Submittee: Charles Doran
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Title: NUMERICAL RICCI FLOW IN COMPUTER SCIENCE, GEOMETRY, AND PHYSICS
Event Type: Conference-Workshop

Location:
UNIVERSITY OF BRITISH COLUMBIA

Dates:
JULY 14-16, 2011

Topic:
This was the first-ever workshop to bring together researchers from Pure and Applied Mathematics, Computer Science, and Theoretical Physics who are exploring applications of (numerical) Ricci flow in their disciplines.

Methodology:
The first day consisted of talks aimed at introducing each of the main research themes of the conference. The subsequent days consisted of research talks (each one hour long).

Organizers:
Charles Doran, University of Alberta (doran@math.ualberta.ca) // David Gu, Stony Brook University (gu@cs.stonybrook.edu) // Robert Gulliver, University of Minnesota (gulliver@math.umn.edu) // Suneeta Vardarajan, IISER Pune (suneeta@iiserpune.ac.in)

Speakers:
Thursday, July 14 9:00 - 10:00 Speaker: David Gu (SUNY Stony Brook) Title: Ricci Flow in Engineering Fields // 10:00 - 11:00 Break // 11:00 - noon Speaker: Huai-Dong Cao (Lehigh U.) Title: Singularity formation in the Ricci Flow // noon - 2:00 Lunch // 2:00 - 3:00 Speaker: Mauro Carfora (U. of Pavia, Italy) Title: Ricci Flow: A Theoretical Physics Perspective // 3:00 - 4:00 Break // 4:00 - 5:00 Speaker: Adam Oberman (SFU) Title: Introduction to Numerical Methods for Nonlinear Elliptic Partial Differential Equations. // Friday, July 15 9:00 - 10:00 Speaker: Dan Knopf (U. of Texas at Austin) Title: Asymptotics of degenerate Ricci flow neckpinches // 10:00 - 11:00 Break // 11:00 - noon Speaker: David Glickenstein (U. of Arizona) Title: Scalar curvature on piecewise flat manifolds and other topics // noon - 2:00 Lunch // 2:00 - 3:00 Speaker: Warner Miller (Florida Atlantic U.) Toward Ricci Flow in Higher Dimensions using Regge Calculus // 3:00 - 4:00 Break // 4:00 - 5:00 Speaker: Guoyi Xu (U. of California at Irvine) Title: Short-time existence of the Ricci flow on noncompact Riemannian manifolds // 5:00-6:00pm Speaker: Jack Gegenberg (UNB) Title: Yang-Mills Flow and Uniformization // 6:00-7:00pm Speaker: James Isenberg (University of Oregon) Title: Ricci Flow on Complete Surfaces // 9:00 - 10:00 Speaker: Ken Stephenson (U. of Tennessee) Title: Curvature Flow via Circle Packing // 10:00 - 11:00 Break // 11:00 - noon Speaker: Mauro Carfora (U. of Pavia, Italy) Title: Ricci flow conjugation and Initial

data sets for Einstein Equations // noon - 2:00 Lunch // 2:00 - 3:00 Speaker Albert Chau (UBC) Title: The complex parabolic Monge Ampere equation and applications // 3:00 - 4:00 Break + poster session // 4:00 - 5:00 Speaker: Huai-Dong Cao (Lehigh U.) Title: Geometry of gradient Ricci solutions // 5:00 - 6:00pm Speaker: Ali Nassar (U. of Lethbridge) Title: Energy functionals for Calabi-Yau metrics // Abstracts ::: (alphabetical) // Numerical Ricci Flow in Computer Science, Geometry, and Physics Mauro Carfora (two seminars): (U. of Pavia, Italy) Expository seminar Title: Ricci Flow: A Theoretical Physics Perspective Abstract: I will touch upon some aspects of the connection between Ricci flow and theoretical physics. Research seminar title: Ricci flow conjugation and Initial data sets for Einstein Equations Abstract: We discuss a natural form of Ricci flow conjugation between two distinct general relativistic data sets given on a compact manifold. // Huai-Dong Cao (Lehigh University) Expository Talk: Singularity formations in the Ricci flow Abstract: This is an expository talk on singularity formation of the Ricci flow on 3-manifolds. Research talk: Geometry of gradient Ricci solutions Abstract: Ricci solutions are natural extensions of Einstein metrics. There are special solutions to Hamiltonâ€™s Ricci flow and arise as possible singularity models. In this talk we shall present some recent progress on classifications of gradient steady and shrinking Ricci solitons. // Albert Chau (University of British Columbia) The complex parabolic Monge Ampere equation and applications Abstract. In this talk I will introduce a class of complex parabolic Monge Ampere equations, then discuss applications to the Kahler Ricci flow on non compact Kahler manifolds. A review of some results for Kahler Ricci in this context will also be given. // Jack Gegenberg (University of New Brunswick) Yang-Mills Flow and Riemannian Geometry Abstract: I will explore the idea of using the Yang-Mills flow to examine uniformization of 2D manifolds. We build the gauge theory connection from the topology of the manifold, and identify the components of the gauge potential with a frame-field and a linear connection on the manifold. I will describe both analytic and numerical solutions to the flow, and discuss its consistency with the uniformization theorem for closed compact 2D manifolds. I will speculate on extending this to higher dimensions. // David Glickenstein: (U. of Arizona) Scalar curvature on piecewise flat manifolds and other topics Abstract: One route to a numerical Ricci flow is to construct spaces that admit nice finite elements, such as triangulated piecewise flat manifolds. Such spaces admit natural scalar curvature measures, proven to converge to scalar curvature measure on a Riemannian manifold by Cheeger, Muller, and Schrader (as conjectured by T. Regge). Using variation of geometric functionals, these scalar curvature measures lead to discrete Yamabe flows (and hence Ricci flows in 2D). We will discuss some of the relationships between these scalar curvatures, conformal variations, circle packings, graph Laplacians, and the Einstein equation. Time permitting, we will also discuss some additional topics relevent to the conference: visualizing abstract piecewise flat manifolds and embedding abstract geometric flows. // David Gu: (SUNY Stony Brook) Ricci Flow in Engineering Fields Abstract: Ricci flow is a powerful tool for designing Riemannian metrics by prescribing curvatures, which plays an fundamental role in many engineering fields. In this talk, we summarize computational algorithms for surface Ricci flow and its main applications, which include surface registration in medical imaging, surface parameterization in computer graphics, 3D face recognition and shape retrieval in computer vision, spline construction and fitting in geometric modeling and design, efficient routing in wireless sensor network and so on. // Dan Knopf : (U. of Texas at Austin) Asymptotics of degenerate Ricci flow neckpinches Abstract: Formation of degenerate (Type-II) Ricci flow neckpinches was studied numerically by Garfinkle and Isenberg in 2005--8. An existence proof for the formation of such singularities from nongeneric rotationally symmetric initial data was supplied by Gu and Zhu in 2008. I will report on recent joint work with Angenent and Isenberg that provides a precise asymptotic profile of these singularities. // Warner Miller (Florida Atlantic U.) Toward Ricci Flow in Higher Dimensions using Regge Calculus Regge calculus (RC) was introduced in 1961 in a paper by T. Regge entitled â€œGeneral relativity without coordinates.â€• This discrete geometry approach provides a beautiful foundation for expressing Einsteinâ€™s geometric theory of gravity on a 4-dimensional simplicial lattice. In RC the geometry within each of the simplicial blocks is assumed to be flat, and curvature is concentrated on its co-dimension two elements. Since its introduction, RC has subsequently developed into a full dynamical theory for gravitation and is used today for problems in both classical and quantum gravity. In this lecture, we will examine how curvature is expressed in a d-dimensional simplicial lattice in and provide explicit expressions for them. In particular, we will apply the RC approach to define the Ricci tensor, and to examine the structure of the Ricci flow equations on a simplicial lattice of arbitrary dimension. // Ali Nassar: (U. of Lethbridge) Energy functionals for Calabi-Yau metrics Abstract: We identify a set of â€œenergyâ€• functionals on the space of metrics in a given Kaehler class on a Calabi-Yau manifold, which are bounded below and minimized uniquely on the Ricci-flat metric in that class. Using these functionals, we recast the problem of numerically solving the Einstein equation as an optimization problem. We apply this strategy, using the â€œalgebraicâ€• metrics (metrics for which the Kaehler potential is given in terms of a polynomial in the projective coordinates), to the Fermat quartic and to a one-parameter family of quintics that includes the Fermat and conifold quintics. We show that this method yields approximations to the Ricci-flat metric that are exponentially accurate in the degree of the polynomial (except at the conifold point, where the convergence is polynomial), and therefore orders of magnitude more accurate than the balanced metrics, previously studied as approximations to the Ricci-flat metric. The method is relatively fast and easy to implement. On the theoretical side, we also show that the functionals can be used to give a heuristic proof of Yauâ€™s theorem. // Adam Oberman (Simon Fraser University) Introduction to Numerical Methods for Nonlinear Elliptic Partial Differential Equations. Abstract: Nonlinear elliptic and parabolic partial differential equations (PDEs) appear in geometric flows, image processing, and mathematical finance. The theory of viscosity solutions has been enormously successful in addressing the problems of existence, uniqueness, and stability for a wide class of such equations. A problem which has not been addressed with as much success is the construction of solutions. In some cases, exact solutions formulas exist, but for the most part, solutions must be found numerically. Viscosity solutions are the correct class of weak solutions for these types of PDE. Correspondingly, monotone methods are the correct class of numerical schemes. We introduce a framework for building monotone schemes which converge to the viscosity solution. This framework allows explicit nonlinear finite difference schemes to be built. In particular we address how to handle: obstacle problems, degeneracy, and singularities in the solutions. We will discuss a number of geometric PDEs which can be solved using Wide Stencil finite difference schemes: Monge-Ampere, Convex Envelope, Infinity Laplace, Mean Curvature, the Porous Medium Equation, and others. In this introductory lecture, Iâ€™ll show how these finite difference schemes can be constructed and implemented. // Ken Stephenson: (U. of Tennessee) Curvature Flow via Circle Packing Abstract: In â€œcircle packingâ€•, it helps to view the circles as imposing a geometry on combinatorial situations. Typically the combinatorics are triangulations and the geometry of â€œpackedâ€• circles is conformal in nature. But circle packing is very maleable: * General combinatorics can be modified to triangulations -even, perhaps, preserving some initial geometry. * Circles need not be â€œpackedâ€•: generic radii impose cone structures. * We all know and understand circles! * Software â€œCirclePackâ€• permits sophisticated and open-ended experimentation, manipulation, and visualization of circle configurations. The computation of circle packing radii itself has been modelled as a discrete Ricci flow by Bennett Chow and Feng Luo. Through serendipity, experiments have uncovered a more mysterious â€œcurvature flowâ€•. The talk is an invitation to consider this approach in studying surface parameterizations. // Guoyi Xu: (U. of California at Irvine) Short-time existence of the Ricci flow on noncompact Riemannian manifolds Abstract: In this talk, using the local Ricci flow, we prove the short-time existence of the Ricci flow on noncompact manifolds, whose Ricci curvature has global lower bound and sectional curvature has only local average integral bound. The short-time existence of the Ricci flow on noncompact manifolds was studied by Wan-Xiong Shi in 1990s, who required a point-wise bound of curvature tensors. As a corollary of our main theorem, we get the short-time existence part of Shiâ€™s theorem in this more general context. //

Links:

Comments / Miscellaneous:

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