Parameterization of Boundary Layer Processes in AGCMs

- The atmospheric boundary layer (ABL) is the region adjacent to the surface of the earth within which the exchange of momentum, heat, moisture, and other constituents between the atmosphere and the surface takes place mainly by turbulent processes.

- Within a sub-layer near the surface vertical fluxes of momentum, heat, and moisture are almost independent of height.

- Within the remainder of the ABL quantities that are typically conserved under adiabatic motion are found to be nearly uniform with height (‘well mixed’)(e.g. potential temperature and specific humidity for cloud-free conditions or equivalent potential temperature and total water mixing ratio in cloudy conditions).

Cartoon of typical structure for a cloud-free convectively Active ABL

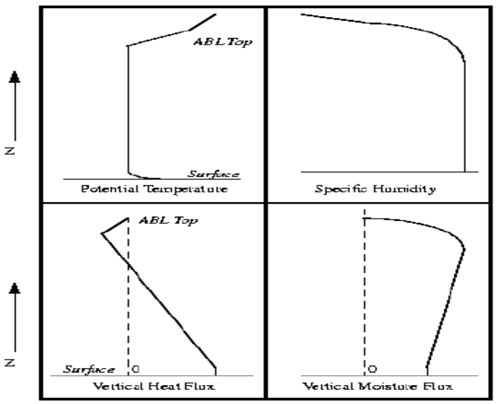

Figure 1. Typical profiles within a clear ABL

Cloud-free ABL :

- neglect effects of water vapour condensation

- ignore (for simplicity) virtual temperature effects (i.e. water vapour is passive)

Basic equations for the large (resolved) scale:

$$\frac{\partial\left(\overline{\rho}\overline{\vec{V}}_H\right)}{\partial t}+\nabla\bullet\left(\overline{\rho}\overline{\vec{V}}\overline{\vec{V}}_H\right)+f\hat{k}\times\overline{\vec{V}}=-\nabla_H\overline{p}-\frac{\partial\left(\overline{\rho}\overline{w'\vec{V}'}\right)}{\partial z} \qquad (1)$$

$$\frac{\partial\left(\overline{\rho}\overline{S}\right)}{\partial t}+\nabla\bullet\left(\overline{\rho}\overline{\vec{V}}\overline{S}\right)-\left(\frac{\partial\overline{p}}{\partial t}+\overline{\vec{V}}\bullet\nabla_H\overline{p}\right)=\overline{Q}-\frac{\partial\left(\overline{\rho}\left(\overline{w'S'}\right)\right)}{\partial z}+\frac{\partial}{\partial z}(\overline{w'p'})-\overline{\rho}\left(\overline{w'B}\right) \qquad (2)$$

$$B = gT_v'/\overline{T_v} \approx gT'/\overline{T} \qquad \text{(buoyancy)}$$

The depth of the ABL (and of turbulent regions in the free atmosphere) is typically small compared to a density scale-height (e.g. $h_{abl}/H \approx .1$). Therefore vertical variations in the background density are often ignored in ABL modelling.

Potential Temperature vs Static Energy

$$\theta = T\left(p_0 / p^{\kappa}\right) \qquad (\kappa = R / c_p)$$

$$S = c_p T + \Phi$$

If departures from hydrostatic conditions are small:

$$\Phi = gz \quad \text{and} \quad \frac{\partial\theta}{\partial z} \cong \left(\frac{p_0}{p}\right)^{\kappa}\left(\frac{\partial T}{\partial z} + \frac{g}{c_p}\right) \cong \frac{1}{c_p}\left(\frac{p_0}{p}\right)^{\kappa}\frac{\partial S}{\partial z}$$

It can also be shown that

$$\frac{d\theta}{dt} = \left(\frac{p_0}{p}\right)^{\kappa}\left[\frac{dT}{dt} - \frac{1}{\rho}\frac{dp}{dt}\right] \cong \left(\frac{p_0}{p}\right)^{\kappa}\left[\frac{dS}{dt} + wB - \frac{1}{\bar{\rho}}\left[\frac{\partial p}{\partial t} + \nabla \bullet (\vec{V}p') + \bar{V} \bullet \nabla_H \bar{p}\right]\right]$$

Also:

$$c_p\theta' \cong \left(\frac{p_0}{\bar{p}}\right)^{\kappa}\left(c_p T' - \frac{p'}{\bar{\rho}}\right)$$

$$\Rightarrow c_p \frac{\partial}{\partial z}\left(\rho w'\theta'\right) \cong \left(\frac{p_0}{\bar{p}}\right)^{\kappa}\left[c_p \frac{\partial\left(\bar{\rho} w' T'\right)}{\partial z} - \frac{\partial\left(w' p'\right)}{\partial z}\right] + g\frac{w'\theta'}{\bar{\theta}}$$

$$\Rightarrow c_p \frac{\partial\left(w'\theta'\right)}{\partial z} \cong \left(\frac{p_0}{\bar{p}}\right)^{\kappa}\left[\frac{\partial\left(w' S'\right)}{\partial z} - \frac{\partial\left(w' p'\right)}{\partial z} + w' B\right]$$

Therefore the R.H.S. of (2) is approximtely $\overline{Q} - c_p \left(\overline{T}/\overline{\theta}\right) \partial\left(\overline{w'\theta'}\right)/\partial z$

Usual current approach: combine a turbulent kinetic energy (*tke*) equation with an eddy diffusivity formulation. Get a *tke* equation by forming an equation for $\vec{V}' \bullet \vec{V}'$ and averaging.

Turbulent Kinetic Energy Equation: $\overline{\mathrm{E}} = \overline{\rho}\left(\vec{\overline{V}} \bullet \vec{\overline{V}}/2 + \overline{\vec{V}' \bullet \vec{V}'}/2\right) = \overline{\rho}\left(\mathrm{E} + \mathrm{e}\right)$

Approximate *tke* (e) equation: (e.g. Stull, 1988)

$$\frac{\partial \mathrm{e}}{\partial t} + \overline{w'\vec{V}'} \bullet \frac{\partial \vec{\overline{V}}}{\partial z} + \frac{\partial}{\partial z}\left(\overline{w'\vec{V}' \bullet \vec{V}'}/2 + \overline{w'p'}/\overline{\rho}\right) = g\,\overline{w'\theta'}/\overline{\theta} - \mathrm{D}_k$$

Eddy diffusion approximation for second moments:

$$\overline{w'\vec{V}'} = -K_m\,\partial\vec{\overline{V}}/\partial z$$

$$\overline{w'\theta'} = -K_H\,\partial\overline{\theta}/\partial z$$

$$\overline{w'\vec{V}' \bullet \vec{V}'}/2 + \overline{w'p'}/\overline{\rho} = -K_e \partial \mathrm{e}/\partial z$$

Diffusivities:

Traditional approach

$$K_{m,H,e} = l_{m,H,e} c_{m,H,e} \sqrt{e}$$

Dissipation:

Physical and dimensional considerations suggest

$$D = e/\tau_d = c_d e^{3/2} / l_d$$

Specifying the lengths $l_{m,H,e,d}$ and coefficients is a closure issue.
Large literature on this topic. Several hypotheses have been explored in recent work (e.g. Sanchez&Cuxart, 2004, Lenderink&Holtslag, 2004, and references therein)

Boundary conditions and constraints

<u>*Matching to the surface layer:*</u>

Monin - Obukhov similarity requires that:

$$|\vec{\tau}|/\overline{\rho} = [(\overline{u'w'})^2 + \overline{(v'w')}^2]^{1/2} = u_*^2$$

$$(\overline{w'\theta'}) = -u_*\theta_*$$

where

$$u_* = \frac{kU_L}{\ln(z_L/z_0) - \psi_m(z_L/L) + \psi_m(z_0/L)}$$

$$\theta_* = \frac{(k/\mathrm{Pr})(\theta_L - \theta_s)}{\ln(z_L/z_t) - \psi_H(z_L/L) + \psi_H(z_t/L)}$$

$$L = -\frac{\theta_v u_*^3}{kg(\overline{w'\theta_v'})_s}$$ (Monin-Obukov length)

k : von Karmen constant,

Pr : turbulent Prandtl number,

U_L, θ_L: wind speed, potential temperature at reference level (z_L)

z_0, z_t roughness heights (where surface values apply).

The functions ψ_m, ψ_H are derived from field campaign observations (e.g. Dyer, 1974) . Moisture and other tracers treated similarly.

Bulk exchange formulae (resulting from fits to non-linear solutions):

$$R_{iB} = g z_L [(\theta_v)_L - (\theta_v)_s] / [(\theta_v)_s U_L^2]$$ (Bulk Richardson Number)

$$u_*^2 = C_D F_m (z_L / z_0 , R_{iB}) U_L^2$$

$$u_* \theta_* = C_H F_H (z_L / z_0 , z_L / z_t , R_{iB}) U_L (\theta_L - \theta_s)$$

$$u_* q_* = C_Q F_Q (z_L / z_0 , z_L / z_q , R_{iB}) U_L (q_L - q_s)$$

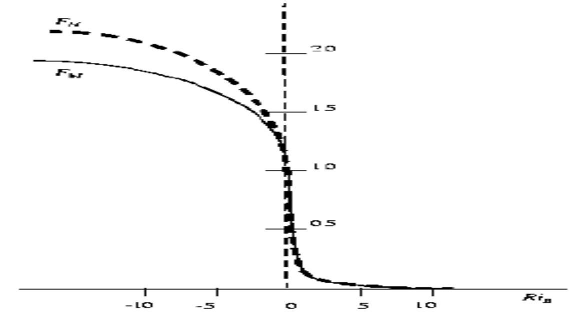

Figure 2. Typical behavior of heat and momentum flux coefficients as a function of bulk Richardson number.

Limitation:

-Dependence of vertical fluxes on local mean gradients does not account for heat transfer in the convectively active ABL where mean gradients are small (slightly stable) but upward heat flux is positive.

-Requires introduction of non-local effects.

For a scalar quantity, χ :

$$\overline{w'\chi'} = -K_{\chi}\frac{\partial\overline{\chi}}{\partial z} + \left(\overline{w'\chi'}\right)_{nl}$$

Approaches:
(a) Introduce prognostic equations for second moments with associated closure Assumptions to derive the nonlocal effects
(e.g. Deardorf, 1966, Mellor& Yamada, 1974, Cuijpers & Holtslag , 1993, Abdella & McFarlane, 1997, Gryanik&Hartmann, 2002, ….).
Simplest formulations give:

$$(\overline{w'\theta'})_{nl} = K_H\gamma_{cg}$$

(b) Represent non-local transfer effects as being associated with plume-like Eddies (e.g. Siebesma et al, 2007)

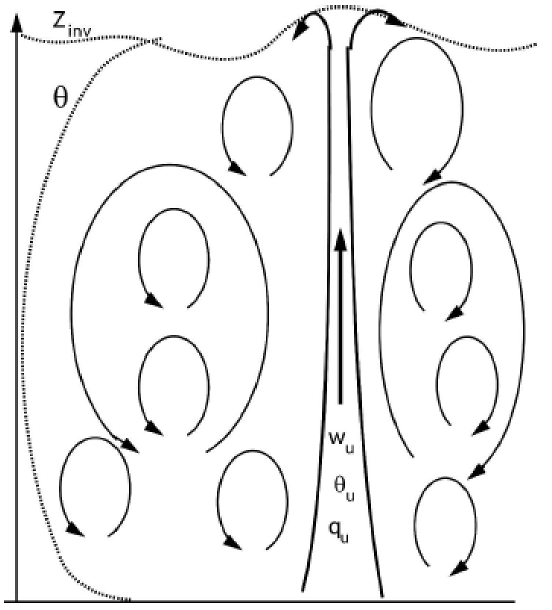

FIG. 1. Sketch of a convective updraft embedded in a turbulent eddy structure.

(From Siebesma et al)

$$\rho\overline{\left(w'\theta'\right)_{nl}} = M(\theta_u - \overline{\theta})$$

$$\frac{\partial M}{\partial z} = (\varepsilon - \delta)M$$

$$\frac{\partial \theta_u}{\partial z} = \varepsilon(\theta_u - \overline{\theta})$$

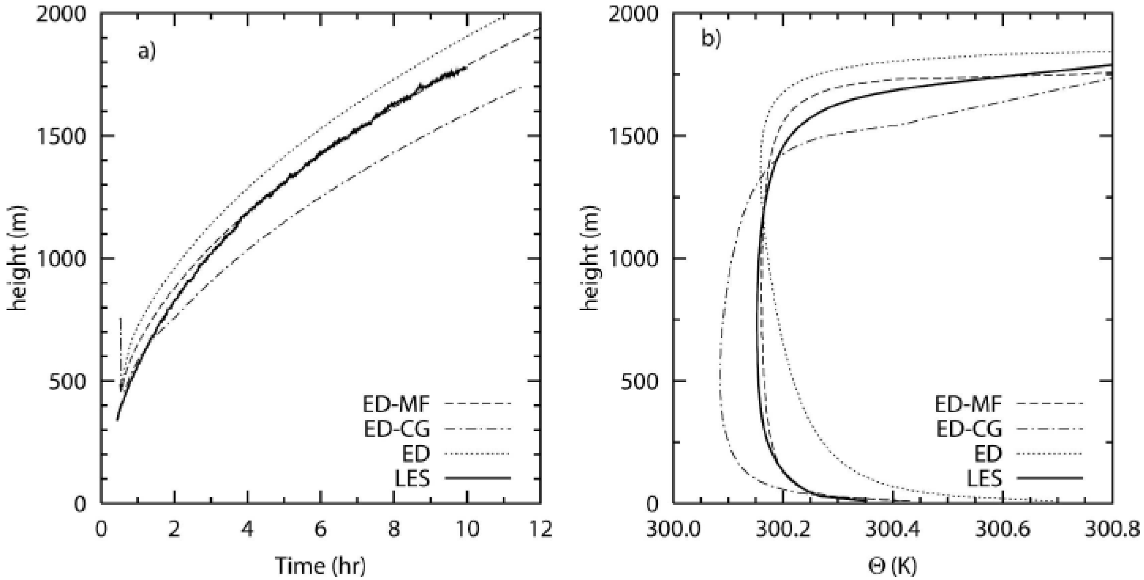

FIG. 10. (a) Time evolution of the inversion height for the three different approaches listed in Table 3 along with LES results as a reference. (b) The mean potential temperature profiles after 10 h of simulation.