Report on the Mathematics of Planet Earth Symposium at the 2013 CMOS/CGU/CWRA congress

TCU Place, Saskatoon, Saskatchewan, May 26-30, 2013

Organizers: Peter Bartello (McGill), Boualem Khouider (UVic), Norm McFarlane (CCCma), Adam Monahan (UVic)

The CMOS/CGU/CWRA 2013 congress gathered approximately 650 delegates and the Mathematics of Planet Earth (MOPE) symposium was one of its highlights. The MOPE symposium ran 4 successive sessions and had a plenary speaker who gave a 45-minute presentation on the challenging problem of uncertainties in climate models in a changing climate. The four sessions had in total 5 invited speakers, 10 contributed talks, and two posters.

The MOPE sessions were well attended and the talks touched upon many subjects and showcased the modern use of mathematics in the area of earth sciences, especially in atmosphere-ocean and climate change sciences. The main topics are, as planned,

Geostatistics and Climate Diagnostics

Stochastic Dynamics in Geosciences

Dynamical Systems and Predictability of Geophysical Flows

Balanced Dynamics and Interactions Across Scales

Numerical Modelling and Data Assimilation in Geophysics

The titles and abstracts of the talks are appended bellow. The invited talks were 30 minutes long including a 5 min period of questions and the contributed talks were 15 minutes long including a minutes period of questions.

Funding:

The expenses for the plenary speaker were taking care of by the Canadian Meteorological and Oceanic Society (CMOS). PIMS funds were used to pay the expenses for the 5 other invited speakers and partially (up to $1000) 8 student and post doc MOPE participants.

Abstracts:

Plenary: Understanding uncertainty in climate models: Robustness of the atmospheric

circulation response to climate change

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Although climate change is often characterised as “global warming”, the impact of climate change will vary greatly from region to region. Regional aspects of climate change are controlled by atmospheric circulation patterns, which moreover exhibit considerable chaotic variability. Model predictions of the atmospheric circulation response to climate change are in many cases highly uncertain, presumably because of systematic errors in the climate models (e.g. the location of the jet stream). The fact that these errors have stubbornly persisted despite increases in spatial resolution suggests that they are somehow linked to unresolved processes, whose effects need to be parameterised in the models. Thus, improving climate models requires a better understanding of multi-scale interactions. There are good reasons to believe that model bias, the divergence of model projections, and chaotic variability are somehow related. This talk will present some examples of these kinds of uncertainties and some potential ways forward.

Invited:

Stochastic Modelling of the Climate System: Past successes and future applications.

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Stochastic modelling of the climate dynamical system is justified mathematically by the fact that the system has a natural fast times scale from the atmosphere and a natural much slower timescale from the ocean. Simple models of various climatic phenomena have been rather successful in accounting for a range of phenomena such as predictability limits, spectra and non-Gaussian effects. We provide a critical review and also show how this perspective can lead to explanations for apparently disconnected climatic phenomena. In particular we shall propose a new stochastic mechanism for the Pacific Decadal Oscillation.

Extreme value theory and its role in understanding observed precipitation changes.

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Extreme value theory is used routinely in climatology to analyse precipitation data, for example, to provide estimates of the magnitude of rare events that might be expected to occur only once during the life of a new building. Such applications typically assume stationary climates. Extreme value theory is also being used, in a slightly more sophisticated way, to determine whether the intensity of extremes has changed over time, and to link those changes to proximal causes. Most studies proceed by attempting to make comparisons between observations and models on a more or less local, point-wise basis, often at the grid box scale, and then aggregating to larger scales. While several models of spatial extremes have been developed by statisticians, they remain difficult to apply, particularly over larger regions. Several other, more basic, challenges also currently limit our ability to better evaluate climate

models and understand observed changes in precipitation extremes. These include so-called "scaling issues" (rain gauges observe point values while models simulate grid-box area averages), limited spatial coverage from an observing network that was not designed primarily for climate purposes, and data inhomogeneity arising from changes over time in spatial coverage, instruments, siting and exposure, and so on. Despite the challenges, there is evidence of an intensification of precipitation extremes globally over the past 60 years, and emerging evidence that anthropogenic emissions of greenhouse gases play a role. This work shows that the overall historical intensification of precipitation extremes observed over global land areas appears to follow the Clausius-Clayeron relation, while climate models appear to have somewhat lower sensitivities to warming over global land areas.

Capturing intermittent and low-frequency variability in high-dimensional data through nonlinear Laplacian spectral analysis

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Nonlinear Laplacian spectral analysis (NLSA) is a method for spatiotemporal analysis of high-dimensional data, which represents spatial and temporal patterns through singular value decomposition of a family of maps acting on scalar functions on the nonlinear data manifold. Through the use of orthogonal basis functions (determined by means of graph Laplace-Beltrami eigenfunction algorithms) and time-lagged embedding, NLSA captures intermittency, rare events, and other nonlinear dynamical features which are not accessible through classical linear approaches such as singular spectrum analysis. We present applications of NLSA to detection of decadal and intermittent variability in the North Pacific sector of comprehensive climate models, and multiscale physical modes of the Madden-Julian Oscillation in infrared brightness temperature satellite data.

Mathematical problems associated with atmospheric data assimilation and weather prediction

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Several sectors of human activities rely on weather forecasts to plan in preparation of high impact weather events like snow storms, hurricanes or heat waves. Climate studies are needed to make decisions about the long-term development in agriculture, transport and land development. Observing and modeling the evolution of the atmosphere is needed to provide key reliable information for both weather prediction and climate scenarios. This paper gives an overview of the scientific research underlying the development and validation of numerical models of the atmosphere and the monitoring of the quality of the observations collected from several types of instruments. A particular emphasis will be given to data assimilation which establishes a bridge between numerical models and observations. The mathematical problems arising in atmospheric research are diverse as the problem is one of stochastic prediction for which errors in both the model and the observations need to be considered and estimated. Atmospheric predictability is concerned with the chaotic nature of the nonlinear equations that govern the atmosphere. Ensemble prediction is one area that has expanded significantly in the last decade. The interest stems from the necessity to evaluate more than just a forecast: it aims at giving an estimate of its accuracy as well. This brings up more questions than answers.

Spontaneous inertia-gravity-wave generation in balanced geophysical flows

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The large-scale dynamics of the mid-latitude atmosphere and ocean is characterised by a time-scale separation between slow balanced motion and fast inertia-gravity waves. As a result of this separation, the two types of motion interact only weakly, and the dynamics can be approximated using balanced models which filter out the fast waves completely. The separation is not complete, however: the evolution of well-balanced flows inevitably leads to the excitation of inertia-gravity waves through the process of spontaneous generation. In the limit of small Rossby number relevant to the mid-latitude atmosphere and ocean, this generation can be shown to be exponentially small in the Rossby number in several idealised models. I will discuss some of these models and show how spontaneous generation can be captured using the techniques of exponential asymptotics.

Contributed talks:

Bayesian Inference for the Stochastic Multicloud Model.

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The state-of-the-art cumulus parametrizations used in global climate models (GCM) are based on the quasi-equilibrium theory, which amounts to assuming a large separation of scales between convective processes and the grid-scale eddies. However, deep convection in the tropics is organized into a hierarchy of scales ranging from the convective cell of roughly one to ten kilometres to mesoscale cloud clusters of hundreds of kilometres to synoptic and planetary scale wave disturbances comprising the Madden-Julian oscillation (MJO) and a spectrum of convectively coupled waves (CCWs). In fact, the failure of GCMs to capture well the tropical variability associated with the MJO and CCWs is often associated with the poor performance of the underlying cumulus parameterizations. Recently, Khouider et al. (2010, Comm. Math. Sci., 8, 187-216) developed a Stochastic Multicloud (SMC) model to represent the missing variability in global climate models due to unresolved features of organized tropical convection. In the SMC model, convective elements are viewed as Markov processes with state transition probabilities that are conditioned on the large scale environmental variables, like the convective available potential energy (CAPE) and middle troposphere moisture content, and a set of cloud timescale control parameters. The model has successfully been tested, for a range of values of the control parameters, in the simple setting of one-column and slab equatorial circulation models (Khouider et al., 2010, Comm. Math. Sci., 8, 187-216). We propose an information theoretic approach to the SMC model based on a Bayesian approach to infer those timescale control parameters from simulated and in situ data. Prior information is combined with data likelihood to construct the parameters' posterior distribution, which are approximated using a Monte Carlo Markov Chain (MCMC) simulation technique. To minimize the associated computational overhead, we developed fully parallelized linear solvers such as a scalable Krylov based method. In this talk, we present validation results of our posterior simulator, and some preliminary inferential tests using the Giga LES dataset, a Large-Eddy Simulation of maritime deep tropical convection. This is a work in progress, part of M. De La Chevrotiere's PhD thesis.

Parametric Estimation of Diffusions for Surface Wind Velocity Data

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Parameter estimation for dynamical models from data is a common problem in many areas of science. The problem is further complicated if the data is assumed to follow stochastic dynamics as distinguishing between the accepted stochasticity and various errors is difficult. In this poster, We present a method that can used to fit stochastic differential equation models to time series data by minimizing the norm of an eigenvalue problem related to the infinitesimal generator for an assumed dynamical model. We first estimate the dynamics of simulated data from simple linear models to help determine ways to make the estimation scheme more robust and reduce biases. Finally, we apply this method to 45 years worth of wind speed data to generate global parameter fields and compare statistics of the original and simulated data to illustrate the success of this method.

Using Hidden Markov Models to Separate Distinct Regimes of Nocturnal Boundary Layer Variability

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It is observed that extremes of near-surface wind speeds are larger at night (relative to the mean wind speed) than during the day. Previous research has suggested that the long tail of the nocturnal wind speed probability distribution is associated with the presence of two distinct regimes in the nocturnal boundary layer (NBL): one in which the stratification and shear are strong and the other in which they are weak. Using data from tall towers at Cabauw, Netherlands and Los Alamos, USA, we will demonstrate how these regimes can be separated using Hidden Markov Model (HMM) analysis. In an HMM, the observed variability is assumed to be influenced by an unobserved variable undergoing transitions between discrete regimes and described by a Markov chain. An HMM analysis estimates both the character of variability within each regime and the trajectory of the "hidden sates". For the data under consideration, the classification of observed states by regime by the HMM analysis allows computations of marginal distributions of wind and temperature, thereby providing a clear picture of the regime dynamics and the physical controls on extreme winds in the NBL.

Extreme learning machines applied to nonlinear regression problems in the environmental sciences

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Environmental scientists have increasingly used nonlinear regression and classification methods developed originally in the field of machine learning/artificial intelligence, with artificial neural networks (NN) being the most commonly used. However, many of the environmental problems, e.g. downscaling general circulation model (GCM) output, involve large datasets, rendering the use of nonlinear machine learning methods such as NN computationally very expensive.

For a much faster alternative approach to NN, the extreme learning machine (ELM) method has been developed recently. The ELM shares the same architecture as the feedforward one-hidden-layer NN, but instead of solving for the model weights using nonlinear optimization, ELM simply assigns random weights for the hidden neurons and solve for the remaining weights using linear optimization with a standard least squares algorithm.

In this work, we tested the ELM on about a dozen environmental science problems. There are three daily precipitation downscaling problems, for (1) Vancouver, BC with 3 predictors from NCEP/NCAR Reanalysis, (2) the station Stave, BC with 25 predictors (from GFS model output and climate indices), and (3) Newton Rigg, UK with 106 predictors from GCM output. The other datasets tested different predictands, including annual maximum daily precipitation, daily air temperature, sulphur dioxide concentration, wind speed, river sediment concentration, forest fire, ammonium concentration in coastal water, equatorial Pacific sea surface temperature, etc.

We compared the ELM results with linear models such as multiple linear regression (MLR) and lasso, and nonlinear models such as feedforward NN, radial basis function NN, support vector regression with evolutionary strategy (SVR-ES), and random forest (RF).

We concluded that ELM is indeed extremely fast (sometimes the speed difference can be orders of magnitude) and in terms of skill scores it is very competitive against the best nonlinear methods over the dozen environmental datasets tested.

At What Time of Day Do Daily-Maximum Near Surface Winds Occur?

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Under clear sky conditions, near-surface winds are characterized by a pronounced diurnal cycle. In the bottom several tens of metres above the surface, mean wind speeds are greatest at mid-day; for some tens to hundreds of metres above this, mean wind speeds are generally greatest at night. Using long time series of 10-min average winds at 10m and 200m from a tall tower at Cabauw in the Netherlands, we will consider the timing of daily wind speed extremes relative to the diurnal cycle, asking: at what time of day do the largest daily wind speeds occur? If a time series has an autocorrelation scale similar or longer than the length of the time window used, such an analysis will be complicated by the tendency for extrema to occur at the beginning and end of the window. As large-scale driving variability (represented by the geostrophic winds) has autocorrelation timescales of a day or longer, this "edge effect" will influence estimates of the timing of daily extrema. We will demonstrate that with appropriate averaging, it is possible to separate this "edge effect" from the physically-driven nonstationarity in the timing of wind speed extremes.

Dynamics and practical predictability of extratropical wintertime low-frequency variability in a low-dimensional system

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Dynamics and practical predictability of extratropical low-frequency variability (LFV) in Northern Hemisphere winter are examined in the framework of a two-dimensional (2D) stochastic differential equation (SDE) on the phase space spanned by two leading empirical orthogonal function modes of low-pass-filtered 500-hPa geopotential height variations. The drift vector and diffusion tensor of the 2D SDE with multiplicative noise are theoretically connected with deterministic and stochastic error growth, respectively; both are statistically estimated from a reanalysis dataset. Projected onto the 2D phase space is the practical predictability of the LFV estimated by the 10-day forecast spread based on the 1-month ensemble prediction operationally conducted by the Japan Meteorological Agency (JMA). It is shown that the forecast spread of the LFV prediction by the JMA model for relatively shorter prediction period when the model bias does not hamper the forecast is primarily explained by the stochastic error growth associated with the diffusion tensor and the deterministic error growth due to the Jacobian of the drift vector plays a secondary role. A non- Gaussian PDF of the LFV is also related to the norm of the diffusion tensor. Hence, the stochastic processes mostly control the dynamics and predictability of the LFV in the 2D phase space.

Four-dimensional tensor equations for a classical fluid in an arbitrary gravitational field

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Tensor equations have the appealing property of keeping their form in all coordinate systems, a property known as covariance. If the equations of motion of a given system are available in tensor form, one may transform those equations from one coordinate system to another by systematically following a set of well-defined transformation rules. A four-dimensional tensor formalism suitable for the equations of motion of a classical fluid in the presence of a given external gravitational field is presented. The formalism allows for arbitrary time-dependent transformations of spatial coordinates. Some well-known conservation laws are derived in covariant form. The metric tensor and the associated Christofel symbols are calculated for coordinate systems useful in meteorology.

Comparison of hydrostatic and nonhydrostatic mesoscale processes.

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This work gives a study of the nonhydrostatic mesoscale processes. Comparison is made between the results of both hydrostatic and nonhydrostatic mesoscale processes. To do so, a stably stratified, two-dimensional, Boussinesq, nonrotating, inviscid fluid experiencing a thermal forcing is considered under both hydrostatic and nonhydrostatic assumptions. Techniques of Fourier and Laplace transform are considered to solve the governing equations. Numerical computing via MATLAB is done to provide us with the numerical results. It has been noticed that the hydrostatic assumption significantly alters the results. For the case of constant background, the propagation of the gravity wave, which doesn't exit under hydrostatic assumption, is present for nonhydrostatic case.

Modelling sediment resuspension and the effect of topography

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The study of lake bottom sediment pickup is an active area of study. Indeed, sediment transport within the bottom boundary layer has been shown to be highly complex. A thorough literature survey reveals that, to date, no model for sediment resuspension exists which captures all of the physical scales of interest. Following the approach of Blanchette et al. (2005), we model the macroscopic properties of particulate ejection into the water column assuming a continuum model for sediment concentration. Using a pseudospectral incompressible Navier-Stokes solver, we numerically model the shear induced pickup of lake bed particulate as a result of the motion of internal solitary waves in a continuously stratified domain. By varying the lake bottom topography, we are able to induce significant ejection into the water column. We observe that given a high enough shear, a separation bubble forms and bursts resulting in sediment resuspension. While previous work has studied shear induced pickup, we are the first to actively couple an internal solitary wave with an explicit model for sediment resuspension. This work has significant implications for the study of biogeochemical cycles and the deformation of small-scale bathymetric features of lakes and oceans.

Blanchette, F., M. Strauss, E. Meiburg, B. Kneller, and M. E. Glinksy (2005), High-resolution numerical simulations of resuspending gravity currents: Conditions for self-sustainment, J. Geophys. Res., 110, C12022, doi:10.1029/2005JC002927.

Adding Newtonian cooling and Rayleigh friction to remove singularities in the katabatic-flow models

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The classical Prandtl and layer-averaged models for katabatic winds are revisited in this study. It is shown that the radiative cooling effect in these models cannot be balanced by the vanishing adiabatic heating effect in the limit of zero slope angles or adiabatic lapse rates. Under such circumstances, a steady state of the katabatic flow is physically inadmissible. Mathematically, the unbalance in the model gives rise to various singularities in the equilibrium solutions. It is demonstrated that these

singularities can be avoided by including a Newtonian cooling and a Rayleigh friction in the models. The physical implications and justifications of including these thermodynamic damping mechanisms are further discussed.

Potential enstrophy in stratified turbulence

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In geophysical flows with strong rotation and stratification, the Ertel potential vorticity is approximately linear in the flow variables. As a result, the integrated square potential vorticity, or potential enstrophy, is an approximately quadratic invariant, a fact that has important implications for energy transfers between scales. However, for flows with Rossby numbers O(1) or larger - as in the atmospheric mesoscale and oceanic sub-mesoscale - the assumption of quadratic potential enstrophy becomes questionable. In this talk, direct numerical simulations of stratified turbulence without rotation will be presented. The potential enstrophy will be shown to be approximately quadratic only when the buoyancy Reynolds number is small, i.e. when the vertical scale of the turbulence is set by viscosity. This regime is common in laboratory experiments, but not in geophysical turbulence. For larger buoyancy Reynolds numbers, the quadratic, cubic, and quartic contributions to the potential enstrophy are all of the same order. These results raise doubts about the applicability of cascade theories based on quadratic potential enstrophy to stratified turbulence in the atmosphere and ocean.

Asymmetries in mode-2 breaking waves

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Using numerical simulations performed with a pseudospectral incompressible Navier-Stokes solver, we describe the asymmetries that arise in the recirculating core of mode-2 internal, solitary-like waves. The waves are generated in a manner consistent with many laboratory studies, namely via the collapse of a region of mixed fluid. Analysis of the simulations reveals that asymmetries across both the wave crest and the pycnocline centre develop in the spatial distribution of density, kinetic energy and a passive tracer transported by the mode-2 waves. The simulations are extended to three-dimensions, to allow for the formation of spanwise instabilities. We find that three-dimensionalization modifies the structure and energetics of the core, but that the majority of the results obtained from two dimensional simulations remain valid. Taken together our simulations demonstrate that the cores of solitary-like mode-2 waves are different then their counterparts for mode-1 waves and that their accurate characterization on both lab and field scales should account for the core asymmetry across the pycnocline centre.

A numerical method for the Taylor-Goldstein equation

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The Taylor-Goldstein (TG) equation determines the linear stability of internal waves in stratified parallel shear flows. This simplified model plays an important role in geophysical fluid mechanics where it is used to identify unstable waves and estimate their initial growth rates. One property of the TG equation is that, for most shear flows and stratifications of practical interest, analytical solutions cannot be found. Numerical solutions of the TG equation are necessary, but made difficult by critical layers. I will describe a novel shooting method for finding unstable solutions of the TG equation. The treatment of critical layers will be described, and the method will be demonstrated using several simple examples.

A Stochastic Parameterization of Cloud Droplet Collision and Coalescence

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Global climate models need accurate representations of cloud droplet radii to adequately model (i) the amount of solar and terrestrial radiation reflected by cloud droplets and (ii) the time until the onset of precipitation. The spectrum of cloud droplet radii can span five orders of magnitude: from less than 1 micron to several millimetres. After initial formation and growth by condensation, droplets increase in size by collision and coalescence before precipitating under the force of gravity. The evolution of the droplet spectrum by the collision and coalescence process can be modelled using the kinetic collection equation (KCE) or evolved by a stochastic process. Since infinitely many sizes of radii cannot be modelled, the equations governing droplet evolution must be discretized, which introduces errors. We highlight the current techniques used to discretize the KCE. We apply mathematical rigour to these techniques and combine them with elements of the stochastic evolution process to gain the advantage of increased accuracy and reduced computational costs.

Posters:

Randomness Characterization in Computing and Stochastic Simulations

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Mathematical randomness has long been studied and in computing, various simulation approaches have been investigated and implemented in all kinds of stochastic computations. Following a brief overview of pseudo, chaotic and quasi-random number generation, some equi-distributed and low discrepancy sequences will be discussed in view of their well-known applications in Monte Carlo simulations of volume integrals. In particular, the numerical representation of irrationals such as π and e will be discussed along with some of their spectral and other characteristics used in stochastic simulations. Some sample planar, spherical and spatial computations will illustrate the potential for applications in practice with indications of the computational efforts required.

Stochastic averaging of heavy tailed processes.

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Gaussian noise has long been the favoured choice for modelling stochastic effects in almost every every of science, including climate dynamics due to analytic tractability and the attractor property of the Gaussian distribution. There are many reasons however to consider that Gaussian noise is not appropriate for modelling several climate phenomena as many processes are subject to extreme events or may have otherwise heavy tailed forcing. This is particularly evident on paleoclimatic time scales, such as the Greenland ice cores. Stochastic averaging offers a way to derive relatively simple phenomenological models for complicated climate processes by reducing dimensionality. I will present some work on stochastic averaging for heavy-tailed processes that may be useful to climate researchers wishing to derive simplier dynamical models for systems displaying heavy tailed forcing.