Exercises 2

1) Show that if G is of type Φ then so is any subgroup. (Show also that type Φ is closed under taking amalgamated free products and HNN extensions, if you know what these mean and how these groups act on graphs)

2) Write out an analogue of the "G acts on a CW-complex..." statement in terms of an exact sequence of certain types of kG-modules.

3) Show that if G acts admissibly on a finite dimensional contractible CW complex with all stabilisers of type Φ and with findim $\leq d$ for some fixed d, then G is of type Φ.

4) If $A \to B \to C$ is exact show

$$\text{projdim}\, B \leq \max\{\text{projdim}\, A, \text{projdim}\, C\}$$
$$\text{projdim}\, A \leq \max\{\text{projdim}\, B, \text{projdim}\, C - 1\}$$
$$\text{projdim}\, C \leq \max\{\text{projdim}\, A + 1, \text{projdim}\, B\}$$

$$\text{projdim}\,(D \oplus E) = \max\{\text{projdim}\, D, \text{projdim}\, E\}$$

5) If G is of type Φ, show

a) If I is injective then $\text{projdim}\, I \leq \text{findim}\, G$

b) If P is projective then $\text{injdim}\, P \leq \text{findim}\, G$.

Hint for (b): i) Show that a projective P is naturally a quotient of $\text{Hom}_k(kG^*, P)$, hence a summand. We can replace P by $\text{Hom}_k(kG^*, P)$ by inj version of (4).

ii) Show that kG^* has a projective resolution Q_* of length $\leq \text{findim}\, G$

iii) Then $\text{Hom}_k(Q_*, P)$ is an injective resolution of $\text{Hom}(kG^*, P)$ once you have shown that each $\text{Hom}_k(Q_i, P)$ is injective.

hint: I is injective $\Leftrightarrow$ $\text{Hom}(-, I)$ is exact.

$$\text{Hom}(A, \text{Hom}(B, C)) \cong \text{Hom}(A \otimes B, C).$$