbb{S}^3$

$$\begin{aligned} H_0 &= \mathbb{Z} \\ H_1 &= \mathbb{Z} \oplus \mathbb{Z} \\ H_2 &= 0 \\ H_3 &= \mathbb{Z} \end{aligned}$$

Pseudomanifold

Conclusions

Conclusions

What we have achieved:

Conclusions

What we have achieved:

- We gave a good discretization procedure of space time manifolds.

Conclusions

What we have achieved:

- We gave a good discretization procedure of space time manifolds.
- We have organized the discretized models in GFT's which have built in scales.

Conclusions

What we have achieved:

- We gave a good discretization procedure of space time manifolds.
- We have organized the discretized models in GFT's which have built in scales.
- GFT sums over all discretizations and metrics.

Conclusions

What we have achieved:

- We gave a good discretization procedure of space time manifolds.
- We have organized the discretized models in GFT's which have built in scales.
- GFT sums over all discretizations and metrics. But they generate singular graphs.

Conclusions

What we have achieved:

- We gave a good discretization procedure of space time manifolds.
- We have organized the discretized models in GFT's which have built in scales.
- GFT sums over all discretizations and metrics. But they generate singular graphs.
- We defined the colored GFT's which eliminate the singular graphs and we discovered an unexpected color symmetry.

Conclusions

What we have achieved:

- We gave a good discretization procedure of space time manifolds.
- We have organized the discretized models in GFT's which have built in scales.
- GFT sums over all discretizations and metrics. But they generate singular graphs.
- We defined the colored GFT's which eliminate the singular graphs and we discovered an unexpected color symmetry.
- We defined a boundary operator and homology groups of colored graphs.

Conclusions

What we have achieved:

- We gave a good discretization procedure of space time manifolds.
- We have organized the discretized models in GFT's which have built in scales.
- GFT sums over all discretizations and metrics. But they generate singular graphs.
- We defined the colored GFT's which eliminate the singular graphs and we discovered an unexpected color symmetry.
- We defined a boundary operator and homology groups of colored graphs.

A short program:

Conclusions

What we have achieved:

- We gave a good discretization procedure of space time manifolds.
- We have organized the discretized models in GFT's which have built in scales.
- GFT sums over all discretizations and metrics. But they generate singular graphs.
- We defined the colored GFT's which eliminate the singular graphs and we discovered an unexpected color symmetry.
- We defined a boundary operator and homology groups of colored graphs.

A short program:

- Build a renormalization transformation such that manifold graphs dominate the IR regime.

Conclusions

What we have achieved:

- We gave a good discretization procedure of space time manifolds.
- We have organized the discretized models in GFT's which have built in scales.
- GFT sums over all discretizations and metrics. But they generate singular graphs.
- We defined the colored GFT's which eliminate the singular graphs and we discovered an unexpected color symmetry.
- We defined a boundary operator and homology groups of colored graphs.

A short program:

- Build a renormalization transformation such that manifold graphs dominate the IR regime.
- Introduce matter fields.

Conclusions

What we have achieved:

- We gave a good discretization procedure of space time manifolds.
- We have organized the discretized models in GFT's which have built in scales.
- GFT sums over all discretizations and metrics. But they generate singular graphs.
- We defined the colored GFT's which eliminate the singular graphs and we discovered an unexpected color symmetry.
- We defined a boundary operator and homology groups of colored graphs.

A short program:

- Build a renormalization transformation such that manifold graphs dominate the IR regime.
- Introduce matter fields.
- Define a graph cohomology.

Conclusions

What we have achieved:

- We gave a good discretization procedure of space time manifolds.
- We have organized the discretized models in GFT's which have built in scales.
- GFT sums over all discretizations and metrics. But they generate singular graphs.
- We defined the colored GFT's which eliminate the singular graphs and we discovered an unexpected color symmetry.
- We defined a boundary operator and homology groups of colored graphs.

A short program:

- Build a renormalization transformation such that manifold graphs dominate the IR regime.
- Introduce matter fields.
- Define a graph cohomology.
- Promote the global color symmetry to a local symmetry.

References

arXiv:0907.2582, 0905.3772