Pacific Institute *for the* Mathematical Sciences

PIMS Marsden Memorial Lecture Series

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An Octahedral Gem Hidden in Newton's Three Body Problem

Fields Institute, July 25, 2012, Time and Location TBD

The three-body problem of Newton has singularities and symmetries. Singularities arise from collisions. McGehee [1974] showed how to eliminate the triple collision singularity through a blow-up procedure. Levi-Civita [1921] showed how to eliminate the binary collision singularities through a regularization procedure. Lagrange [1772], followed by a host of others, showed how to eliminate symmetries through a reduction procedure. We sketch how a systematic and democratic (no body selected as being 'special') application of all three procedures leads to a regularized reduced three body problem which consists of a complete but non-Hamiltonian vector field on a manifold with boundary. Upon composing the transformations associated to reduction and regularization we discover a certain degree 4 map of the two-sphere which is related to the octahedron. The vertices of the octahedron represent binary collisions. When all masses are equal, this octahedron leads to a discrete symmetry group of order 48 for the regularized reduced vector field. Does a global understanding of this completed vector field, the manifold on which it lives, or of the role of the octahedron, help us to answer any of the long-outstanding basic open questions regarding the three body problem? How much of this story generalizes to N bodies moving in d dimensions? *(report on joint work with Rick Moeckel)*

About this series: This lecture series is dedicated to the memory of Jerrold E Marsden (1942-2010), a world-renowned Canadian applied mathematician. Marsden was the Carl F Braun Professor of Control and Dynamical Systems at Caltech, and prior to that he was at the University of California (Berkeley) for many years. He did extensive research in the areas of geometric mechanics, dynamical systems and control theory. He was one of the original founders in the early 1970's of reduction theory for mechanical systems with symmetry, which remains an active and much studied area of research today.

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