Zonal Jets, Dipole EOFs, and Annular Modes

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  - zonal winds: zonal index, "jet shift"
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- Obtained as EOFs, often treated as interchangeable
- Present study: how much of all of this can be understood from the kinematics of a fluctuating jet (without invoking complex dynamics)?

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Empirical Orthogonal Functions (aka PCA)

- Covariance matrix of field $u(x,t)$

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- EOFs orthogonal, PCs uncorrelated

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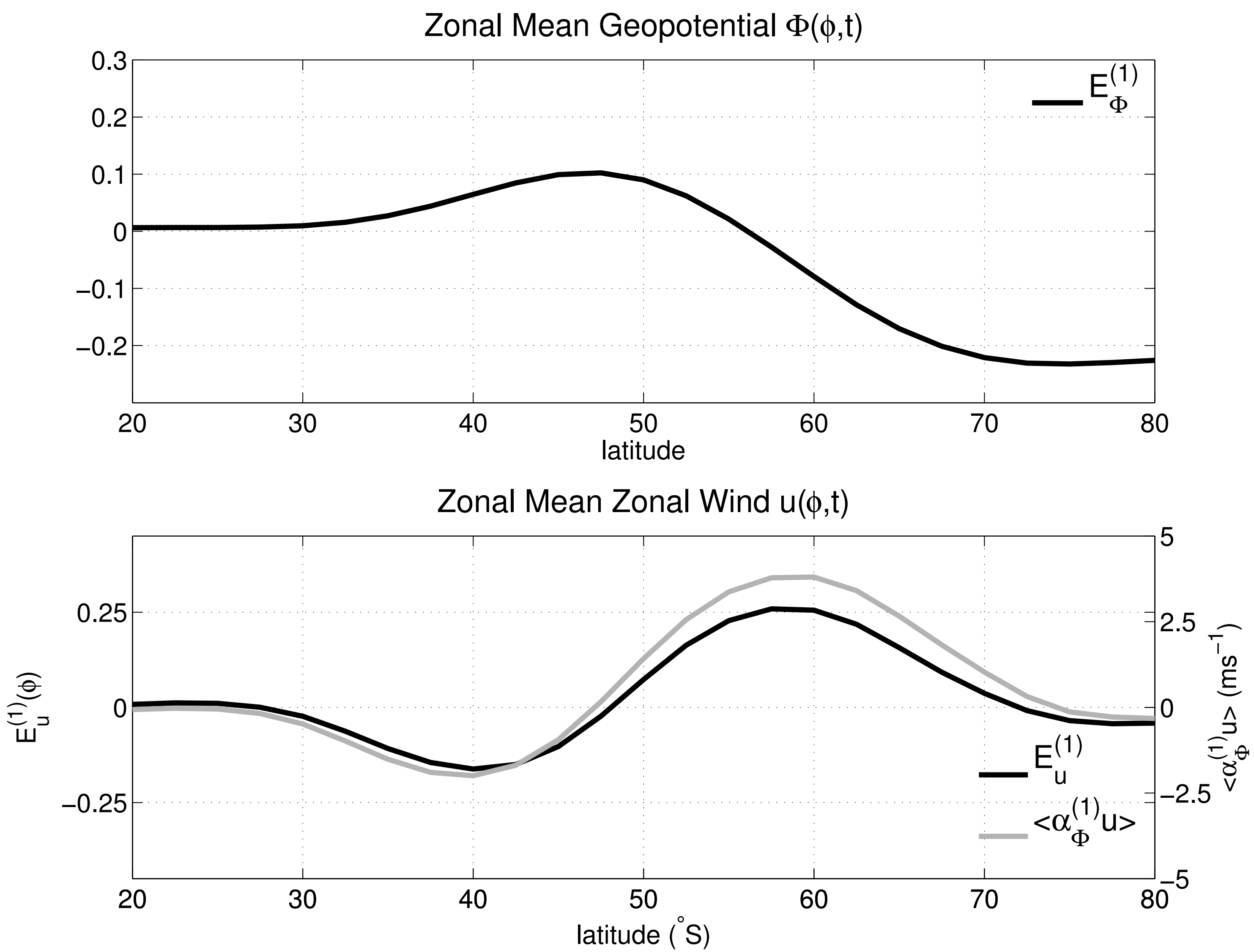
Observed EOFs: Zonal Index and Annular Mode



The Idealised Zonal Jet

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$$u(x,t) = U(t)\mathcal{F}\left(\frac{x - x_c(t)}{\sigma(t)}\right)$$

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	$U(t) = U_0(1 + l\xi(t))$	jet strength
where	$x_c(t) = h\lambda(t)$	jet position
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(where second term imposes mass conservation)

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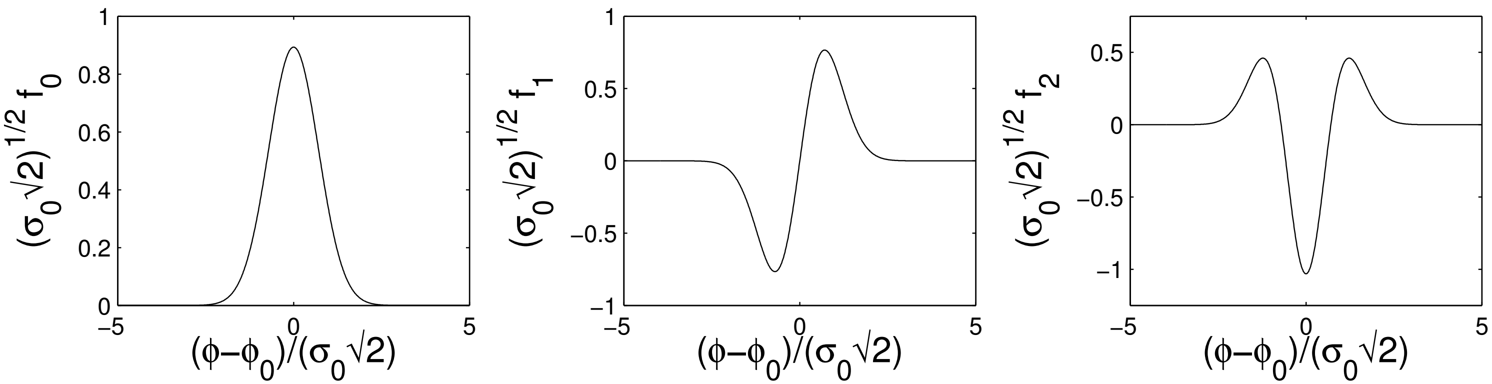
  F_0 is a "monopole", F_1 a "dipole", and F_3 a "tripole"



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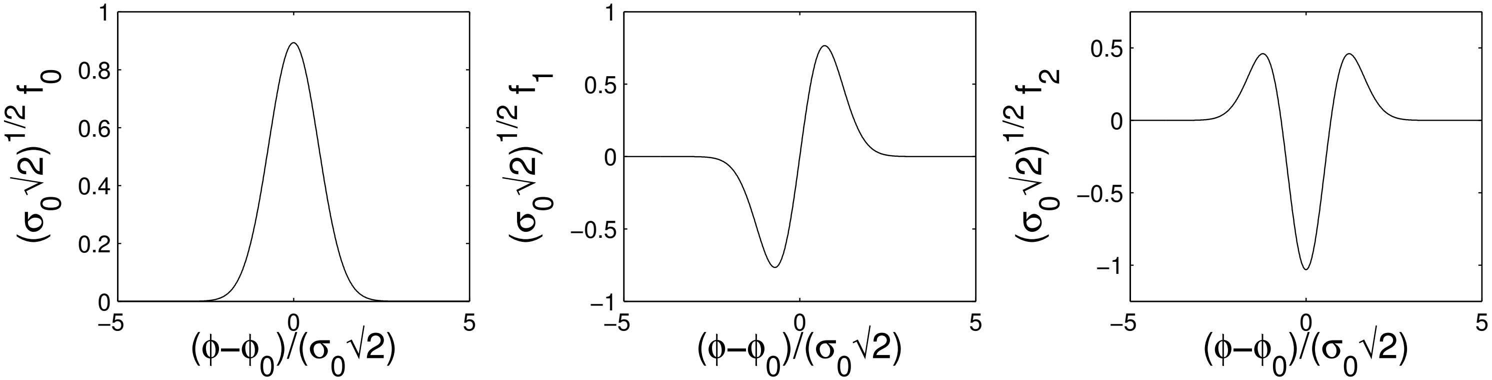

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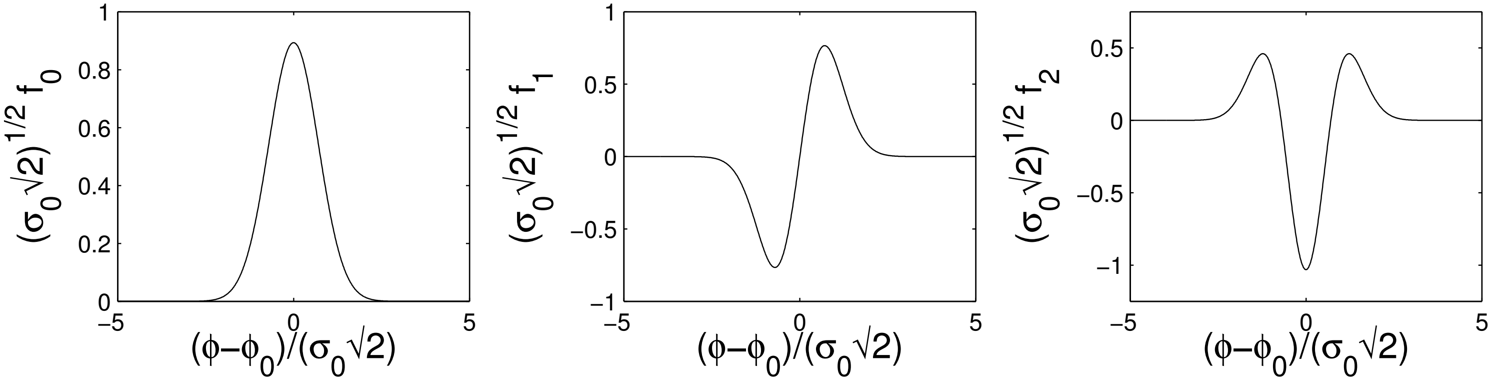

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- By definition:

$$\frac{d}{dx}F_j(x) = \frac{N_{j+1}}{N_j}F_{j+1}(x)$$

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$$u'(x,t) = u(x,t) - \mathsf{E}\{u(x,t)\} = U_0 N_1 h\lambda F_1(x) + \frac{1}{2} N_2 U_0 (h\lambda)^2 F_2(x) + \ldots$$

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and so

$$\begin{aligned} C(x,x') &= U_0^2 h^2 N_1^2 F_1(x) F_1(x') + \frac{U_0^2}{2} N_1 N_2 h^3 s_\lambda [F_1(x) F_2(x') + F_2(x) F_1(x')] \\ &+ \frac{U_0^2}{4} N_2^2 h^4 (\kappa_\lambda + 3) F_2(x) F_2(x') + \ldots \end{aligned}$$

(where s_λ, κ_λ skewness and kurtosis of λ)

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- $s_\lambda \neq 0$ couples these basis functions in EOFs

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- Leading PC time series couple position & strength fluctuations:

$$
\alpha_u^{(1)}(t) \sim (U_0 + \xi(t))\lambda(t) + h.o.t.
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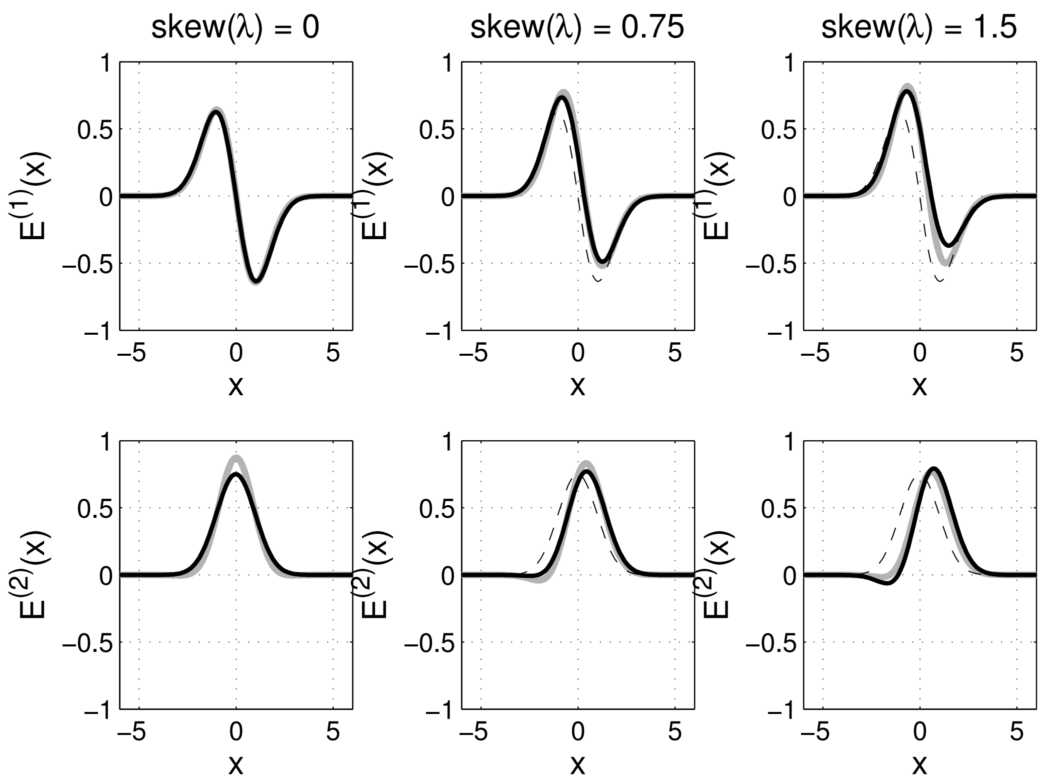
- For Gaussian jet with $h = 0.3$, $l = 0.185$



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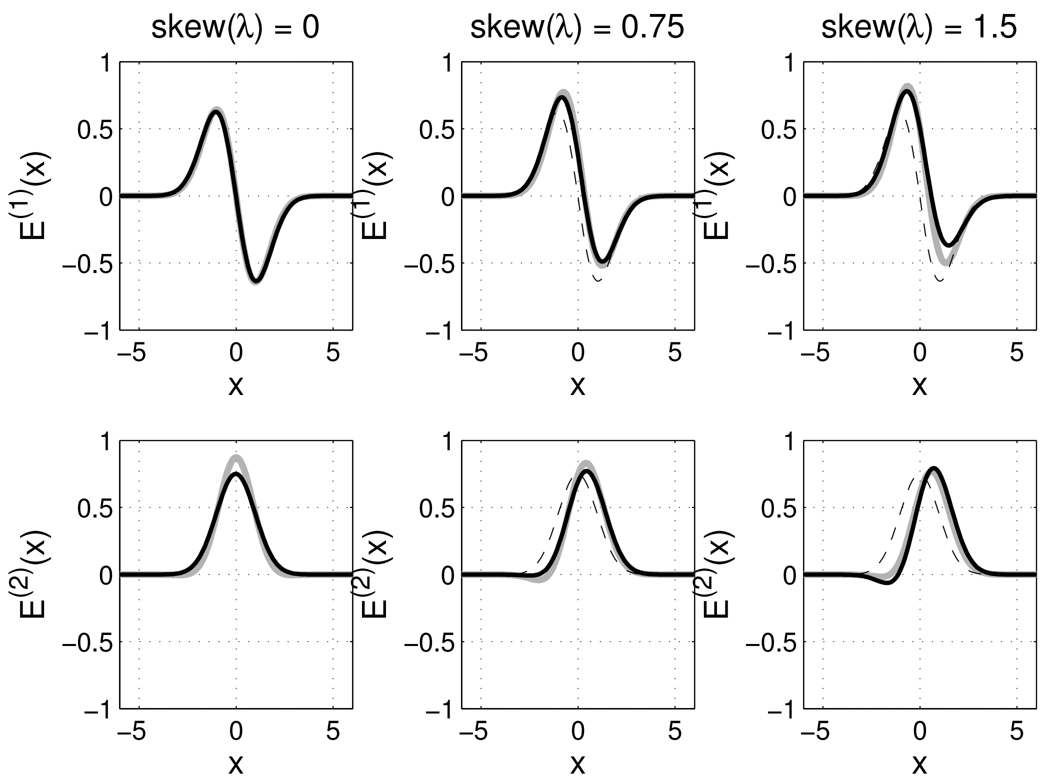

- Skewness in $\lambda \Rightarrow$ EOFs asymmetric around jet axis; dipole still dominates even for strong skewness

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- For position, strength fluctuations of comparable width, coupling will be strong

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  - position fluctuations stronger than width
- That these fairly general facts are characteristic of the tropospheric jet in models & observations $\Rightarrow$ explanation for dipole as generic feature of zonal wind EOFs
- While dipole *arises* because of position fluctuations, the associated EOF mode bundles together variability in all jet degrees of freedom

EOFs of Dynamically Related Fields

- Geopotential related to zonal wind through linear transformation

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- Consider $\mathbf{y} = L\mathbf{x}$; covariances related by

$$C_{yy} = LC_{xx}L^T$$

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$$\Phi(x,t) = -\int_{x_1}^{x} f(x')u(x',t)\,dx' + \int_{x_1}^{x_2}\left(\int_{x_1}^{x} f(x')u(x')dx'\right)\mu(x)\,dx$$

- Dynamically related but distinct fields will not generally have same EOFs
- Consider $\mathbf{y} = L\mathbf{x}$; covariances related by

$$C_{yy} = LC_{xx}L^T$$

- EOF decomposition of $\mathbf{x}$ $\quad C_{xx} = \mathcal{U}\Lambda\mathcal{U}^T$

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so

$$C_{yy} = (L\mathcal{U})\Lambda(L\mathcal{U})^T$$

and EOFs of $\mathbf{y}$ only EOFs of $\mathbf{x}$ if rows of $L\mathcal{U}$ orthogonal (which they won't be in general)

Geopotential EOFs

- Can expand $u'(x,t) = u(x,t) - \langle u(x) \rangle$ over EOF basis:

$$u'(x,t) = \sum_{j=1}^{J} \alpha_u^{(j)}(t) E_u^{(j)}(x)$$

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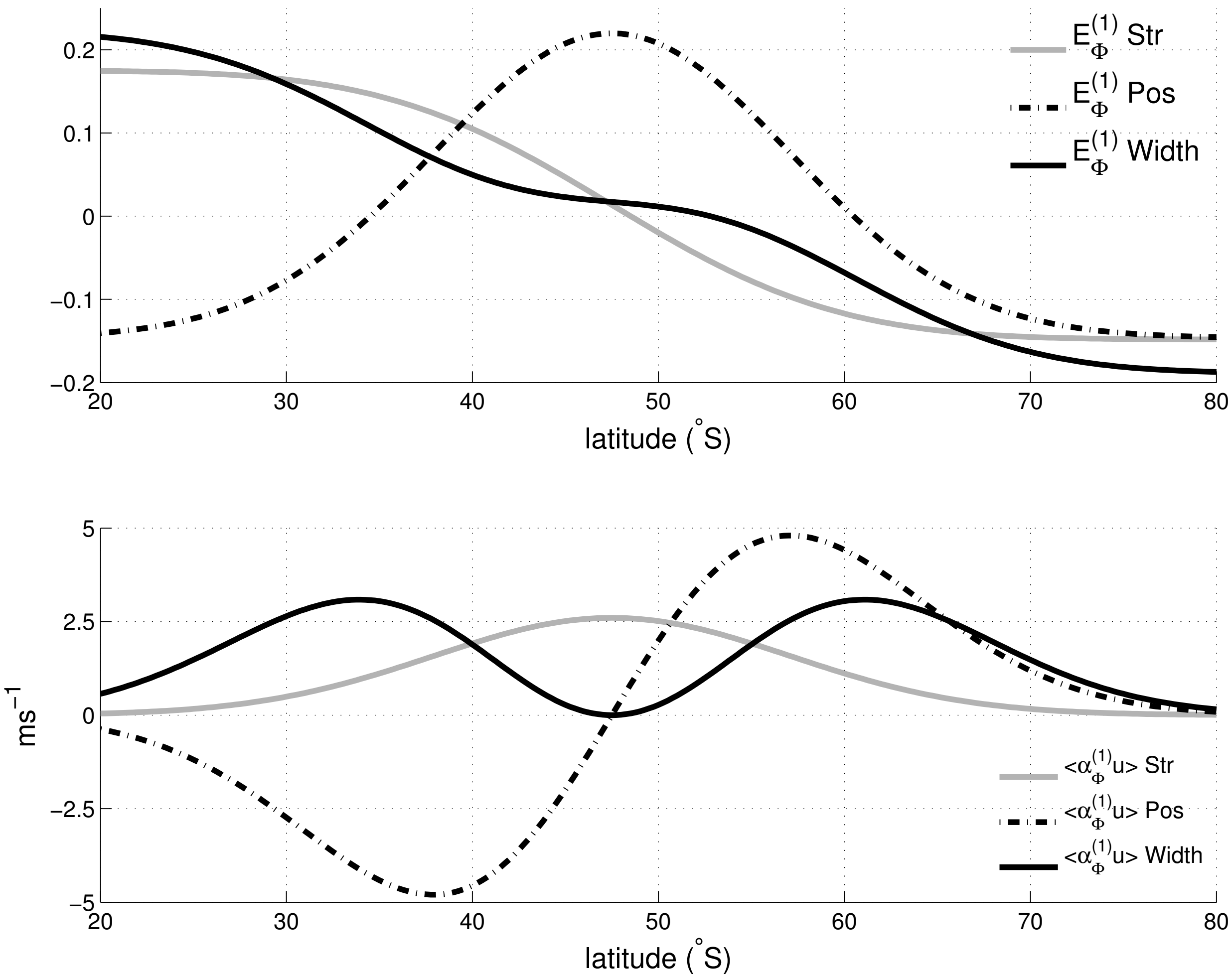
- Mass conservation constraint influences non-orthogonality

Geopotential EOFs: Strength, Position, & Width

- Gaussian jet, neglecting sphericity of Earth ($\mu(x) = f(x) = 1$)



Geopotential: Strength & Position Fluctuations (Flat)

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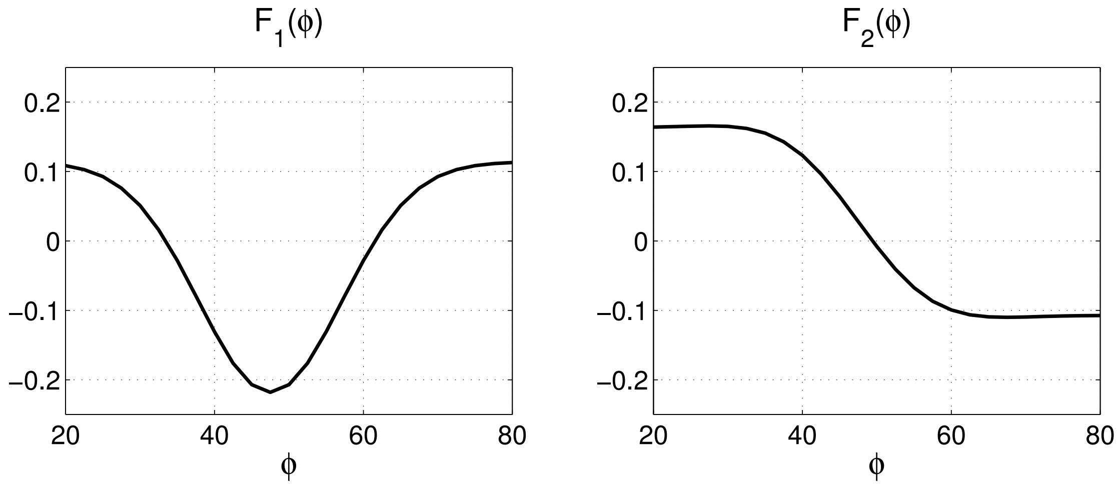

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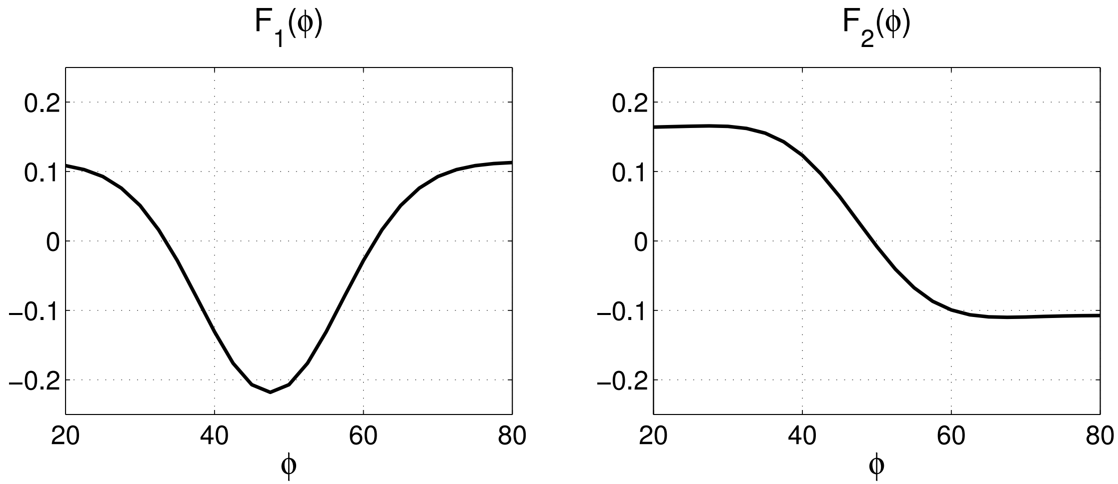

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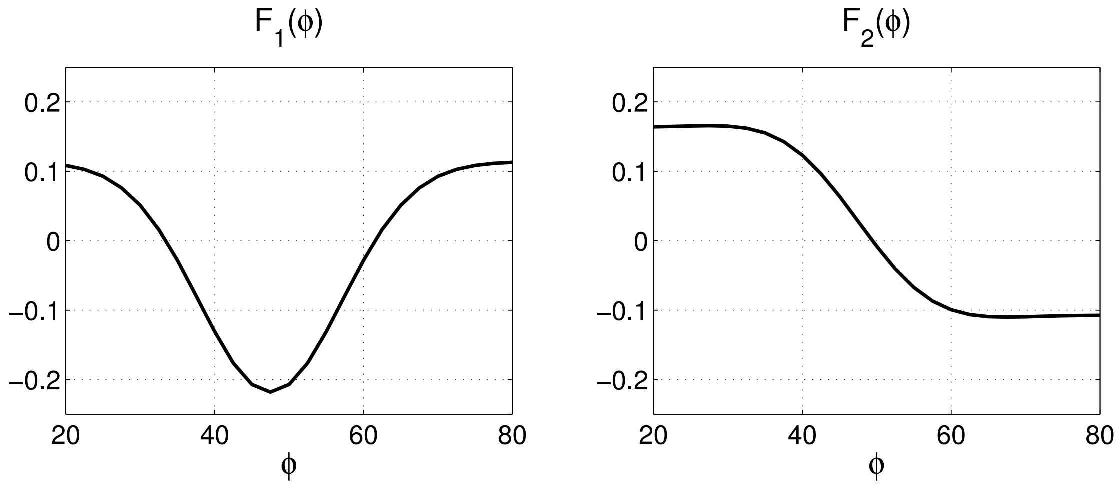

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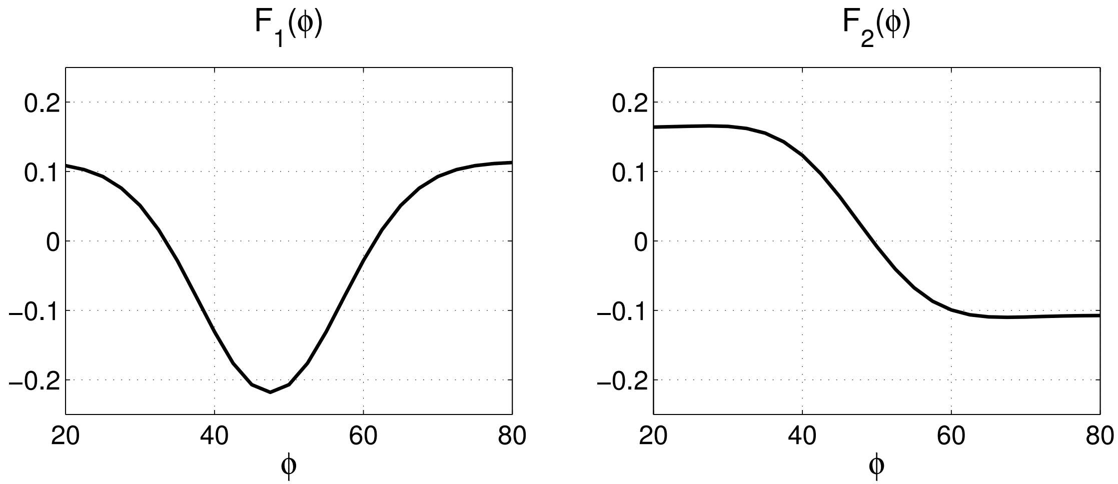

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- Leading EOF of $\Phi(x,t)$ mixes EOFs of $u(x,t)$ (because of mass conservation)

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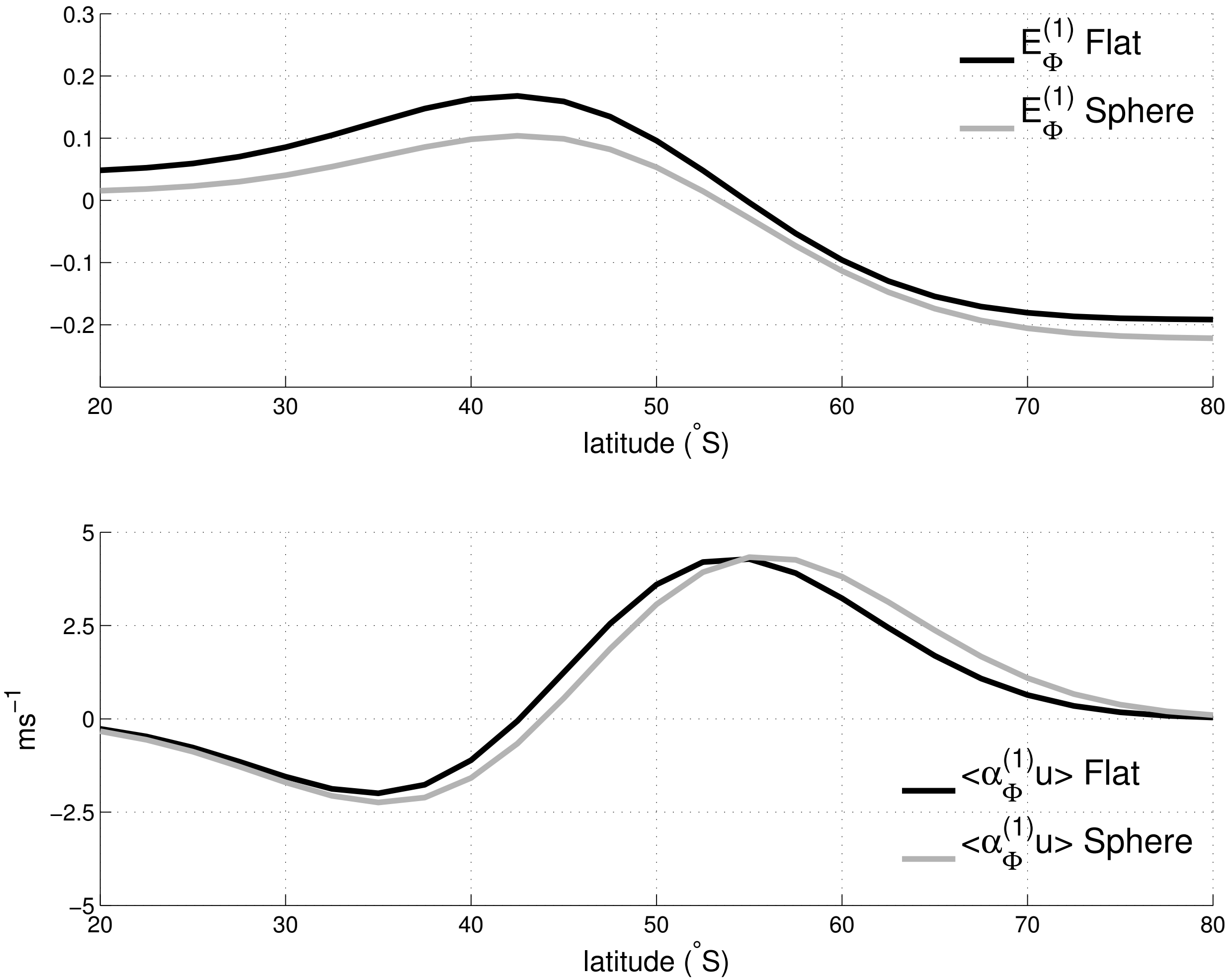
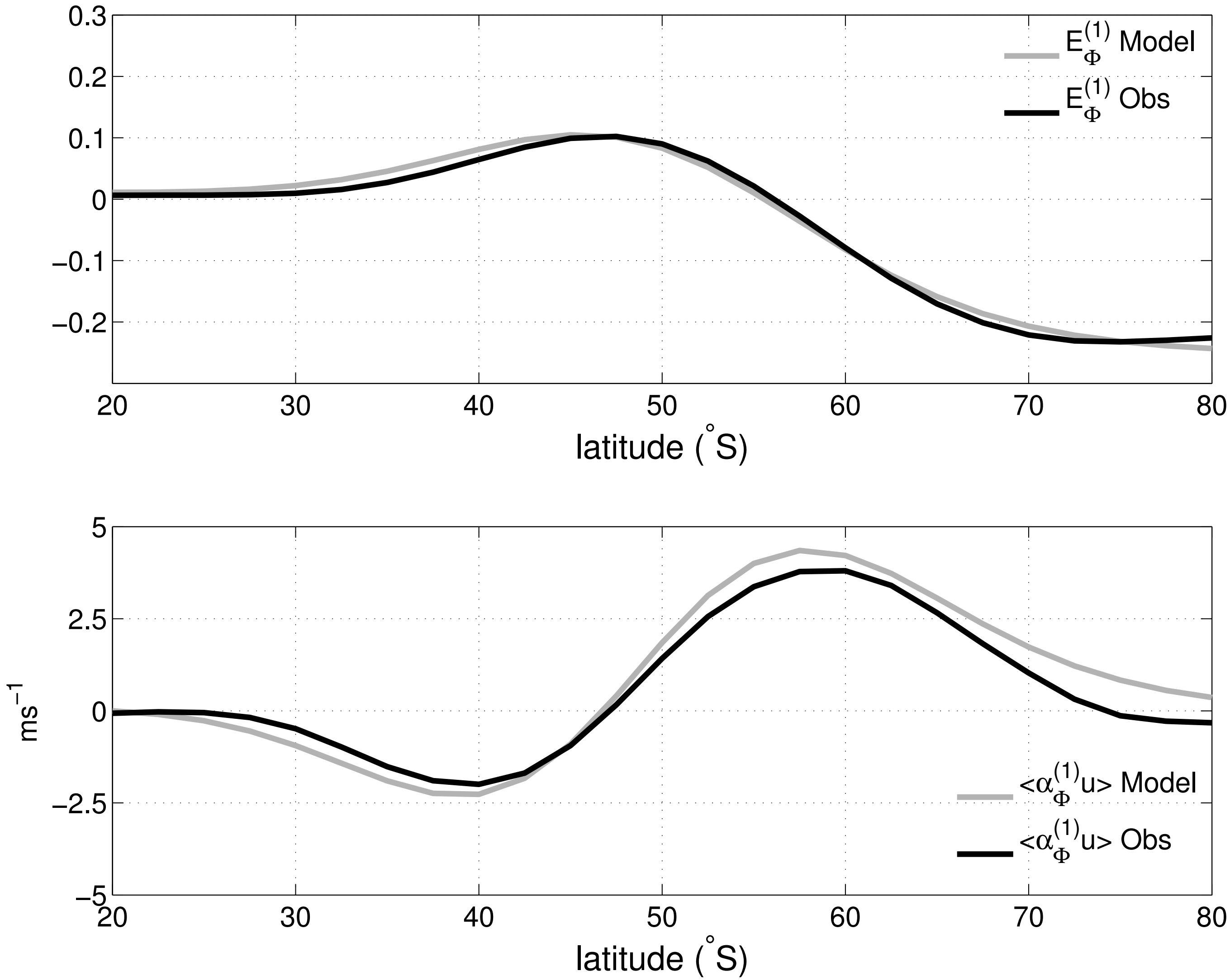
Geopotential: Strength & Position Fluctuations

- Annular mode structure requires both strength and position fluctuations and mixes leading two EOFs of zonal wind



Geopotential: Strength, Position, & Width (Sphere)



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