Abstracts

PRIMA Conference

July 20-30, 2010

Mini-Course 1

Variational and Ricci flow techniques in the study of positive curvature

Richard Schoen **(Stanford University)**

In this series of four lectures we will focus on methods which have had some success in the study of Riemannian manifolds under positive curvature assumptions. After introducing some basic focus questions and results from Riemannian geometry, we plan to spend the first two lectures on minimal submanifold methods. The final two lectures will focus on Hamilton's Ricci flow method for positive curvature including preservation of certain curvature conditions and convergence results for the flow. We intend to make the lectures accessible for students who have had a course in Riemannian geometry and who have some background in PDE. Descriptions of and references for more specialized background material will be given in the course of the lectures.

Mini-Course 2

Curvature flows in complex geometry

Gang Tian **(Beijing University & Princeton University)**

Mini-Course 3

Dirac Operators on Non-Compact Manifolds

Werner Ballmann (Max Planck Institute for Mathematics & University of Bonn)

In the first part of the course, I will discuss model problems for elementary differential operators. We will encounter related problems in the general case in the later lectures. In the second part, I will introduce Dirac operators and explain some important examples of such operators. After a reminder on the results in the compact case, I will discuss some problems that arise in the case, where the base manifold is non-compact. They are related to the question whether the operator is a Fredholm operator and, if so, what we can say about its index.

Localized Scalar Curvature Gluing

Justin Corvino **(Lafayette College)**

We discuss some results about gluing Riemannian manifolds with the same constant scalar curvature, to produce a new metric with constant scalar curvature that agrees with the original metrics away from the gluing region. An application is the construction of solutions to the Einstein constraint equations which model initial data for the gravitational N-body problem.

Asymptotic behavior of solutions to the σ_k Yamabe equation near isolated singularities

Zheng-Chao Han **(Rutgers University)**

The large genus limit of the infimum of the Willmore energy

Ernst Kuwert **(Albert-Ludwigs-Universität Freiburg)**

The Willmore energy of a surface immersed into $\mathbb{R}^n$ is the integral of its squared mean curvature. It is known that the infimum β_p^n of the Willmore energy among all closed oriented surfaces of genus p in $\mathbb{R}^n$ is attained by a smooth embedded surface, and that $4\pi < \beta_p^n < 8\pi$ for $p \geq 1$. We show that β_p^n converges to 8π as p goes to infinity.

This is joint work with Reiner Schätzle and Yuxiang Li

Parabolic systems with rough initial data

Tobias Lamm (University of British Columbia)

Geometrization of orbifolds via Ricci flow

John Lott (University of California, Berkeley)

A three-dimensional compact orbifold (with no bad suborbifolds) is known to have a geometric decomposition from the work of Perelman along with earlier work of Boileau-Leeb-Porti/Cooper-Hodgson-Kerckhoff. I'll describe a unified proof of the geometrization of orbifolds, using Ricci flow. The emphasis will be on the aspects that are particular to orbifolds. This is joint work with Bruce Kleiner.

Deformations of the hemisphere that increase scalar curvature

Fernando Coda Marques (IMPA)

The following rigidity result is well-known: if g is a metric of nonnegative scalar curvature on Euclidean space, and g coincides with the Euclidean metric outside a compact set, then g is flat. This is a consequence of the celebrated Positive Mass Theorem, proved by Schoen and Yau'79 (for dimensions less than or equal to 7), and by Witten'81 (for spin manifolds).

Inspired by that, Min-Oo proved the analogous rigidity statement for the hyperbolic space in 1989, and later made a conjecture for the spherical setting: if g is a metric of scalar curvature greater than or equal to n(n-1) on the standard hemisphere, and g coincides with the standard metric in a neighborhood of the equator, then g has constant sectional curvature 1. Since then, partial results have been obtained by several people.

In this talk, we will describe our construction of counterexamples to Min-Oo's conjecture in any dimension greater than or equal to three. This is joint work with Simon Brendle and Andre Neves.

An extremal Kähler version of Donaldson-Tian-Yau's Conjecture

Toshiki Mabuchi **(Osaka University)**

In this talk, clarifying the concepts of relative K-stability (by Szekelyhidi) and asymptotic relative Chow stability (by myself), I propose a program to solve an extremal Kähler version of Donaldson-Tian-Yau's Conjecture on the existence of extremal Kähler metrics on polarized algebraic manifolds.

Special holonomy metrics on certain conifolds

Min-Oo Maung **(McMaster University)**

Kobayashi-Hitchin correspondence for D-module

Takuro Mochizuki **(Kyoto University)**

Classical Kobayashi-Hitchin correspondence has provided us interesting interactions between global analysis and algebraic geometry. Rather recently, we obtained a variant connecting global analysis and algebraic analysis, in some sense. Namely, we established the correspondence between polarized wild pure twistor D-modules and semisimple holonomic D-modules. It made us possible to prove a deep result on holonomic D-modules. We would like to give an overview of this story.

A compactness theorem for complete Ricci shrinkers

Reto Mueller (Pisa)

We prove precompactness in an orbifold Cheeger-Gromov sense of complete gradient Ricci shrinkers with a lower bound on their entropy and a local integral Riemann bound. We do not need any pointwise curvature assumptions, volume or diameter bounds. In dimension four, under a technical assumption, we can replace the local integral Riemann bound by an upper bound for the Euler characteristic. The proof relies on a Gauss-Bonnet with cutoff argument. This is a joint work with Robert Haslhofer.

Lower Ricci Curvature, Convexity and Applications

Aaron Charles Naber (Massachusetts Institute of Technology)

We prove new estimates for tangent cones along minimizing geodesics in GH limits of manifolds with lower Ricci curvature bounds. We use these estimates to show convexity results for the regular set of such limits. Applications include the proofs of several conjectures dating back to the work of Cheeger/Colding and the ruling out of certain limit spaces, including the so called generalized trumpet spaces. We construct new examples which exhibit various new behaviors and show sharpness of the new theorems. This work is joint with Toby Colding.

Finite time singularities for Lagrangian mean curvature flow

Andre Neves (Imperial College)

On any compact Calabi-Yau and given any smooth Slag sphere N, I will show the existence of a smooth embedded Lagrangian sphere in the hamiltonian isotopy class of N which develops a finite time singularity under mean curvature flow. This shows that simplified versions of the Thomas-Yau conjecture do not hold.

Regularity results for the mean curvature flow

Natasa Sesum **(University of Pennsylvania)**

We will present several results that extend Huisken's result about extending the flow having the second fundamental form bounded. We show that in the case of a type I singularity the mean curvature controls the flow. We present also some improvements for the surfaces for any kind of singularities.

L^2 curvature flow near the round sphere

Jeff Streets **(Princeton University)**

Given a metric on S^4 with the L^2 norm of the traceless cuvature tensor sufficiently small, we show that the gradient flow of the L^2 norm of curvature exists for all time and converges exponentially to the round metric.

Uniqueness and Nonuniqueness for Ricci flow

Peter Topping **(Warwick Mathematics Institute)**

Yamabe invariants and limits of self-dual hyperbolic monopole metrics

Jeff Viaclovsky **(University of Wisconsin-Madison)**

Consider the self-dual conformal classes on n CP^2 discovered by LeBrun. These depend upon a choice of n points in hyperbolic 3-space, called monopole points.

I will discuss the limiting behavior of various constant scalar curvature metrics in these conformal classes as the points approach each other, or as the points tend to the boundary of hyperbolic space. There is a close connection to the orbifold Yamabe problem (which I will show is not always solvable, in contrast with the case for compact manifolds).