Stochastic Models for Convective Momentum Transport

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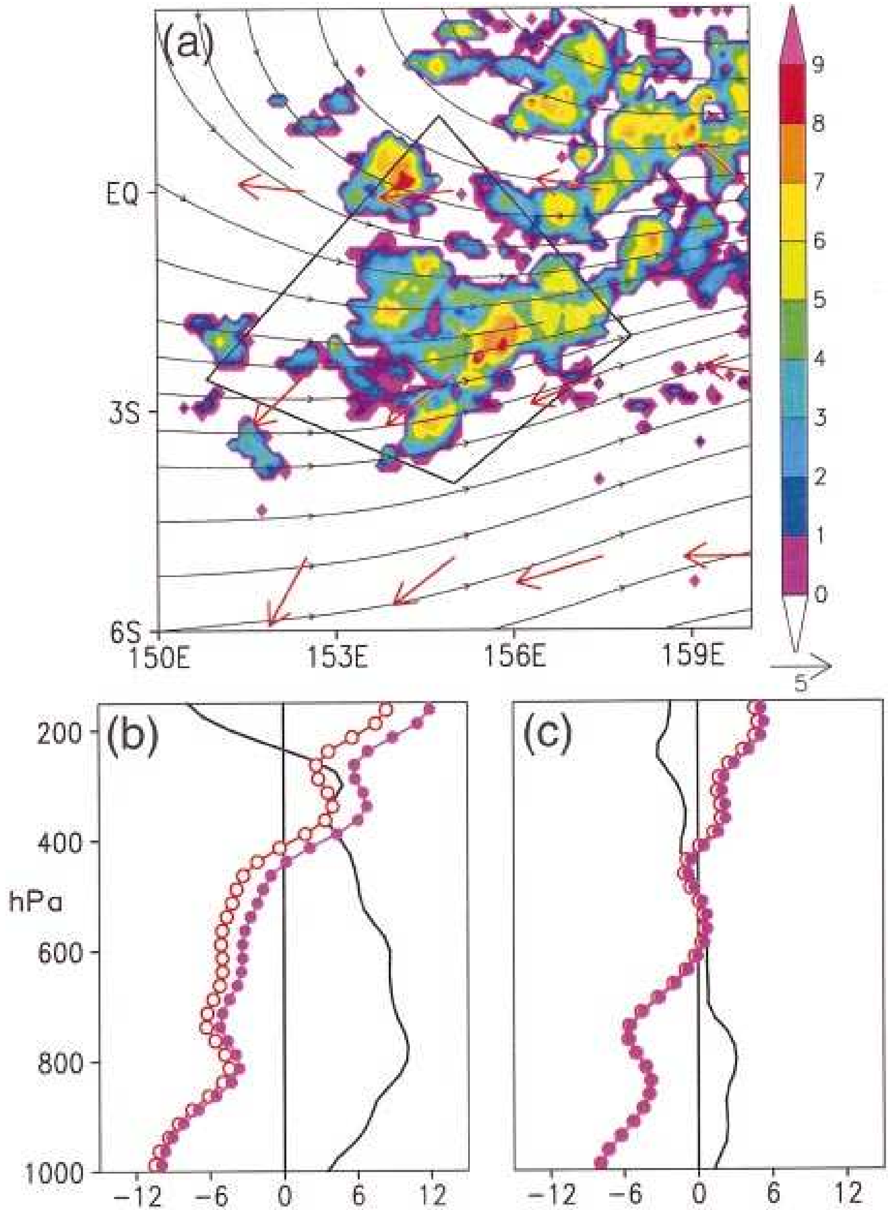
Examples of convective momentum transport (CMT)



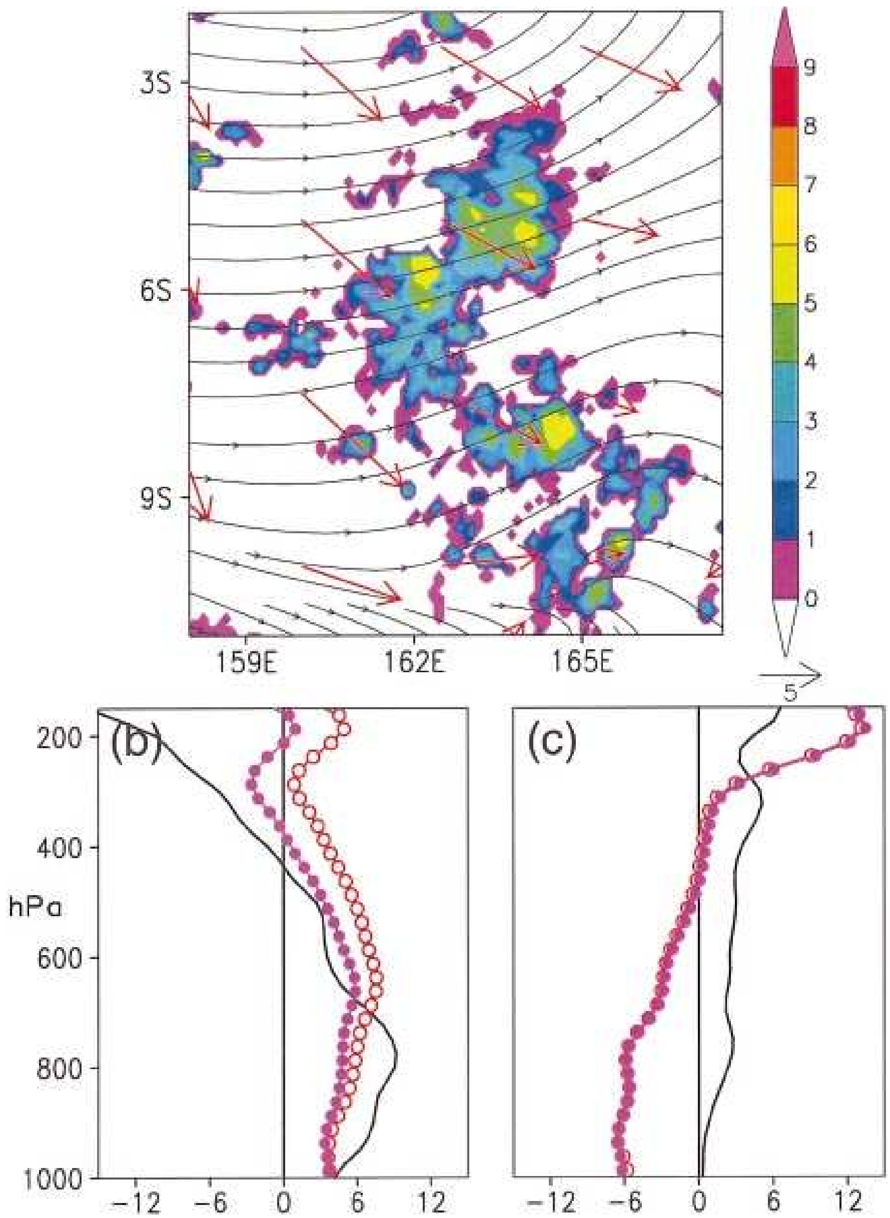
Tung and Yanai (2002b)

Non-squall convective system:

- CMT decelerates mean wind

(b) $\bar{U}$ (black), $-(\overline{w'u'})_z$ (color)
(c) $\bar{V}$ (black), $-(\overline{w'v'})_z$ (color)

Examples of convective momentum transport (CMT)



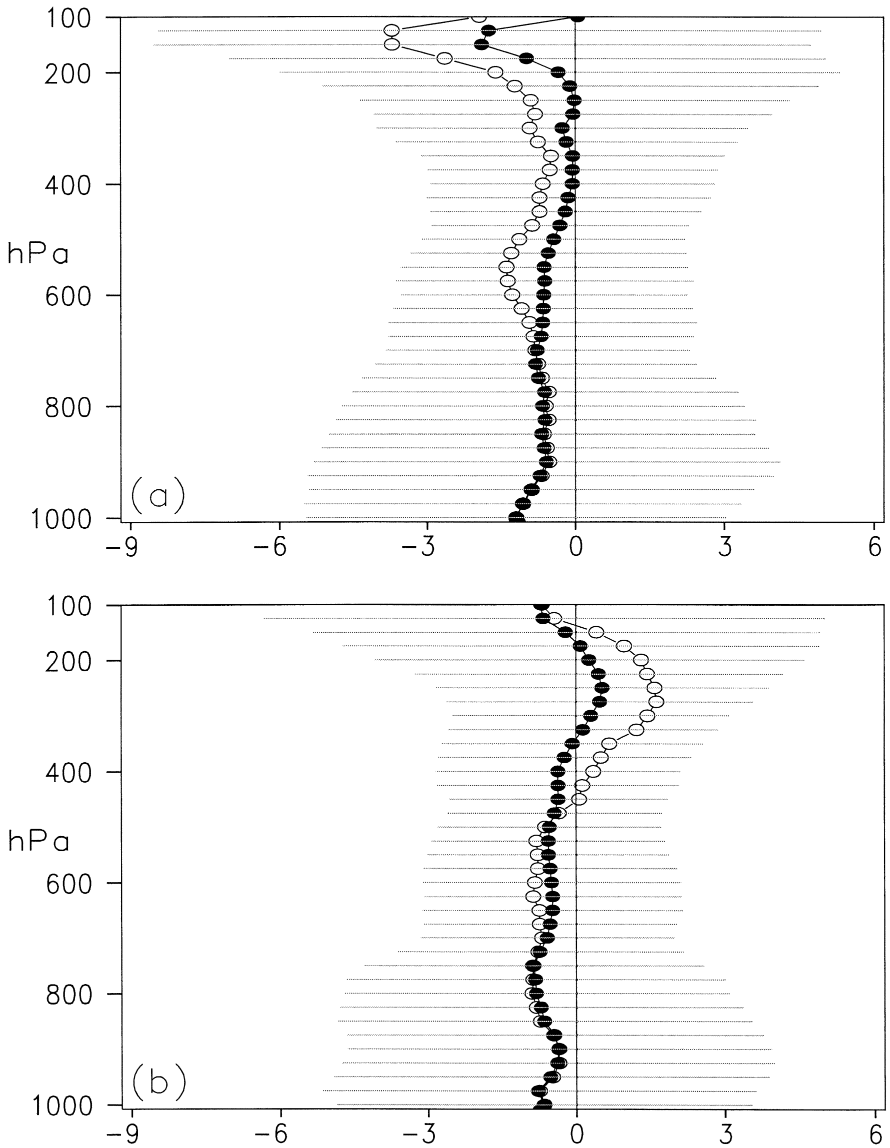
Tung and Yanai (2002b)

Squall line:

- CMT accelerates $\bar{U}$

(b) $\bar{U}$ (black), $-(\overline{w'u'})_z$ (color)
(c) $\bar{V}$ (black), $-(\overline{w'v'})_z$ (color)

Statistics of convective momentum transport (CMT)



Tung and Yanai (2002a)

Top: $-(\overline{w'u'})_z \frac{\bar{U}}{|\bar{U}|}$

Bottom: $-(\overline{w'v'})_z \frac{\bar{V}}{|\bar{V}|}$

Circles: IOP mean

Horizontal lines: standard deviation

- Mean CMT: weak damping (cumulus friction)
- But standard dev. of CMT is huge!
- Examples demonstrate that both acceleration and deceleration can be intense

Motivation for stochastic models for CMT

1. Convective parameterizations in GCMs usually include only cumulus friction:

$$\partial_t u + \partial_x(u^2) + \partial_z(wu) + \partial_x p = -\partial_z(\overline{w'u'})$$
$$\approx -d_c(u - \hat{u})$$

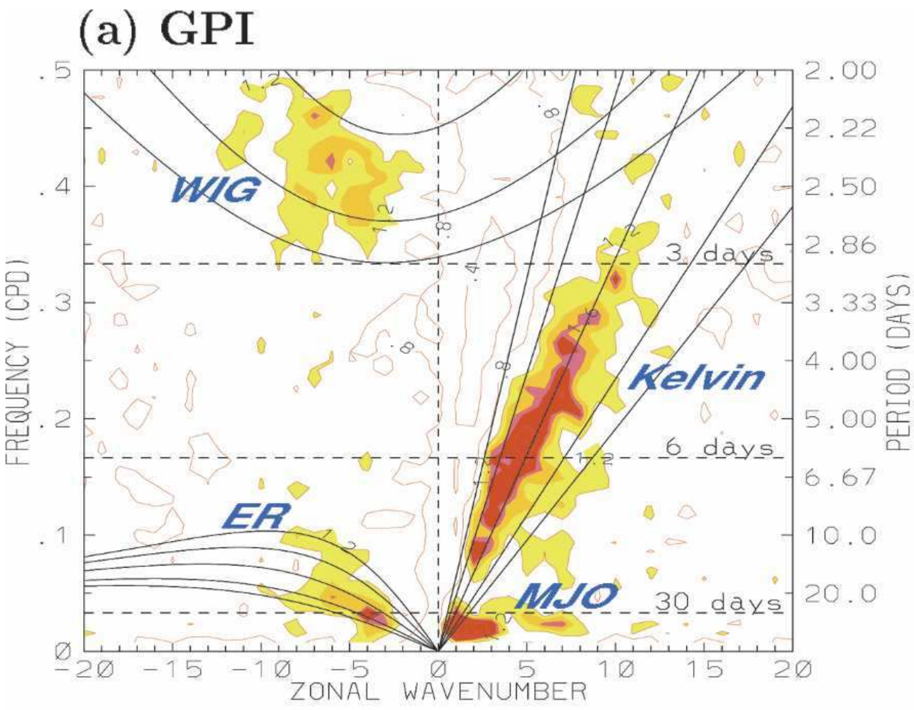
   - Wu et al. (2007) include a deterministic CMT parameterization and improve the mean climatology
   - Goal of present work: to develop a simple stochastic CMT model that includes intermittent intense bursts of CMT as in observations

2. GCMs fail to capture realistic variability of tropical convection

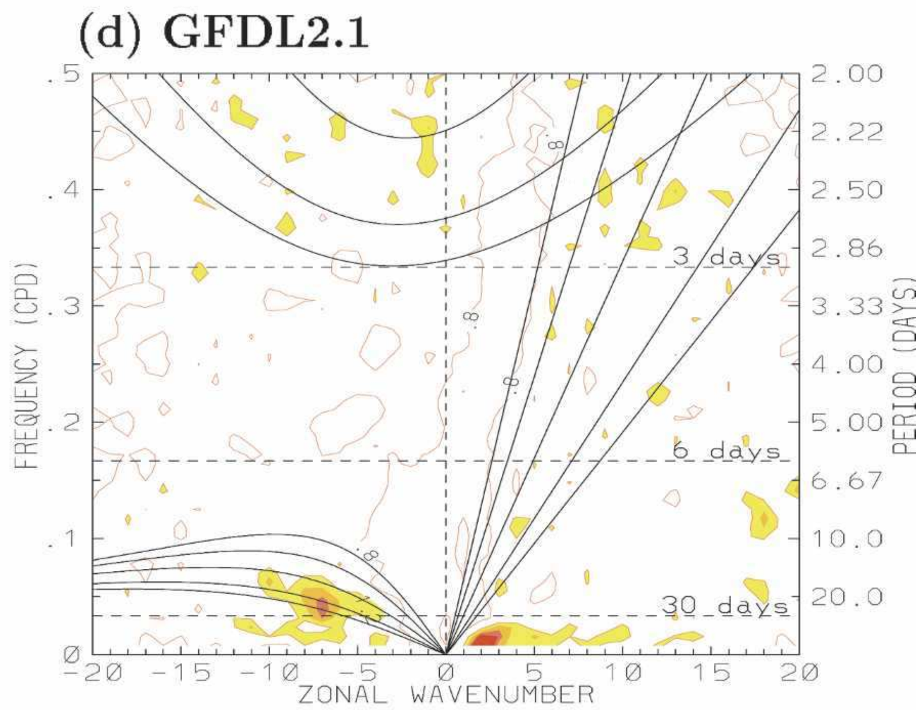
   - A stochastic parameterization of convection could improve this

Spectral Power of Tropical Precipitation in Observations and GCMs

Observations



GCM



From Lin et al. (2006)

Stochastic models to capture the intermittent impact of smaller scale events on the larger scales

- Majda A, Khouider B (2002) Stochastic and mesoscopic models for tropical convection. Proc. Natl. Acad. Sci. 99:1123 1128.
- Katsoulakis M, Majda A, Vlachos D (2003) Coarse-grained stochastic processes for microscopic lattice systems. Proc. Natl. Acad. Sci. 100:782–787.
- Khouider B, Majda A J, Katsoulakis M A (2003) Coarse-grained stochastic models for tropical convection and climate. Proc. Natl. Acad. Sci. 100:11941–11946.
- Katsoulakis M, Majda A, Sopasakis A (2006) Intermittency, metastability and coarse graining for coupled deterministic stochastic lattice systems. Nonlinearity 19:1021–1047.
- Majda A J, Franzke C, Khouider B (2008) An applied mathematics perspective on stochastic modeling for climate. Phil. Trans. Roy. Soc. A, in press.
- Majda A J, Stechmann S N (2008) Stochastic models for convective momentum transport. Proc. Natl. Acad. Sci., to appear.

Stochastic Models for Convective Momentum Transport

Outline

1. Description of simple stochastic model for CMT
2. Test case: column model
3. Test case: convectively coupled wave

3 Convective Regimes with Different CMT

1. Dry regime.
   - Weak or no cumulus friction.
   - Favored for dry environments, regardless of shear.

2. Upright convection regime.
   - Stronger cumulus friction.
   - Favored for moist, weakly sheared environments.

3. Squall line regime.
   - Intense CMT, either upscale or downscale depending on the shear.
   - Favored for moist, sheared environments.

Markov jump process for transitions between regimes

3-state continuous-time Markov jump process

- at each large-scale spatio-temporal location (x, t)
- with transition rates depending on local values of large-scale variables at (x, t)

Denote the discrete, stochastic regime variable by

$$r_t = 1 \text{ (dry)}$$
$$r_t = 2 \text{ (conv.)}$$
$$r_t = 3 \text{ (squall)}$$

T_{ij}: transition rate from regime i to regime j

based on observations such as LeMone, Zipser, & Trier (1998)

Transition rates

$$T_{12} = \frac{1}{\tau_r}\mathcal{H}(Q_d)e^{\beta_\Lambda(1-\Lambda)}e^{\beta_Q Q_d} \qquad \text{dry} \rightarrow \text{conv.}$$

$$T_{13} = 0 \qquad \text{dry} \rightarrow \text{squall}$$

$$T_{21} = \frac{1}{\tau_r}e^{\beta_\Lambda \Lambda}e^{\beta_Q(Q_{d,ref}-Q_d)} \qquad \text{conv.} \rightarrow \text{dry}$$

$$T_{23} = \frac{1}{\tau_r}\mathcal{H}(|\Delta U_{low}| - |\Delta U|_{min})e^{\beta_U|\Delta U_{low}|}e^{\beta_Q Q_c} \qquad \text{conv.} \rightarrow \text{squall}$$

$$T_{31} = T_{21} \qquad \text{squall} \rightarrow \text{dry}$$

$$T_{32} = \frac{1}{\tau_r}e^{\beta_U(|\Delta U|_{ref}-|\Delta U_{low}|)}e^{\beta_Q(Q_{c,ref}-Q_c)} \qquad \text{squall} \rightarrow \text{conv.}$$

- Exponentials capture sensitive dependence on large-scale variables
- τ_r, β: model parameters
- Q: cloud heating
- Λ: measures dryness of lower-mid troposphere relative to boundary layer
- ΔU: vertical wind shear

Different convective regimes have different CMT

$$F_{CMT} = -\partial_z(\overline{w'u'}) = \begin{cases} -d_1(U - \hat{U}) & \text{for} \quad r_t = 1 \\ -d_2(U - \hat{U}) & \text{for} \quad r_t = 2 \\ F_3 & \text{for} \quad r_t = 3 \end{cases}$$

$$F_3 = -\partial_z(\overline{w'u'}) = \kappa[\cos(z) - \cos(3z)].$$

$$\kappa = \begin{cases} -\left(\frac{Q_d}{Q_{d,ref}}\right)^2 \frac{\Delta U_{mid}}{\tau_F} & \text{if} \quad \Delta U_{mid}\Delta U_{low} < 0 \\ 0 & \text{if} \quad \Delta U_{mid}\Delta U_{low} > 0 \end{cases}$$

Formulas for F_3 and κ motivated by observations, CRM simulations, and a simple multi-scale model ...

Formula for $F_3 = -\partial_z(\overline{w'u'}) = \kappa[\cos(z) - \cos(3z)]$

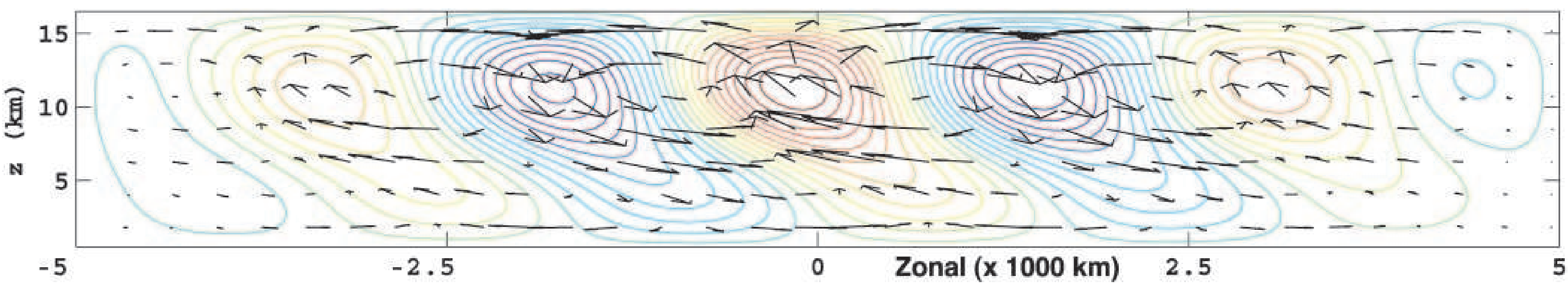
Exactly solvable multi-scale model (Majda and Biello, 2004; Biello and Majda 2005; Majda, 2007)

$$w' = S'_\theta$$

$$u'_x + w'_z = 0$$

Choose S'_θ to include stratiform heating lagging deep convective heating:

$$S'_\theta = k\cos[kx - \omega t]\,\sqrt{2}\sin(z) + \alpha k\cos[k(x + x_0) - \omega t]\,\sqrt{2}\sin(2z)$$

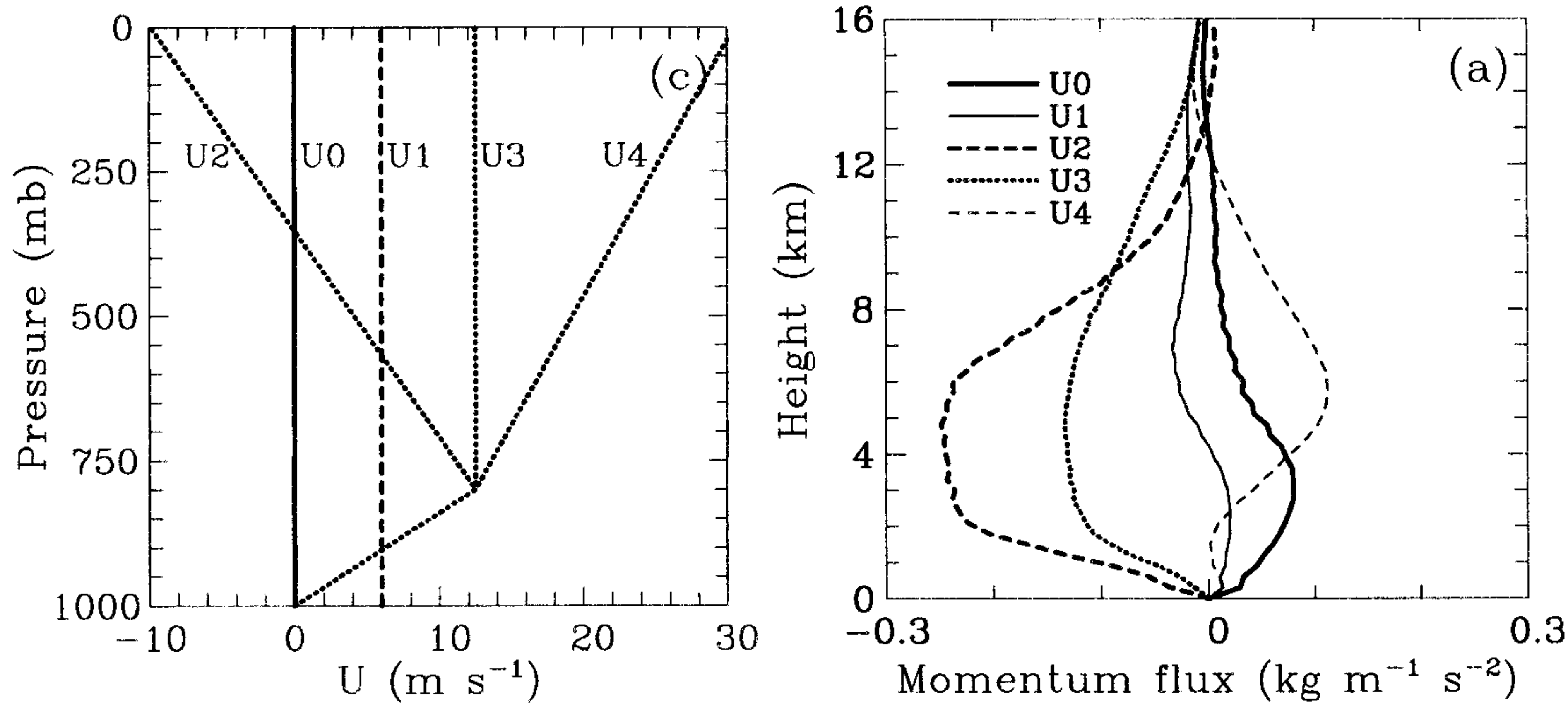


Exact solution: 1st, 2nd mode heating generates CMT in the 1st, 3rd modes

$$\partial_z(\overline{w'u'}) = \frac{3\alpha k}{2}\sin(kx_0)[\cos(z) - \cos(3z)]$$

$$\text{Formula for } \kappa = \begin{cases} -\left(\frac{Q_d}{Q_{d,ref}}\right)^2 \frac{\Delta U_{mid}}{\tau_F} & \text{if} \quad \Delta U_{mid} \Delta U_{low} < 0 \\ 0 & \text{if} \quad \Delta U_{mid} \Delta U_{low} > 0 \end{cases}$$

CRM results: Liu and Moncrieff (2001)



Vertical tilts of squall lines, and their CMT, depend on the mid-level shear

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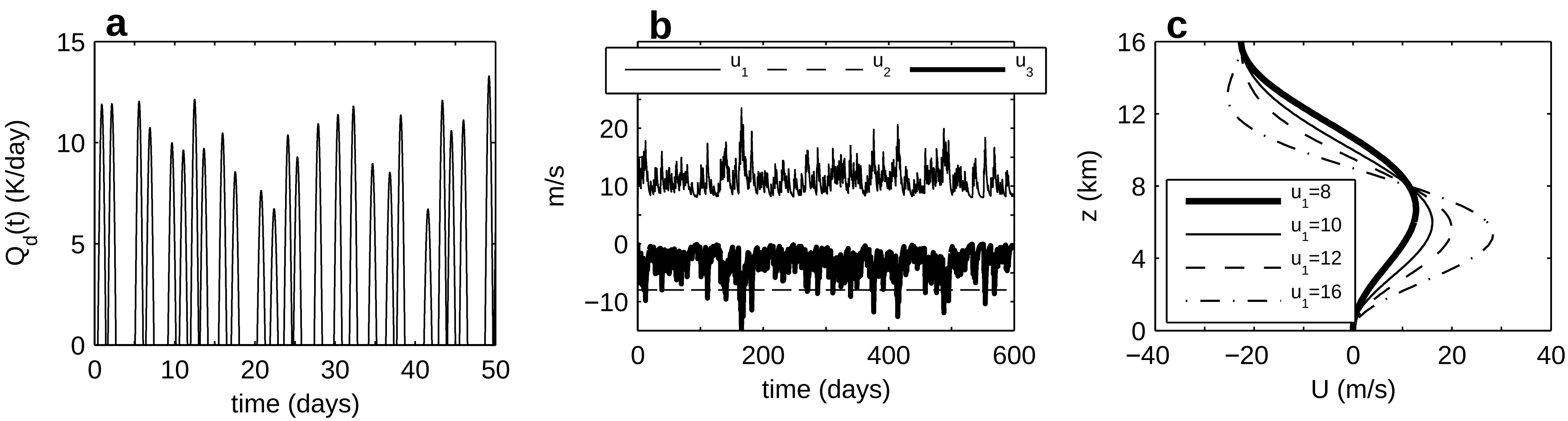
3. Test case: convectively coupled wave

Test case: column model

$$\frac{\partial u}{\partial t} = F_{CMT}$$

$$u(z,t) = \sum_{j=1}^{3} u_j(t)\,\sqrt{2}\cos(jz)$$

Other model variables are imposed



Demonstrates intermittent bursts of CMT

Useful for calibration of model parameters

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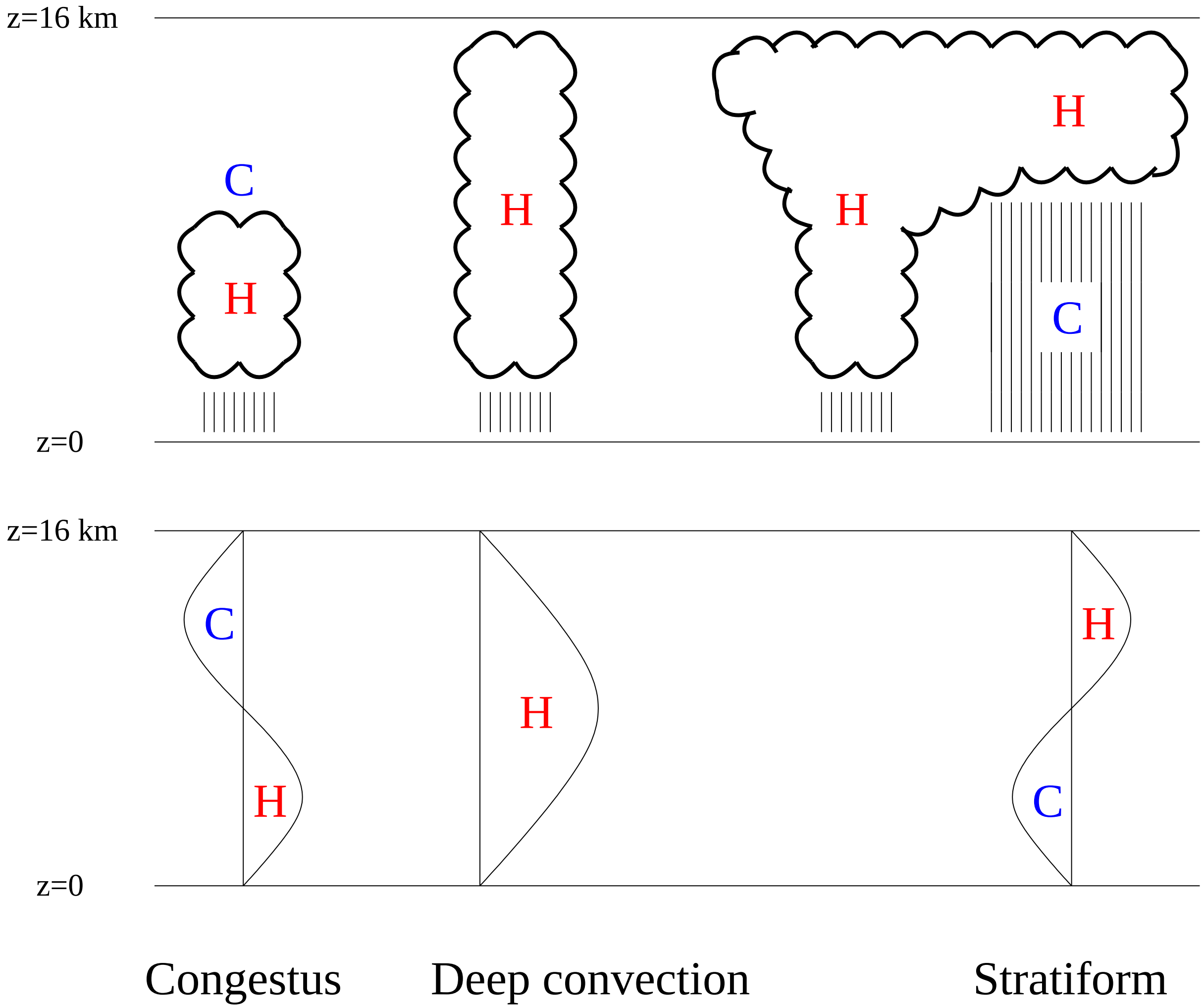
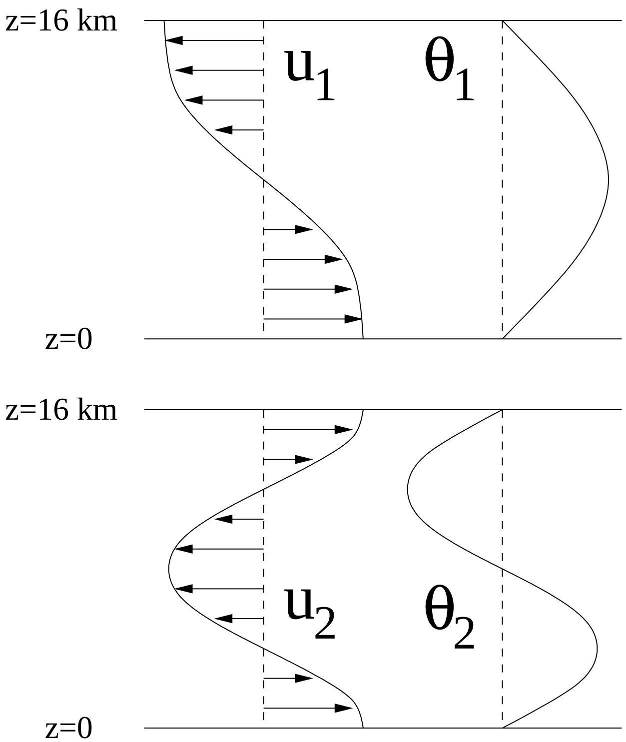
Model for convectively coupled waves

Multi-cloud Model of Khouider & Majda (2006,2008)



Model for convectively coupled waves

Multi-cloud Model of Khouider & Majda (2006,2008) + Stochastic CMT Model



$$\frac{\partial u_1}{\partial t} - \frac{\partial \theta_1}{\partial x} = F^1_{CMT}$$
$$\frac{\partial u_2}{\partial t} - \frac{\partial \theta_2}{\partial x} = F^2_{CMT}$$
$$\frac{\partial u_3}{\partial t} = F^3_{CMT}$$
$$\frac{\partial \theta_1}{\partial t} - \frac{\partial u_1}{\partial x} = H_d + \xi_s H_s + \xi_c H_c - R_1$$
$$\frac{\partial \theta_2}{\partial t} - \frac{1}{4}\frac{\partial u_2}{\partial x} = H_c - H_s - R_2$$

+ evolution equations for θ_{eb}, q, H_s

and formulas for nonlinear interactive source terms

such as convective heating, downdrafts, etc.

Convectively coupled wave simulation

6000-km periodic domain

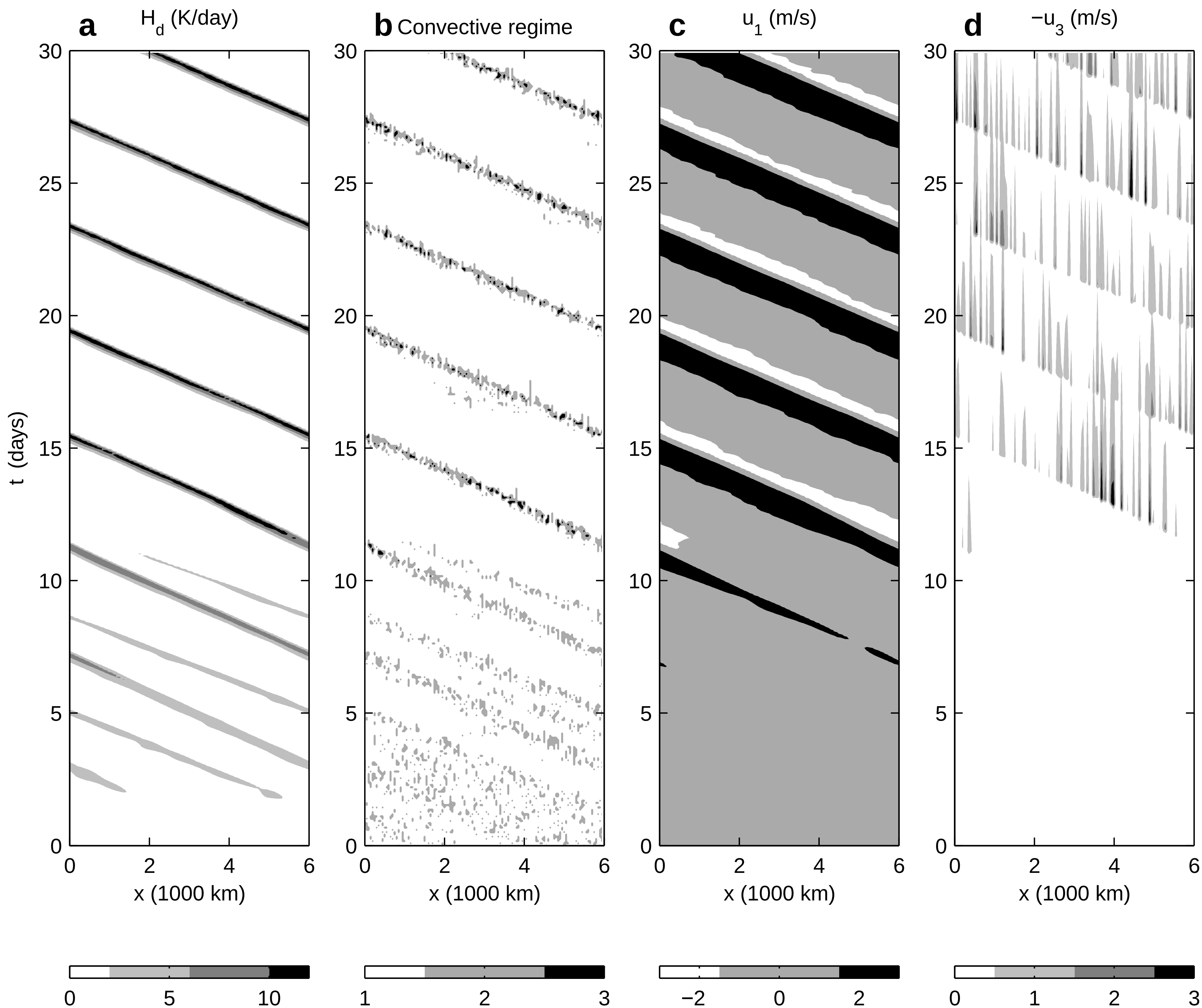
- to capture a single convectively coupled wave

$\Delta x = 50$ km

- representative of a GCM's grid spacing

Initial conditions:

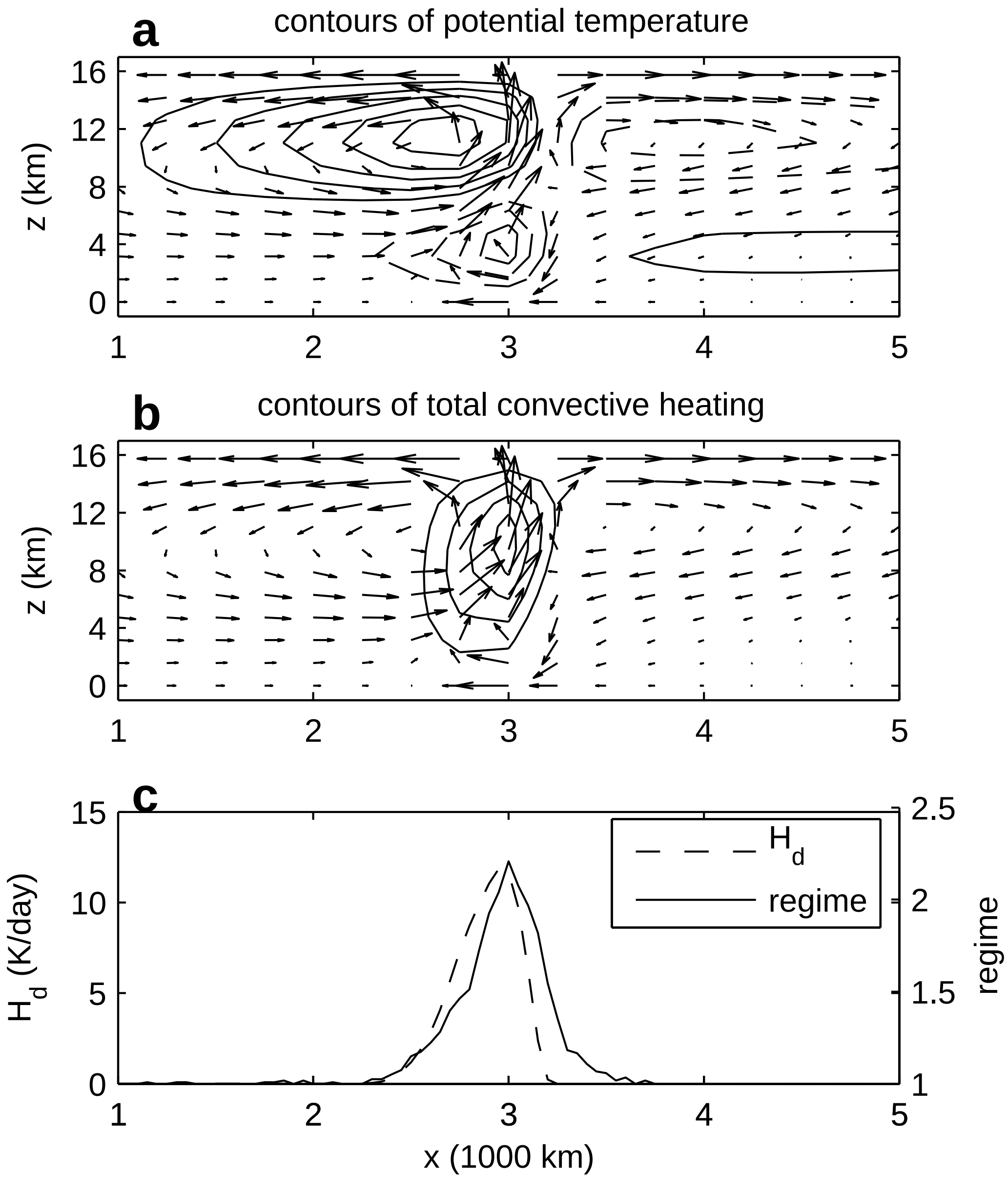
- small perturbation to uniform radiative–convective equilibrium solution



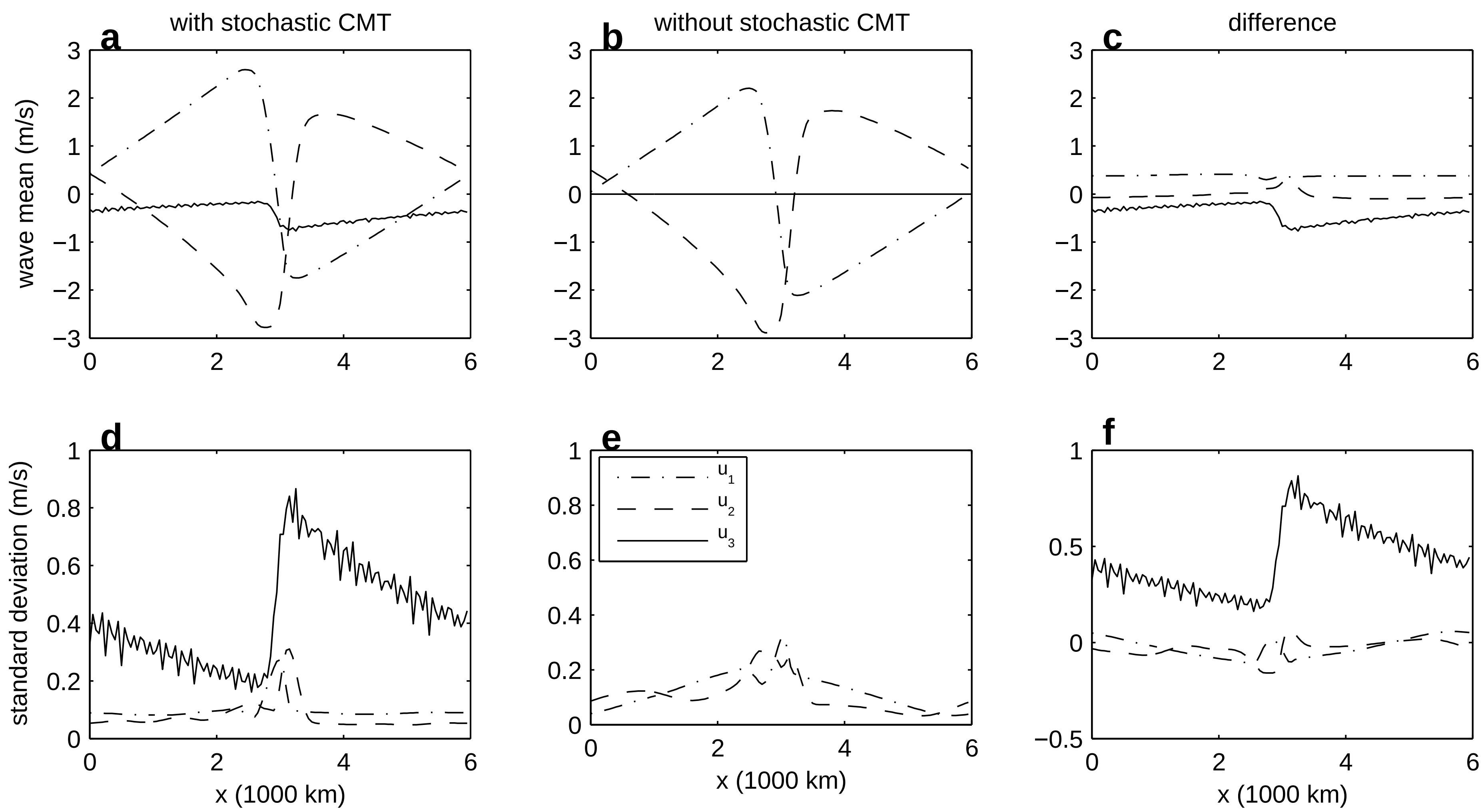
a
H_d (K/day)
b Convective regime
c
u_1 (m/s)
d
$-u_3$ (m/s)
t (days)
x (1000 km)
0
5
10
15
20
25
30
0
2
4
6
0
5
10
1
2
3
−2
0
2
0
1
2
3

Wave-mean structure

Average in a reference frame moving with the wave at -17.5 m/s



Comparison: with and without stochastic CMT



u_1: dash-dot

u_2: dash

u_3: solid

Stochastic CMT generates a nontrivial mean flow that can interact with the wave (see Majda and Stechmann (2008) J. Atmos. Sci., in press)

Summary

- A simple stochastic model for CMT was developed and tested
  - 3-state continuous-time Markov jump process, r_t, represents the convective regime at each large-scale spatio-temporal location (x, t)
    (dry regime, upright convection regime, and squall line regime)
  - Transition rates depend on large-scale resolved variables
    (cloud heating, wind shear, etc.)
  - CMT from unresolved scales acts on large-scale spatio-temporal location (x, t) in different ways depending on the convective regime at (x, t)

- Test case: column model
  - Intermittent bursts with physically reasonable values
  - Useful for calibrating model parameters

- Test case: convectively coupled wave
  - Stochastic CMT creates nontrivial mean flow that can interact w/ wave