1) Let \( e \in \text{End}_{kG}(k) \) be an idempotent. Show that if \( P \leq G \) is a finite \( p \)-subgroup then \( \text{res}^G_P(e) = 0 \) or 1 and that if \( P' \) is in the same component of \( N(G)/G \) as \( P \) (i.e. you can get from one to the other by a chain of inclusions and conjugations of non-trivial \( p \)-subgroups) then 
\[ \text{res}^G_P(e) = \text{res}^{P'}_P(e). \]
Deduce that the idempotents corresponding to the components of \( N(G)/G \) are primitive.

2) Calculate \( \text{End}_{k\mathbb{Z}/p\mathbb{Z}}(k) \) and \( \text{Aut}_{k\mathbb{Z}/p\mathbb{Z}}(k) \).

3) Calculate \( T(\text{SL}_2(\mathbb{Z})) \) at different primes (\( \text{SL}_2(\mathbb{Z}) \cong \mathbb{Z}_2 \times \mathbb{Z}_2 \)).

4) Calculate \( T(C_2 \times C_2 \times C_2) \).

5) There is an obvious surjection \( C_4 \times C_2 \to \mathbb{Z}/8 \). Calculate the inflation map \( T(C_8) \to T(C_4 \times C_2) \).

6) Calculate \( T(C_2 \times \mathbb{Z}) \) (it is an HNN extension). What happened to the 1-dim representations of \( \mathbb{Z} \)?

7) Calculate \( T(C_2 \times \mathbb{Z} \times \mathbb{Z}) \). Is it finitely generated?

8) Calculate \( T(\mathbb{Z}/p^n) \). (\( \mathbb{Z}/p^n \) means the \( p \)-torsion in \( \mathbb{Q}/\mathbb{Z} \); it acts on a graph with finite stabilizers).

9) For \( G = A \rtimes B \), \( M \) a \( kB \)-module, \( N \leq kB \)-module such that \( M \rtimes_c N \leq_c \) stably, show that \( M \leq M', N \leq N' \) such that \( M \rtimes_c N \leq_c N' \); a genuine isomorphism.

Hint: Make \( M, N \) Gorenstein projective. Let \( F \) be a very big free module for \( G \) and set \( M' = F \oplus F, N' = N \oplus F \), use the Eilenberg trick.

10) Show that any stable automorphism \( \phi: M \to M \) can be realised as a genuine automorphism \( \phi': M' \to N' \).

Hint: take \( M \) Gorenstein projective. Find a projective
Consider $\text{M} \otimes \mathbb{F}^n$ and matrix

$$
\phi' = \begin{bmatrix}
\varphi & e \\
0 & 1 \\
1 & 0 \\
1 & 0 \\
\end{bmatrix}
$$

(This is not strong enough for the construction $\text{E}(\text{M})$ in general, but it suffices for Q6.7 above. You could try and formulate and prove the more general version).

11) Check that $\phi \mapsto C(k, k; \phi)$ is a group homomorphism (or use D).

Hint: use Exercises 3, Q6.

12) Show that if $M$ is endotrivial then the natural map

$$
\text{M} \otimes \mathbb{F}^n \rightarrow \text{End}_k(M)
$$

is a stable isomorphism, hence $\text{End}_k(M) \cong k$ stably.