SCHEDULE

MONDAY: Optimal Transportation

09:30—10:00 *Overview of the conference themes* (coord. Cominetti-Ekeland)
COFFE BREAK
10:30—11:30 McCann
11:30—12:30 Nazaret
LUNCH
14:00—15:00 Buttazzo
15:00—16:00 Queyranne

Presentations
McCann – *Extremal doubly stochastic measures and optimal transportation*
Nazaret – *A new class of transport distances between measures*
Buttazzo – *Optimal location of Dirichlet regions for elliptic PDEs*
Queyranne – *Optimum transportation: the discrete optimization point of view*

TUESDAY: Networks and Congestion

09:00—10:00 Carlier
COFFE BREAK
10:30—11:30 Silva
11:30—12:30 Baillon
LUNCH
14:00—15:00 Cominetti
15:00—16:00 Sorin
16:15—17:15 **OPEN SESSION**: *Alternative optimal transport models* (coord. Buttazzo-Carlier-McCann)

Presentations
Carlier – *Wardrop equilibria in a continuous framework, theoretical & numerical aspects*
Silva – *Continuum equilibrium for massively dense ad-hoc networks*
Baillon – TBA
Cominetti – *Equilibrium and learning in traffic network games*
Sorin – TBA

WEDNESDAY: Equilibrium and Hedonic Models

09:00—10:00 Ekeland
COFFE BREAK
10:30—11:30 L'Espérance
11:30—12:15 Lachapelle
12:15—13:15 **OPEN SESSION**: *Learning dynamics in repeated games* (coordinator Sorin)
LUNCH

AFTERNOON: Guided Visit to Anthropology Museum (UBC)

Presentations
Ekeland – TBA

L'espérance – *Application of optimal transportation to equilibrium in hedonic markets*
Lachapelle – *Optimal transportation and optimal control in a finite horizon framework*

THURSDAY: Matching/Optimal allocation

09:00—10:00 Queyranne
COFFE BREAK
10:30—11:30 Piazza
11:30—12:30 Peypouquet
LUNCH
14:00—15:00 Figueroa
15:00---16:00 Santambrogio
16:15—17:15 **OPEN SESSION**: *Optimal transport and equilibrium in economics* (coord. McCann-Ekeland)

Presentations
Queyranne – *Monotone parametric Min Cuts revisited: structures and algorithms*
Piazza – *Optimal dynamic allocation of a fixed resource*
Peypouquet – *Alternating prox algorithm for hierarchical selection of optimal points*
Figueroa – *Mechanism design and applications to auctions and networks*
Santambrogio – *A transport approach to the Hotelling problem and other applications of Kantorovich potentials derivatives*

FRIDAY: Identification/Statistics/Decision Theory

09:00—10:00 Chernozhukov
COFFE BREAK
10:30—11:30 Galichon
11:30—12:30 Pioner
LUNCH
14:00—15:00 Henry
15:15—16:15 **OPEN SESSION**: *Optimal transport in statistics and econometrics* (coord. Chernozhukov-Galichon-Henry)

Presentations
Chernozhukov – TBA
Galichon – *Risk aversion and optimal transportation*
Pioner – *Semiparametric identification of multidimensional screening models*
Henry – *Optimal transportation and inference in models with multiple equilibria*

ABSTRACTS

Giuseppe Buttazzo (U. di Pisa)
Optimal location of Dirichlet regions for elliptic PDEs

We consider an elliptic problem in a given domain W and a given right hand side *f*. The Dirichlet region is the unknown of the problem and has to be chosen in an optimal way, in order to minimize a cost functional, and in a class of admissible choices. The cost we consider is the compliance functional and the class of admissible choices consists either of all sets of *N* points or of one-dimensional connected sets (networks) of a given length *L*. Then we let *N* (respectively *L*) tend to infinity and look for the asymptotical distribution of the Dirichlet regions. The asymptotically optimal shapes are discussed as well and links with average distance problems are provided.

Jean-Bernard Baillon (U. Paris 1)
TBA

Guillaume Carlier (U. Paris-Dauphine)
Wardrop equilibria in a continuous framework: theoretical & numerical aspects

We'll present an optimal transportation problem with congestion effects and will relate it to the concept of Wardrop equilibrium. Alternative and dual formulations will also be considered. We will also describe a numerical consistent scheme and present some simulations.

Victor Chernozhukov (MIT)
Improving Point and Interval Estimates of Monotone Functions by Rearrangement

Suppose that a target function $f_0: \Bbb{R}^d \to \Bbb{R}$ is monotonic, namely, weakly increasing, and an original estimate $\hat f$ of this target function is available, which is not weakly increasing. Many common estimation methods used in statistics produce such estimates $\hat f$. We show that these estimates can always be improved with no harm using rearrangement techniques: The rearrangement methods, univariate and multivariate, transform the original estimate to a monotonic estimate $\hat f^*$, and the resulting estimate is *closer* to the true curve f_0 in common metrics than the original estimate $\hat f$. The improvement property of the rearrangement also extends to the construction of confidence bands for monotone functions. Let ℓ and u be the lower and upper endpoint functions of a simultaneous confidence interval $[\ell, u]$ that covers f_0 with probability 1-a, then the rearranged confidence interval $[\ell^*, u^*]$, defined by the rearranged lower and upper end-point functions ℓ^* and u^*, is shorter in length in common norms than the original interval and covers f_0 with probability greater or equal to 1-a. We further illustrate these theoretical results with a computational example and an empirical example dealing with age-height growth charts.

Joint work with Ivan Fernandez-Val and Alfred Galichon

Roberto Cominetti (U. de Chile)
Equilibrium and learning in traffic network games

Traffic in congested networks is often described as an equilibrium or steady state that emerges from the adaptive behavior of drivers. While this tends to be confirmed in experiments, traditional concepts such as Wardrop or Stochastic User Equilibrium are formulated as *static* equilibrium equations which are not tied to an underlying adaptive dynamics. The notion of Markovian equilibrium incorporates some dynamical features by assuming that passengers proceed to their destination through a sequential process of arc selection based on a random discrete choice model at every intermediate node: route selection is the outcome of a sequential decision process while network flows are the invariant measures of the corresponding Markov chain. Despite its dynamical flavor, this notion remains essentially static and not tied to an adaptive behavior of players.

After reviewing the static equilibrium concepts and a recent application in the context of internet traffic (TCP/IP control protocol), we discuss a discrete time stochastic process that represents a plausible model for the adaptive behavior of finitely many users in a simple traffic network. The dynamics are based on a minimal piece of information: each player observes only the travel time for the specific route chosen on any given day, and future decisions depend on the history of past individual observations. Since travel times depend on the congestion imposed collectively by all the player's decisions, the process progressively reveals to each player the congestion in the network. In the long run, the temporal evolution of the dynamics lead the system to coordinate on a steady state that may be characterized as a Nash equilibrium for a particular steady state game, providing a unified framework in which a stochastic model for user behavior gives rise to a continuous dynamical system and leads ultimately to a consistent notion of equilibrium.

Joint work with: J-B. Baillon, E. Melo and S. Sorin

Ivar Ekeland (U. British Columbia)
TBA

Nicolas Figueroa (U. de Chile)
Mechanism design and applications to auctions and networks

We present a tutorial on mechanism design, highlighting some open problems at the interface of mechanism design, optimal control and networks.

Alfred Galichon (E. Polytechnique)
Risk aversion and optimal transportation

Marc Henry (U. of Montreal)
Optimal transportation and inference in models with multiple equilibria

The empirical study of game theoretic models is complicated by the presence of multiple equilibria, which generally leads to a failure of identification of the structural parameters governing the model. In the existing literature, identification of structural parameters is achieved through equilibrium refinements, shape restrictions, informational assumptions or the specification of equilibrium selection mechanisms. An alternative approach is to eschew identification strategies and base inference purely on the identified features of the models with multiple equilibria, which are sets of values rather than a single value of the structural parameter vector. Call identified set the set of parameter values compatible with the true distribution of observed data. We propose to develop efficient ways to compute this identified set and confidence regions for the latter based on an equivalent formulation of the identified set as the set of solutions to an minimum cost transportation problem, where what is transported is probabilities of observable equilibrium outcomes to probabilities of predicted combinations of equilibria. In the case of discrete games, the dual of the minimum cost transportation problem is a maximum flow problem, for which there exist many efficient algorithms and implementations. These efficient algorithms apply directly to the deterministic problem of computing the identified set of parameter values in models with multiple equilibria in the ideal situation where the probability of observable outcomes is known. When the true probability of outcomes is not known and needs to be estimated from a sample of observations, the deterministic problem needs to be relaxed (to increase the flow in the the network) in order to ensure that a value of the parameter selected by the procedure belongs to the identified set with a pre-determined probability. Given the efficiency of maximum flow algorithms that exist in the literature, this procedure will be applied to models of interaction with realistic parameter dimensions, which is not possible now.

Joint work with A. Galichon.

Aimé Lachapelle (U. Paris-Dauphine)
Optimal transportation and optimal control in a finite horizon framework

We use optimal transportation problems in a continuous time period, with stochastic diffusion or optimal control, to modelize commitments in a micro-labour market. We show that the stochastic case corresponds to the Monge-Kantorovich problem and we give the dual formulation and some optimality conditions for the deterministic case with optimal control.

Bruno L'Espérance (U. British Columbia)
Application of optimal transportation to equilibrium in hedonic markets

We prove the existence of equilibrium in hedonic markets when consumers and producers have utility functions separable with respect to the price. Throughout this paper we consider the particular setting of a labor market. In this model, the workers have to choose their level of consumption of good and to choose which jobs they want to execute. We allow the workers to stay out of the labor market and get their reservation utility. On the other hand, the employers need to

decide which jobs they want to get executed and how many workers they want to hire to execute theses jobs. In our model, a job is a multidimensional vector of characteristics. We consider the equilibrium in the labor market only and take the price of the good as given, indeed the model can be seen as partial equilibrium model. The main results are obtained using standard tools of functional analysis, convex analysis and optimal transportation.

Robert J. McCann (U. of Toronto)
Extremal doubly stochastic measures and optimal transportation

Square N by N matrices with non-negative entries whose columns and rows all sum to one form a convex set with N! vertices. Each vertex corresponds to a permutation on N letters, according to a theorem of Birkhoff and von Neumann.

In 1948 Birkhoff asked what the infinite dimensional analog of this theorem should be. We survey related developments geared towards finding conditions on the support of a measure on the square which characterize extremality among non-negative measures with the same x- and y-marginals. We use such conditions to derive new sufficient conditions for uniqueness of minimizer in Kantorovich's optimal transportation problem --- especially for smooth costs on compact manifolds, when Monge's problem has no solution, meaning the optimizer does not generally concentrate its mass on the graph of a single map. Our sufficient conditions for uniqueness are expressed in terms of the topology (Morse structure) of the transportation cost.

Bruno Nazaret (U. Paris-Dauphine)
A new class of transport distances between measures

We introduce a new class of distances between nonnegative Radon measures in Rd. They are modeled on the dynamical characterization of the Kantorovich-Rubinstein-Wasserstein distances proposed by BENAMOU-BRENIER and provide a wide family interpolating between the Wasserstein and the homogeneous W-1,p-Sobolev distances. From the point of view of optimal transport theory, these distances minimize a dynamical cost to move a given initial distribution of mass to a final configuration. An important difference with the classical setting in mass transport theory is that the cost not only depends on the velocity of the moving particles but also on the densities of the intermediate configurations with respect to a given reference measure. We study the topological and geometric properties of these new distances, comparing them with the notion of weak convergence of measures and the well established Kantorovich-Rubinstein-Wasserstein theory. An example of possible applications to the geometric theory of gradient flows is also given

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Juan Peypouquet (UTFSM)
Alternating prox algorithm for hierarchical selection of optimal points

We apply an alternating proximal point algorithm to approach the set

$$\text{Argmin} \{ F(x) : x \in \text{Argmin}(Y) \}$$

If the function Y represents a collective cost and F is the individual cost, the algorithm can be interpreted as the actions of a player (or a set of players) who alternatively tries to minimize his own and the collective costs giving more and more importance to the latter at each iteration.

Adriana Piazza (CMM, U. de Chile).
Optimal dynamic allocation of a fixed resource

A fixed resource (e.g. land) must be optimally allocated to a finite set of activities (e.g. different forest species) over an infinite time horizon. We briefly review same of the most used models and main results known up to date (in a discrete time framework) and present in particular the case where each activity blocks the resource for a specific number of periods after which is liberated and can be re-allocated to a different activity. We present a discrete-time optimal control model and discuss the asymptotic behavior of the optimal solutions both in the discounted and undiscounted settings. After comparing with the classical results for the "undiscounted" or "weakly discounted" cases, we put forward a number of questions on the possible extension to other "structured" optimal dynamic allocation problems in economics. Are these results also valid for continuous-time models?

Heleno Pioner (U. Chicago)
Semiparametric identification of multidimensional screening models

This paper considers the identification of a class of models for the pricing of differentiated products. It shows how one can use the information presented in the characteristics of the consumers, in the attributes of the goods being sold by a multiproduct monopolist and in the theoretical predictions of the model to recover preference and cost parameters. The monopolist optimally groups consumers with different characteristics by offering them the same good. By using consumers in this bunching region, one can infer the distribution of unobserved attributes, such as quality.

Maurice Queyranne (Sauder School of Business at UBC)
Optimum transportation: the discrete optimization point of view.

Transportation problems, usually credited to Kantorovich, Hitchcock and Koopmans, form one of the bases on which discrete optimization has developed.

We review the main results on solving transportation problems with finite sets of origins and destinations, and more general minimum cost network flow problems. We also review related computational complexity results on single-sourcing problems, i.e., under the Monge restriction.

On the other hand, so-called “Monge transportation problems” are a special class of discrete transportation problems with costs defined by a submodular function, which can be solved very easily by a simple greedy algorithm.

We show that this property extends to certain multidimensional transportation problems with three or more sets of elements (not just origins and destinations) to be connected, and to related search and optimization problems in Monge arrays.

Maurice Queyranne (Sauder School of Business at UBC)
Monotone parametric minimum cuts revisited: structures and algorithms

We consider the minimum cut problem, one of the fundamental network optimization problems, when capacities depend on a parameter.

Some classes of this parametric min cut problem are known to have the nice structural (monotone comparative statics) property that min cuts are nested in the parameter, and the nice algorithmic property that all min cuts can be computed in the same asymptotic time as a single min cut. We extend these results in several directions: we describe much more general classes of problems for which these two nice properties continue to hold, and we extend other results with the same flavor as well.

Joint work with: Frieda Granot and S. Thomas McCormick (Sauder School of Business at UBC), and Fabio Tardella (Universitá La Sapienza, Rome, Italy).

Filippo Santambrogio (U. Paris-Dauphine)
A transport approach to the Hotelling problem and other applications of Kantorovich potentials derivatives

The Hotelling problem is a double-step equilibrium problem for the strategic location of *N* firms trying to maximize their incomes from a given distribution m of consumers, by properly chosing prices and positions in a domain W. It is interesting to focus the attention on the unknown demand distribution on firms, which is an atomic measure n with *N* dirac masses in W. Writing down the optimality conditions in terms of n involves the Kantotovich potential in the transport from m to n and its derivatives with respect to variations of n. Some new relations can be obtained through this method but much more could be obtained if one knew how to compute these Kantorovich potentials derivatives. This is the case for other problems as well, and I will address possible applications of this kind of results.

Alonso Silva (U. de Nice)
Continuum equilibrium for massively dense ad-hoc networks

Computing optimal routes in massively dense ad-hoc networks becomes intractable as the number of nodes becomes very large. One recent approach to solve this problem is to use a fluid type approximation in which the whole network is replaced by a continuum plain. Various paradigms from physics have been used recently in order to solve the continuum model.

We consider massively dense ad-hoc networks and study their continuum limits as the node density increases and as the graph providing the available routes becomes a continuous area with location and congestion dependent costs. We study both the global optimal solution as well as the non-cooperative routing problem among a large population of users where each user seeks a path from its source to its destination so as to minimize its individual cost. We seek for a (continuum version of the) Wardrop equilibrium. We first show how to derive meaningful cost models as a

function of the scaling properties of the capacity of the network as a function of the density of nodes. We present various solution methodologies for the problem: (1) the viscosity solution of the Hamilton-Bellman-Jacobi equation, for the global optimization problem, (2) a method based on Green Theorem for the least cost problem of an individual, and (3) a solution of the Wardrop equilibrium problem using a transformation into an equivalent global optimization problem.

Sylvain Sorin (U. Paris 6)
TBA